The robustness of VaR models during the Ukrainian crisis and the impact of ESG scores on the results of US ETF backtests.

Auteur : Litaudon, Orlane
Promoteur(s) : Hambuckers, Julien
Faculté : HEC-Ecole de gestion de l'Université de Liège
Diplôme : Master en sciences de gestion, à finalité spécialisée en Banking and Asset Management
Année académique : 2022-2023
URI/URL : http://hdl.handle.net/2268.2/17674

The robustness of VaR models during the Ukrainian crisis and the impact of ESG scores on the results of US ETF backtests.

How did the different VaR models of US ETFs perform during the extreme market movements caused by the Ukraine invasion by Russia?

Is there a downward link between the ESG score of US ETFs on the robustness of VaR models during the extreme market movement caused by the Ukraine invasion by Russia?

Promoter: Julien HAMBUCKERS

Reader: Sophie MOINAS

Dissertation by ORLANE LITAUDON

Academic year 2022/2023

Words: 24673

Acknowledgement

I am very grateful for the opportunity that HEC Liège and Toulouse School of Management gave me of studying respectively Banking and Asset Management and Financial Markets and Risk Evaluation. This master thesis allowed me to deepen a subject of market risk, which is a field I am particularly interested in.

I would like to acknowledge and express gratitude to my thesis supervisor, Professor Julien Hambuckers, and my reviewer, Professor Sophie Moinas, for their guidance and suggestions throughout my work. Their expertise and advice have undoubtedly contributed to enhancing my financial knowledge, particularly in the areas of risk management and the transition to Basel III standards.

Additionally, it is important to recognize the support and motivation I received from my friends during this project. Special thanks to Lucas Orlandi, with whom I collaborated extensively and who provided valuable insights for my Master thesis. Their contributions and teamwork have been instrumental during my master's program.

Regarding the content of the paper, the research focuses on comparing the robustness of different VaR models (historical, Monte Carlo, and parametric models) during the Ukrainian crisis. The study specifically examines US ETFs during the period from February 2022 to January 2023. The aim is to provide practitioners with additional insights into the performance of these models in crisis scenarios, considering the Basel II and III regulations.

Furthermore, this research seeks to explore the relationship between a fund's ESG score and the robustness of its downside risk model. While previous studies have identified a downward effect on VaR levels or other risk indicators for assets with high ESG scores, no research has established a link with the robustness of the risk models.

In terms of the technical aspects, RStudio was used for identifying the parameters of the VaR model, while VBA (Visual Basic for Applications) was employed for backtesting the results.

Executive summary

Under the supervision of the Commission de Surveillance du Secteur financier (CSSF), the financial regulator in Luxembourg, the management of tail risk for UCITS funds relies on the daily computation of Value at Risk (VaR), which is then backtested monthly to ensure its reliability. Consistency in VaR models is determined by the level of exceptions aligning with the confidence interval and their independence from each other. The robustness of VaR models has faced challenges during the Ukrainian crisis and the subsequent announcements by central banks regarding interest rate hikes. During extreme market events, VaR may not fully capture the risk, leading to the rejection of VaR models in the backtesting process.

This paper argues that parametric VaR methodologies applied to US ETFs exhibit greater robustness compared to historical and Monte Carlo methods during the shocks caused by the Ukrainian war. By comparing backtesting results, it has been demonstrated that the exceptions were independent from each other according to the Haas test, and the proportion of failure aligned more coherently with the estimated confidence level. This is attributed to the ability of parametric models to select the skewed student-t distribution, which effectively captures extreme downward movements in asset returns.

Previous research has examined the relationship between ESG scores and VaR levels, particularly during financial crises, and indicated a downward effect. This study aimed to assess whether there is a downward relationship between ESG scores and backtesting results, as a reduction in VaR level and lower sensitivity to extreme market shifts should correspond to a lower number of exceptions and an accepted VaR model. Surprisingly, the variables related to backtesting results, such as the number of exceptions and the results of the Kupiec proportion of failure test, were positively correlated with ESG scores and their sub-pillars (Economic, Social, and Governance). Furthermore, the fund with the lowest ESG score exhibited the fewest exceptions, and its VaR models were generally accepted by the different backtesting procedures, unlike other ETFs with higher scores. Therefore, for US ETFs during the Ukrainian crisis, no downward relationship between ESG scores and backtesting results was found.

Contents

Glossary

Akaike information criterion: AIC
Asymmetric Power ARCH model: APARCH
Autocovariance function: AcovF
Auto-Regressive: AR
Autoregressive Conditional Heteroskedastic Models: ARCH
Autocorrelation function: ACF
Banks' distance to default: DTB
Committee of European securities: CESR
European Securities and markets authority: ESMA
Environmental, social and governance: ESG
Exchange traded fund: ETF
Expected Shortfall: ES
Exponentially Weighted Moving Average: EWMA
Federal Reserve Board: FED
Generalized Autoregressive Conditional Heteroskedastic Models: GARCH
Lower partial moment: LPM
Moving average: MA
Partial autocorrelation function: PACF
Proportion of failure: POF
Profit & Loss: P&L
Proportion of failure test: POF test
Solvency test: SST
Student t distribution: std
Skewed student t distribution: sstd
Time until failure: TUFF
VaR: Value at risk
VaR-99%: The Value at risk level, with a 99% confidence interval
United States: US
Volatility Index: VIX

List of figures and tables

Introduction

To be compliant with the European regulation, UCITS funds have to manage their risk levels and in particular their downside risks. Value at risk (VaR) reports have to be created on a daily basis to ensure that the risk level is managed and does not exceed the regulatory level. In Luxembourg, the CSSF (*Commission de surveillance du secteur financier*), the Luxemburgish financial regulator, requires to have a VaR report on a daily basis with a 99% confidence interval for all UCITS funds. Indeed, alternative investment are not forced to follow this rule. In this way, the authorities can control the level of risk the fund is exposed and establish their risk profiles.

The VaR is a measure of potential losses with a given confidence interval and a given time horizon. This is a very popular market risk indicator because it summarizes the risk level of a fund in one single value. Moreover, it offers a probabilistic answer to the question: "What is the most I could lose, in percentage or value, on my portfolio in a single day with a given confidence interval that I will not lose more than that amount?" It can be seen as a tail risk on a normal distribution curve, if it is assumed that returns are normally distributed. In this context, was born the VaR . Mathematically, it can be seen as following:

$$Probability(Loss_t \geq VaR_{1-\alpha}) = \alpha$$

With 1-α the confidence level of the VaR model at time t. VaR corresponds to a percentile of the distribution of portfolio Profit and Loss (P&L). It is expressed as a potential loss of the current value of the portfolio. In case a 99% confidence level, the VaR is the amount that one should not lose more than except in 1% of the cases. VaR is the most widely used statistic measure for portfolio managers because that measure the potential risk of financial losses and that is a required measure under Basel II. It provides the likelihood that a loss greater than a certain amount would be realized.

To be compliant with the European regulation, the VaR model used for the funds have to be backtested on a monthly basis to assess its validity. Probability tests are used such as the Kupiec, Christoffersen or the Haas one. The results of these tests have to be chi-squared distributed to accept the corresponding property of the model: the unconditional coverage and the independence of exceptions. An exception occurs when the returns of one trading day, for the daily VaR, is worse than the VaR level computed. The VaR is an attractive measure because it is easy to ensure its model validity.

Both regulatory compliant VaR valuation methodologies, the historical and the Monte Carlo have limitations. The historical VaR is indeed easier to implement than other models and requires no assumptions on the return distribution. The main limitations are that it uses the past performance as a predictor of future, so the results are highly dependent on the historical sample selected and it assumes that distribution of changes is stationary through time. The Monte Carlo VaR is powerful and flexible as it allows to address the fat tailed distribution of returns. The problem is that it needs an accurate random generator to include the stochastic part. It also takes longer to implement.

The financial markets were very volatile since February 2022 because of the Russian invasion of Ukraine. In fact, some particular market events disrupted the market. On the February 24th, the market crashed in particular in the euro zone where investors considered that the risk became higher due to the political conflict. Following this events, the European Commission has voted few rounds of sanctions against Russia and economic sanctions have been adopted. It resulted from these decisions a global inflation which impacted a lot the commodities and energy prices. Indeed Ukraine and Russia are very large exporters of oil, gas, fertilizer and wheat. The United States (US) were impacted by this inflation as well, forcing the Federal Reserve Board (FED) to increase their interest rates. Actually, most central banks around the world increased their interest rates in order to fight the inflation. Chronologically, on March 15-16, the FED approved a 0.25 percentage point rate hike, the first increase since December 2018, and members also lowered expectations for economic growth this year and had sharply raised their inflation outlook. On May 3-4, the FED raised its benchmark interest rate by 0.5%, in line with market expectations. The central bank also presented a program under which it will eventually reduce its bond holdings by $95 billion per month. On June 14-15, the Federal Reserve raised its benchmark interest rates 0.75% in its most aggressive increase since 1994. Officials also significantly reduced their economic growth outlook for 2022, now anticipating GDP growth of only 1.7%, down from 2.8% in March. The US central bank announced on July 27 that it was raising interest rates by 0.75%. This was the fourth increase in five months. The July rate increase brings the target range for the federal funds rate to 2.25%-2.5%.

The global inflation came mainly from the energy prices which increased a lot during the studied period. Indeed, the gas and oil prices were very volatile and this situation disturbed the financial markets. The bond market became riskier than the equity ones because of interest rates increase. On the *Figure 1*, the magnitude of oil and gas returns appears to be greater from March 2022 onwards. The gas returns correspond to the NYMEX Henry Hub Natural Gas Electronic Energy Future returns and the oil returns to the NYMEX Light Sweet Crude Oil Electronic Energy Future ones.

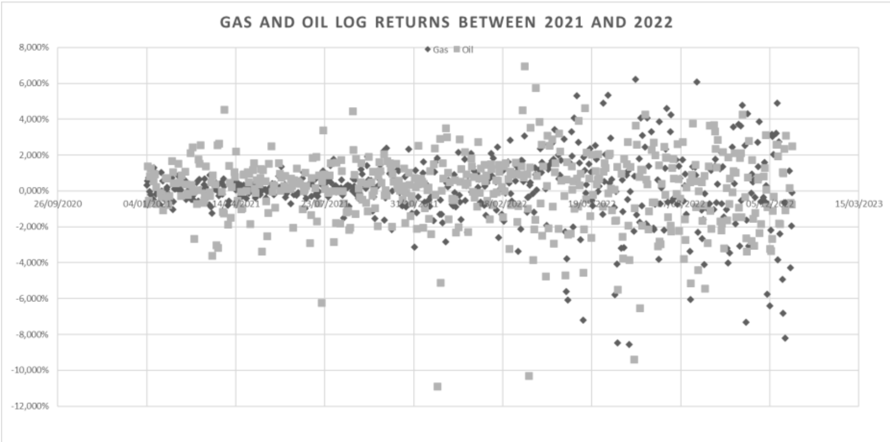


Figure 1: Gas and oil returns between 2021 and 2022

To confirm that log returns were more volatile in 2022 than in the previous year, one can observe descriptive studies conducted on these two commodities as well as on the main US stock market indexes:

Table 1: Descriptive study of commodities and equity indexes between 2021 and 2022

	Evolution between 2021 and 2022									
	Gas	Oil	VIX	Dow Jones Industrial Average(Dia)	Russell 1000 Growth ETF(IWF)	Nasdaq-100 Index(QQQ.O)	CRSP US Mid Cap(VO)	S&P 500(VooiV.P)	Russell 2000 Index(VTWO.O)	MSCI US Prime Market Growth Index(VUG)
Mean	-0,014%	-0,083%	0,190%	-0,105%	-0,235%	-0,254%	-0,172%	-0,193%	-0,147%	-0,257%
Median	0,273%	0,231%	0,012%	-0,126%	-0,370%	-0,364%	-0,193%	-0,330%	-0,091%	-0,341%
Standard deviation	1,799%	0,743%	-0,978%	0,468%	0,860%	0,880%	0,686%	0,706%	0,384%	0,961%
Skewness	-30,452%	98,957%	-18,395%	31,169%	33,109%	35,482%	37,448%	40,110%	4,698%	41,418%
Kurtosis	-96,544%	-793,494%	-120,715%	-31,652%	-41,501%	-80,360%	-38,954%	-39,800%	-2,902%	-41,630%

Overall, the mean of these equities and commodities indexes declined between both periods and the standard deviation increased (except for the VIX). Therefore, the returns were broadly more volatile in 2022, knowing that the Ukrainian war started in February of this year. The skewness has become more negative for gas and the VIX, which is not the case for the stock indexes and oil. Graphically, one can observe that the magnitude of log(returns) for the main US equity indexes increased as well.

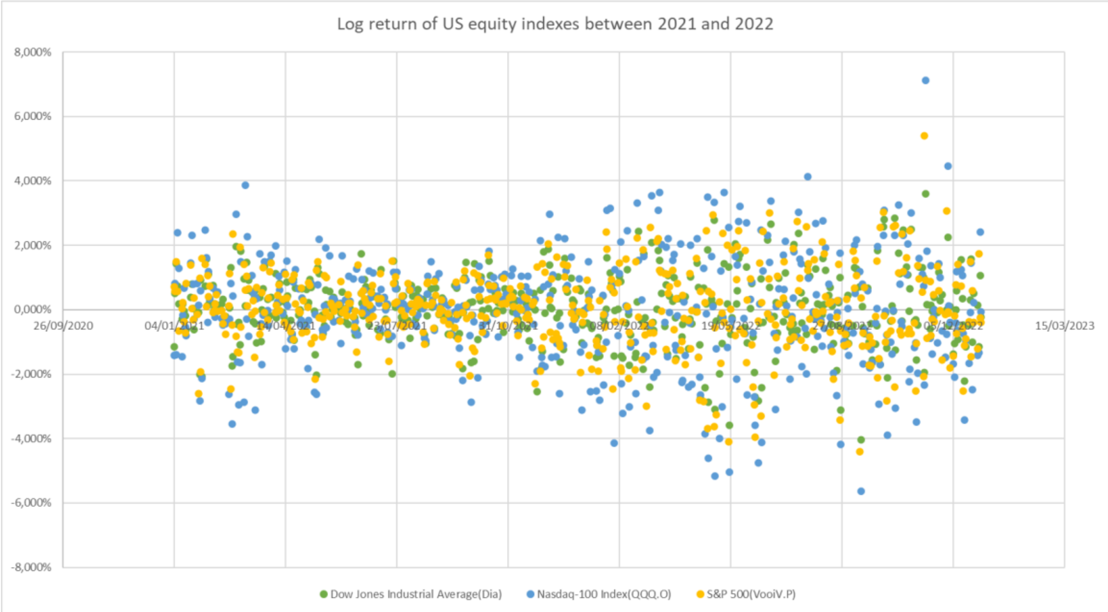


Figure 2: Returns of the Dow jones, Nasdaq and S&P 500 between 2021 and 2022

This imply that the financial returns became less predictable and therefore, the VaR level difficult to estimate. This situation could lead to exceptions to the VaR models and if there are too many, the models could be rejected with the backtests.

As previously mentioned, since February 2022 the market conditions involved because of the Ukrainian war. The commodities prices became more volatile and this lead to a global inflation forcing the different central banks around the world to increase their interest rate. In this context, the market risk management departments and firms have seen their VaR model rejected, which implied that the risk level of regulated funds was underestimated, which may technically lead to an intervention by financial regulators. Nevertheless, VaR is a risk indicator which structurally fail to capture extreme market movements. There are three methodologies to compute it: historical, Monte Carlo and parametric. The first is dependent on the historical sample selected and the legislator requires that the sample be of 250 observations. The previous researches tend to show that the sample must be larger to correctly capture market risk. Indeed, the markets were subject to very high volatility during the previous market disturbances caused by the Covid-19 and it would have influenced the VaR level downwards to include them in the sample. This would have lead to less exceptions, because of the very negative returns inclusion in the sample. The Monte Carlo method includes a stochastic part, which limits the influence of the sample size, without removing it. The parametric VaR, it allows to choose the distribution model and to forecast the future volatility according to the previous data. Overall, the VaR models are more or less dependent on the previous returns, even if this tend to be less true for the Monte Carlo simulation which include a random part.

In this paper, the robustness of the VaR models will be examined and compared. Monte Carlo, historical and parametric methods will be applied to American Exchange Traded Funds (ETFs) to compare the effectiveness of the models in capturing downside risk. This study is limited to examining the impact of the crisis on the robustness of US equity VaR models. ETFs are financial instruments designed to closely replicate changes in an index. It is an asset that instantly offers a "basket" of equity positions without the investor having to rebalance the weight of his portfolio to replicate a payoff. They can be created directly with stocks or with derivatives that replicate the index concerned. Choosing ETFs as the object of study allows to eliminate the step of creating and diversifying a portfolio and to directly study the VaR level of US indexes. The sample size requested by the CSSF for the establishment of the VaR model and the backtest will be respected. The aim is that the results of this study can be used by practitioners. That is the reason why the confidence interval backtest will be 99%.

Nevertheless, the Committee of European securities (CESR), the European legislation, does not allow the market risk analysts to select themselves the model to forecast the volatility for UCITS which hold financial derivatives with non-linear risk features, such as options. In other words, this means that the parametric VaR models are not recognized for a significant part of the regulated funds. Therefore, a regulated fund could choose to have a parametric VaR model if the managers estimate that it is more appropriate, but they would not be allowed to buy any option. In fact, UCITS choose rather the Historical simulation or the Monte Carlo one to compute their VaR level. Nevertheless, to explore the diversity of the models, the parametric method will also be tested. In addition, only the US stock are in the scope of the study, so they are not concerned by the non-linearity of the payoffs.

The purpose of this study is to determine which VaR model was the least likely to be rejected by the backtest during the war between Russia and Ukraine in order to guide practitioners on the optimal choice of model in case of a crisis. This is the primary motivation for this study, but the 99% confidence interval VaR is the primary risk measure under Basel II legislation. It is not a coherent measure of risk. The Basel III regulation, which will soon be implemented, suggests using a 10-day Expected Shortfall (ES) at a level of 97.5%. This is going to be the main risk measure for setting trading books capital. It describes the potential loss in 2.5% of the case, therefore what happen in the tail. However, the ES cannot be backtested in the same manner as the VaR. For this reason, a multinominal test will be performed, to estimate the ES backtest results. This will allow to comment on the results of the VaR models obtained and get ahead of the legislation that will evolve.

The research problem can be formulated as follow: How did the different VaR models of US ETFs perform during the Ukrainian crisis? Specifically, which model, between the historical model, the Monte Carlo model or the parametric model, had the least number of exceptions for a confidence level of 99% and were these independent? This question will also be processed with a multinomial backtest to see if the 97.5% parametric Monte Carlo or historical ES has been rejected for the American ETFs.

In addition, the Environmental, Social and Governance (ESG) scores will be linked to the results of the VaR model backtests to examine the potential correlation between the ESG score and the effectiveness of the VaR model during the war between Russia and Ukraine. In a context of volatile energy prices, it is legitimate to ask what impact the ESG score has on asset tail risk. The ESG scores calculated by Thompson Reuters take into account for the environmental pillar: the resources used (water, energy, sustainable packaging and environmental supply chain), the emissions, the waste of energy and the green revenue from the research and development and the capital expenditure. Overall, the companies which does not efficiently manage the energy are penalized in the ESG score and financially because they are more exposed to commodities indexes. They can protect their positions exposed with derivatives, but that is often costly which leads to financial performance reduction. This could affect their returns and their VaR level. Thompson Reuters does not provide access to ESG data for selected ETFs over a long enough period to be able to run a regression, but it is possible to compare backtesting results to the ESG scores of funds before and during the Ukrainian war to see if it is possible that the ESG score has an impact on the robustness of VaR models.

The second research problem consist in the investigating the nature of the link between ESG scores and the VaR models robustness of US ETFs. Previous researches established a downside risk reduction for best in class assets, because of the legislative and financial risk decrease. This could have an impact on the robustness of the models in the event of a crisis because the number of exceptions would have to be lower. Is there a downward link between the ESG score of US ETFs on the robustness of VaR models during the extreme market movement caused by the Ukraine invasion by Russia? Indeed, while many papers have looked at the impact of ESG scores on downside risk, no one has asked what impact this has on the relevance of the models used to calculate it. Indeed, the VaR levels tend to be higher (less negative) for “best in class” assets even during financial crisis.

These questions directly address the problem faced by market risk companies, because they are challenged by the robustness of their risk indicators during crises and when markets are particularly volatile. It is difficult to build a model which fully capture the risk a portfolio faces. In one hand, it should not overestimate the risk, so the VaR 99% should have some exceptions (1%) when the markets are stable, but it must be reliable enough even during particular market events. It is worthwhile to determine the advantages and disadvantages of the models to meet these constraints. Moreover, exploring the link between the ESG score and the robustness of the downside risk indicator can guide investment choices in the event of a crisis in order to maintain a regulatory VaR level without being out of compliance.

It is a matter of determining the VaR levels at different confidence intervals, and then checking that the number of exceptions is consistent with it. For the historical value, one has to look at the previous observations to take the 1% of the worst return for the VaR-99% case. The Monte Carlo methodology uses the hypothesis that the returns can be forecasted using the average and the standard deviation of the past ones and a stochastic part. This last one is obtained with an accurate random number generator. Then, some simulations are runed with the random number generated and the results are classified in an increasing order to take the value corresponding to the desired quantile. The parametric VaR is quite similar but the volatility is forecasted with an accurate model.

There three kinds of VaR in a portfolio: the marginal, the incremental and the absolute one. The last is mainly used for static reports, but it could be useful to know the impact of each asset one the total VaR to see which can be considered as more or less risky. Marginal VaR (MVaR) measures how much risk a position adds to a portfolio. It describes, for example, how much will the VaR of an entire portfolio change if we change the weights of the assets which composed it. The Incremental VaR (IVaR) is defined as a risk measurement with an additive property which allow the decomposition of the total portfolio VaR. It assesses the sensitivity of the portfolio to each position’s VaR. In other words, this risk measure answers the question: which positions/assets level contributes the most to the overall risk? Its additive property comes from the fact that the total portfolio VaR is equal to the sum of each asset IVaR. The change in the overall VaR could be approximated by the IVaR for small change (0% to 5% approximately) in assets proportion. The fact that the total VaR number is reduced or increased depends on the whether the position’s VaR contribution is positive or negative. A negative one means that an increase in the holding of the corresponding position (or set of position) will actually decrease the overall VaR. Only the absolute VaR is in the scope of the studies, because the assets are ETFs and will be studied separately and not inside a portfolio.

To backtest the VaR-99%, the Kupiec test named proportion of failure test (POF test) will be used to check if the number of exception is coherent with the confidence level. To be compliant, the backtest should correspond to one working year, so approximately 250 observations. The traffic light test will also be performed before the Kupiec one, because it allows by only counting the number of overshoots to have an insight about the validity of the model. This test is going to be performed at a multinominal model as well, to approximate the backtest results of the ES-97.5%. This means that the overshoots will be counted for few VaR confidence level. To verify if the exceptions hold their independence property, the Christoffersen and Haas test will be computed. Indeed, they must have a coherent elapsed time between each other to accept the model, even if their number are coherent with the confidence level.

The ESG will be linked to the backtest results by comparing the evolution of the number of exception and the tests results to the ESG score of the funds during the Ukrainian crisis. The corelation between these variables will be studied as well. As explained before, the number of ESG data available on Thompson Reuters is not sufficient to perform a regression and study precisely the impact of ESG score on the robustness of the VaR model. Nevertheless, it is feasible to study whether funds with high ESG score are likely to have their VaR models rejected compare to the ones whit low scores.

Literature review

The Value at risk is one of the most widely used measure to assess the financial assets' tail risk. There are several ways to compute this value. This measure has been challenged during crises when the markets are very volatile. Indeed, it could fail to capture the entire asset's risk and this leads to an exception to the model. Firstly, this study will examine the ability to different VaR models to capture the American ETFs risk during extreme market movements caused by the Ukrainian crisis in 2022. The robustness of three models, historical, parametric and Monte Carlo, is studied, therefore they are backtested with different methodology. In a context of volatile energy prices, it is legitimate to ask if there is a link between the ESG scores and asset risk and its measures. Before analysing this potential link, between the ESG score and the robustness of the Value at Risk models, it is necessary to define how these scores are established.

The origin of ESG scores

ESG issues have become increasingly important in financial markets. Several factors contribute to this trend. On September 13th 1970, Milton Friedman published an essay on corporate purpose. The notion of Corporate Social Responsibility (CSR) was created. According to Friedman[1], the firm has a bilateral relationship with the shareholders: they provide financial resources so that the company conducts properly its activity and the firm provides dividends to them. The other stakeholders have a unilateral relationship with the company. The suppliers provide materials and the necessary equipment for production. The employees provide workforce and skills and the customers receive the finished products or the services. Nevertheless, Friedman does not include the ability of these three stakeholders to give feedback or to have a negotiating power. This suggests that the only goal of a business is to maximize the profits. Another view broader emerged including the new elements: Public Authorities, citizens and non-governmental-organization (NGO). They are considered as stakeholders because they can affect the operations of the firm and be affected as well. The firm remains at the center of this complex network but has bilateral relations with every stakeholder including the suppliers, customers and employees. This means that the firm can affect and be affected by these stakeholders. This new theory is called the stakeholders network approach. The firm can also affect indirect stakeholders through indirect relationship with them. For example, its suppliers can be firms which have their own stakeholders. This broadening allows a more global view on the situation of the company concerned.

The bargaining power of stakeholders is based on three attributes: power, legitimacy and urgency. The power is the potential influence of the stakeholder on the company's activities and the resources. The legitimacy is a subjective concept which depends on the managers' point of view. That is how they consider a stakeholder legitimacy in the conduct of the business' activities. The urgency consists in whether a stakeholder ask something urgently from the firm. The stakeholders which combines these three attributes are considered as definitive stakeholders. The managers have to be careful to their needs because they can influence the firm. The second category is the expectant stakeholders, they are not as prominent as the previous ones, but they have two attributes and the situations can evolve, and as a manager it is important to pay attention to this category and monitor them. The third category are latent stakeholders. They are limitations to this framework. The first one is the fixed situation of this model. It is specific to an environment and a context that will inevitably change, meaning that the analysis have to be conducted several times in order not to miss an important evolution. The second limitation is the subjectivity of the analysis and especially when it

[1] Friedman M., A Friedman Doctrine: The Social Responsibility of Business is to Increase Its Profits, New York Times, 1970

is conducted by a single person. Therefore, the consideration of stakeholders has its limits when it comes to ethical management, but they became an official concern for the company.

Business Ethic Management is a formal way to manage this through specific programs to achieve more sustainable goals. These can be classified in different categories according to their origin and approach. The origin can be autonomous, which means that the rules are defined internally in the company. Heteronomous means that the equipment, the policies are defined by external actors. Concerning the approach, for most company it will be procedural, meaning that there will be a lot of processes and new equipment to improve the ethicality of the organizational behaviour. The substantive approach differs by the fact that the business ethics management is the core of the company. This last one has a social mission, a strategy based on positive impact.

In practice the autonomous and procedural mechanisms are translated through value statements, code of ethics, stakeholders consultation and mapping, ethical audit and ethic committee for example. The company can choose to do it or not. The heteronomous mechanisms can be illustrated by the sustainable development goals adopted by all United Nations Member States in 2015. The Global Compact is another example of this mechanism. It is a United Nations initiative launched in 2000 to motivate companies around the world to adopt a socially responsible attitude by committing to integrate and promote several principles relating to human rights, international labour standards, the environment and the fight against corruption. Other initiatives that aim to guide companies to be more socially responsible have emerged such as the ISO 26000 or the Global reporting initiatives. But their main limitations were that they were not controlled by an external auditors, so the SA8000 and AA1000 AS were created to certify that companies respected both ISO 26000 norms and the Global reporting initiatives respectively. It is also in this context of heteronomous and procedural mechanisms that ESG scores were created. The first one was created by Innovest, a research firm[2].

The ESG scores are based on three pillars : environment, social and governance. It reveals the organization's environmental footprint through the quantity of energy and raw materials used and the waste of resources. The social pillar take into account the employees wellbeing at work, for example. The companies are often asked to disclose information such as gender parity, working hours per employee and wages. The donations to social organizations can be taken into account as well. The governance concerns mainly the firms' policy toward bribery and corruption, the management, leadership and the corporate social responsibility strategy.

The ESG concept has a much shorter history than the social responsible investment and the corporate social responsibility ones. It appeared for the first time in a United nations Global Compact in 2004. The main goal was "*to develop guidelines and recommendations on how to better integrate environmental, social and corporate governance issues in asset management, securities brokerage services and associated research functions*"[3]. In 2005, large bank, asset management companies and financial stakeholders began to include ESG standards into their policy. The investors increased their demands for ESG data and ESG complying investments. Despite this, limitations are highlighted by researchers regarding the validity of this kind of data. For example, Chatterji et al (2016) [4] acknowledge that corporate evaluators play an important role in assessing areas ranging from sustainability to corporate governance, but they note a lack of convergence in data related to the social pillar. Even when adjusting for differences in the definition of corporate social responsibility, the social assessments appear to differ in their results. Delmas et al (2013)[5] used a unique dataset

[2] Eccles, R. G., Lee, L. E., & Stroehle, J. C. (2020). The social origins of ESG: An analysis of Innovest and KLD. *Organization & Environment*, *33*(4), 575-596.

[3] Recommendations by the financial industry to better integrate environmental, social and governance issues in analysis, asset management and securities brokerage, United Nation Global Compact 2004: p. 5

[4] Chatterji, A. K., Durand, R., Levine, D. I., & Touboul, S. (2016). Do ratings of firms converge? Implications for managers, investors and strategy researchers. *Strategic Management Journal*, 37(8), 1597-1614.

[5] Delmas, M. A., Etzion, D., & Nairn-Birch, N. (2013). Triangulating environmental performance: What do corporate social responsibility ratings really capture?. *Academy of Management Perspectives*, *27*(3), 255-267.

combining environmental assessments of three large suppliers, and identified the main components of firms' environmental performance. They found that firms' environmental outcomes that their processes generate explain 80% of the variance in financial performance. In other words, financial performance is not perfectly correlated with environmental results, but rather with the processes in place.

As mentioned previously, the first company which created an ESG score was Innovest. This company and KLD (another research firm) were acquired by RiskMetrics in 2009, a risk management company created by J.P Morgan. Innovest already had work on environmental and social assessments just as KLD. They were the largest ESG research providers at that time. Both company had a different approach in data processing, collection and aggregation. Innovest considered that different industries have different exposure levels to ESG issues, when KLD considered the same approach of data collection regardless of the industry involved. The first one had an industry ranking approach rather than an absolute one. When RiskMetrics was acquired by MSCI in 2010, the KDL approach was chosen, creating a single data set with a single set of data collection processes. However, MSCI was facing a growing market demand for ESG data with a more rational financial materiality, which also required greater data coverage (number and geographic scope of companies assessed) for reliable benchmarking. So MSCI decided that the Innovest methodology performed better because it was the product that best met its clients' needs. Furthermore, its industry-specific information gathering approach was likely easier to adapt quickly, thus meeting the growing market demand.

KLD STATS (Statistical tool for analysing trends in social and environmental performance) published a set of data of the environmental, social, and governance performance of companies. Each annual KLD STATS spreadsheet contained: the company name, ticker and identifying information, the strength and concerns ratings for the corresponding indicators. These indicators were classify by qualitative issue area. RiskMetrics defined them as followed:

COMMUNITY

STRENGTHS
Charitable Giving: The company has consistently given over 1.5% of trailing three-year net earnings before taxes to charity, or has otherwise been notably generous in its giving.
Innovative Giving : The company has a notably innovative giving program that supports non-profit organizations, particularly those promoting self-sufficiency among the economically disadvantaged. Companies that permit non-traditional federated charitable giving drives in the workplace are often noted in this section as well.
Non-US Charitable Giving : The company has made a substantial effort to make charitable contributions abroad, as well as in the U.S. To qualify, a company must make at least 20% of its giving, or have taken notably innovative initiatives in its giving program, outside the U.S.
Support for Housing. The company is a prominent participant in public/private partnerships that support housing initiatives for the economically disadvantaged, e.g., the National Equity Fund or the Enterprise Foundation.
Support for Education : The company has either been notably innovative in its support for primary or secondary school education, particularly for those programs that benefit the economically disadvantaged, or the company has prominently supported job-training programs for youth.
(…)

CONCERNS
Investment Controversies : The company is a financial institution whose lending or investment practices have led to controversies, particularly ones related to the Community Reinvestment Act.
Negative Economic Impact: The company's actions have resulted in major controversies concerning its economic impact on the community. These controversies can include issues related to environmental contamination, water rights disputes, plant closings, "put-or-pay" contracts with trash incinerators, or other company actions that adversely affect the quality of life, tax base, or property values in the community.
(…)

The KLD reports were presented as binary summaries of positive and negative points (1 or -1) on each of the 80 indicators in seven major qualitative issue areas. When the company is neutral on an indicator (no strength or controversy), the score noted was 0 and when no data can be reported, "NR" (not rated) was written for the category. This was published yearly. The controversial trade issues acted as a do-not-call list. Alcohol, tobacco, gambling, firearms, military, and nuclear weapons were excluded from the ratings.

Innovest's ESG rating differed because it took into account the industry in which the company is located and not just to create an exclusion list. The MSCI ESG Ratings model identifies which ESG risks are the most liable to affect an industry or a sector. The model has been refined to identify the key issues important to each pillar (environmental, social, governance) for each industry. These have therefore varied over the years since Innovest was acquired by MSCI. The companies were rated with letters.

AAA AA	Leader in its industry in managing the most significant ESG risks and opportunities.
A BBB BB	Mixed or unexceptional track record of managing the most significant ESG risks and opportunities relative to industry peers.
B CCC	Companies which fail to manage their ESG risks.

Thompson Reuters uses the industry group to calculates the environmental and social category scores, and the country is used as a reference because governance best practices tend to be more consistent within countries according to their documentation. It takes into account the number of company which are worse than the current one, the companies which have the same value and the companies which have any value. The percentile rank methodology is used to compute 11 categories scores. The final score is therefore not very sensitive to outliers because it is based on a ranking.

$$score = \frac{nbr\ of\ companies\ with\ a\ worst\ value + \frac{nbr\ of\ companies\ with\ the\ same\ value\ included\ the\ current\ one}{2}}{nbr\ of\ companies\ with\ a\ value}$$

To calculate the overall Thomson Reuters ESG score, the weight of each category is determined by the number of checkpoints that it contains. This number is then related to all the indicators used in the form of a ratio. Therefore, categories that contain multiple checkpoints, such as management (composition, diversity, independence, committees, compensation, etc.), will have a higher weight than other categories. The number of measures per category determines the weight of the respective category. The table below details the numbers and weights:

PILLAR	CATEGORY	INDICATORS IN SCORING	WEIGHTS
ENVIRONMENTAL	Resource Use	20	11%
	Emissions	22	12%
	Innovation	19	11%
SOCIAL	Workforce	29	16%
	Human Rights	8	4.5%
	Community	14	8%
	Product Responsibility	12	7%
GOVERNANCE	Management	34	19%
	Shareholders	12	7%
	CSR Strategy	8	4.5%
TOTAL		178	100%

Source: Thompson Reuters

The link between ESG and the financial risk

ESG investments are not only related to environmental or societal commitments of companies, but have a real impact on financial risks. Albuquerque, Koskinen, and Zhang (2019)[6] published a paper presenting an industry equilibrium model where firms that engage in CSR activities tend to diversify their products and this increases their profit margins. Moreover, their systematic risk decreases because they are present in several markets and this increases the value of the company. This findings are coherent with the ones from the asset management companies which consider that ESG

[6] Albuquerque, R., Koskinen, Y., & Zhang, C. (2019). Corporate social responsibility and firm risk: Theory and empirical evidence. *Management Science*, 65(10), 4451-4469.

investments can mitigate portfolio risks (Blackrock and Ceres, 2015)[7]. This is because ESG investment acts as an insurance mechanism against harmful and risk-creating events. Indeed, the risk of damage to a company's reputation is lower when it is actively engaged in ESG practices. The likelihood of regulatory, legislative or consumer action against companies is therefore reduced. In addition, Giese G. and al. (2019)[8] showed that companies' ESG information was transmitted to their valuation and performance, both through their systematic risk profile (lower costs of capital and higher valuations) and through their idiosyncratic risk profile (higher profitability and lower exposures to market or tail risks). Research suggests that changes in a company's ESG can be a useful financial indicator. ESG ratings can also be incorporated into policy benchmarks and financial analyses.

A study based on an alternative approach to ESG asset returns was conducted to examine the utility that ESG scores provide to investors[9]. A model that captures the implications for investment if ESG is valued by the investor as well as the wealth created. Additional empirical evidence that investors who value ESG factors have improved utilities that do not come at the expense of return performance was provided by this study. On the other hand, Luo and Balvers (2017)[10] argue that there is a systematic "boycott risk premium" that has a substantial financial impact because most investors now choose assets with ESG scores in mind. The boycott effect cannot be replaced by litigation risk, a negligence effect, liquidity considerations, or industry dynamism and concentration. To keep their investors, companies investing in fossil fuels have to give higher than average dividends to their shareholders and this would explain a higher return on investment.

Regarding financial risk measures, several academics have highlighted that ESG criteria have an impact on the downside risk of the investment. Downside risks may be particularly important for a number of investors. Pension funds must match their assets to their liabilities and, therefore, face downside risk constraints. Banks and insurance companies face regulatory capital requirements for equity positions. These two types of players are very vigilant about their VaR level, such as all regulated funds. Nevertheless, due to the large number of assets they hold, these market participants are generally very well diversified and therefore do not face idiosyncratic ESG risk. Markowitz demonstrated in 1987[11], with his capital asset pricing theory, that perfectly diversified portfolios only face systemic risk. But systematic ESG risk can be drastically reduced by investing in companies with high ESG scores.

The relationship between investors' ESG commitments to their portfolio companies and the subsequent downside risks of those companies was analysed by Andreas G.F. Hoepner, Ioannis Oikonomou, Zacharias Sautner, Laura T. Starks and Xiao Y. Zhou in 2022[12]. They used two measures of downside risk to determine whether shareholder engagement on ESG issues can reduce downside risk: lower partial moment (LPM) and the VaR.

The LPM is computed as follow:

$$LPM = \sqrt{\frac{1}{N_1 - 1}\sum_{i=1}^{N_1}(r_{n,i} - \overline{r_{n,l}})^2}$$

[7] Blackrock and Ceres (2015): 21st Century Engagement, Investor Strategies for Incorporating ESG Considerations into Corporate Interactions.

[8] Giese, G., Lee, L. E., Melas, D., Nagy, Z., & Nishikawa, L. (2019). Foundations of ESG investing: How ESG affects equity valuation, risk, and performance. *The Journal of Portfolio Management*, 45(5), 69-83.

[9] Ahmed, M. F., Gao, Y., & Satchell, S. (2021). Modeling demand for ESG. *The European Journal of Finance*, 27(16), 1669-1683

[10] Luo, H. A., & Balvers, R. J. (2014). Social screens and systematic boycott risk. Forthcoming, *Journal of Financial and Quantitative Analysis*.

[11] M Harry Markowitz. (1952). Portfolio Selection, Journal of Finance, 7 (1), 77-91..

[12] Hoepner Andreas G.F., Oikonomou Ioannis, Sautner Zacharias, Starks Laura T. and Zhou Xiao Y., 2018, ESG Shareholder Engagement and Downside Risk, American Finance Association, Philadelphia

$r_{n,i}$: $negative\ daily\ return\ of\ firm\ i\ during\ a\ given\ month$
$\overline{r_{n,l}}$: $mean\ value\ of\ r_{n,i}$
N_1: $number\ of\ observed\ negative\ daily\ returns\ for\ firm\ I\ during\ a\ given\ month$

The authors choose to use the VaR, calculated at the firm-month level from daily log stock returns with a 5% confidence level in absolute value. It turns out that shareholder engagement on ESG issues leads to downside risk reduction for both risk measures.

Using the net monthly returns of MSCI Emerging Markets ESG Indices and non-ESG MSCI Emerging Markets Indices from August 2007 through December 2016, Matthew W. Sherwood and Julia L. Pollard[13] discovered that ESG assets have better risk indicators than their non-ESG counterparts. Overall, the Sharpe, Sortino and Omega ratio are higher for MSCI emerging market ESG indexes. Concerning the tail risk, the expected shortfalls are overall larger for the non-ESG assets and their skewness are more negative. This implies that for emerging markets, non-ESG assets are more likely to have negative returns than others and that they may be further from their average. In other words, they have a higher probability of exceeding their VaR level in the event of a market disruption. Karoline Baxa, Özge Sahinb, Claudia Czadob and Sandra Paterlinia[14] confirmed the importance of the ESG criterion in determining the tail risk for another dataset. Nevertheless, they point out that these results may be induced by the fact that investors tend to shy away from assets with low scores.

Banks with high ESG scores experienced greater stability during the 2008 financial crisis and the European sovereign debt crisis. Chiaramonte, L., Dreassi, A., Girardone, C., & Piserà, S. (2022)[15] also investigates the impact of each ESG pillars and they found that they are positively correlated with the reduction of bank fragility during periods of financial distress. This is particularly true for the social pillar. To determine it, they did a regression analysis to study the banks' distance to default (DTB) as the dependent variable. They used the global ESG scores from Thomson Reuters and they studied the impact of each sub pillars as well. When the components of each ESG pillar are examined, the results exhibit that larger effects are attributable to environmental innovation, fair treatment of labour, product responsibility, and equal treatment of shareholders. ESG strategies could act as an insurance-like risk mitigation device for banks in times of financial distress. Nevertheless, only the largest European banking groups appear to achieve financial stability benefits in times of crisis. Thus, depending on regulatory standards, the effectiveness of ESG strategies to mitigate risk differs.

The dependant variable is the Merton's distance to default, which is used to designed the bank stability in the previous study. Greater is this value, better is the bank stability. The Merton's model supposed that the firm's equity (E) is traded as a call option which use as underlying the firm's asset's value (V). The strike price is considered as the firm's zero-coupon bond debt. The firm's value follow a geometric Brownian motion with a drift term μ and a stochastic term σ.

$$\frac{\partial V}{V} = \mu dt + \sigma_V db$$

Because the equity price is considered as a call option, its value can be computed using the Black-Scholes formula:

$$E = VN(d_1) - De^{-rT}N(d_2)$$

[13] Sherwood, M. W., & Pollard, J. L. (2018). The risk-adjusted return potential of integrating ESG strategies into emerging market equities. Journal of Sustainable Finance & Investment, 8(1), 26-44.
[14] Bax, K., Sahin, Ö., Czado, C., & Paterlini, S. (2023). ESG, Risk, and (tail) dependence. International Review of Financial Analysis, 102513.
[15] Chiaramonte, L., Dreassi, A., Girardone, C., & Piserà, S. (2022). Do ESG strategies enhance bank stability during financial turmoil? Evidence from Europe. *The European Journal of Finance*, 28(12), 1173-1211.

$$d_1 = \frac{ln\left(\frac{V}{D}\right) + (r + 0.5\sigma_v^2)T}{\sigma_v\sqrt{T}}$$

$$d_2 = d_1 - \sigma_v\sqrt{T}$$

E is the market equity price, D the firm's debt value, N is the normal law, r the risk free rate, σ_v the firm's asset volatility and T the time until the debt maturity.

The other equation used is the Ito's Lemma one:

$$\sigma_E E = \frac{\partial E}{\partial V}\sigma_v V = N(d_1)\sigma_v V$$

Solving these equation, Merton found that the distance to default can be computed as follow:

$$DD = \frac{ln\left(\frac{V}{D}\right) + (\mu + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}$$

Merton's probability of default:

$$\pi_{Merton} = N(d_2)$$

Cox and Ross (1976)[16] have demonstrated that, under the risk neutrality assumption, the option valuation problem can be solved. This allows to compute the expected return on a given stock using a stochastic part as well:

$$E(S_{t+1}/S_t) = e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W(T)}$$

We can write:

$$S_{t+1} = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W(T)}$$

Where: $W(T) = \int_0^T dW(s) = \tilde{Z}\sqrt{T}\ with\ \tilde{Z} \sim N(0,1)$

$S_t = the\ current\ stock\ price\ at\ time\ t$
$\mu =$ the drift of the process, interpreted as the expected return of the risk factor
$\sigma =$ the diffusion of the process, interpreted as the standard deviation of the return of the risk factor
$T = time$

The last formula is used to perform Monte Carlo simulation. Indeed, it links the current expected return and standard deviation with a stochastic part. By simulating, with a random number generator, the asset price in t+1 a large number of times, one can rank these values in an increasing order and determine the VaR level with the desired confidence level. Moreover, the drift and the diffusion of the process can be determined using a model and this will lead to the parametric VaR.

The previous researches on ESG scores and their link to financial risks tend to show that high ESG scores decrease the assets' downside risks and the distance to default of European banks. This seemed to be true even and particularly during financial crisis. Although there are no studies that directly address the link between ESG score and VaR model robustness, papers tend to show that skewness is less negative for assets with high ESG scores, which may imply that they are less sensitive to extreme market movements and therefore less likely to have exceptions to their VaR model. This hypothesis will have to be verified. Nevertheless, a boycott risk premium may exists because of the lack of investors available for assets with low ESG scores. The regulations standards seem to play an important role in the effectiveness of ESG assets in reducing downside risk. Therefore, this effect may not have the same magnitude depending on the studied region. Moreover, the equity value could be determined using the Black Scholes formula, which imply a drift part and a stochastic one. This finding led to the creation of the Monte Carlo simulation, which is a way to determine the future price of an asset and therefore its VaR level. It is not the only way to determine asset's returns as a lot of other models exist, including risk factors or past returns.

[16] Cox, J. C., & Ross, S. A. (1976). The valuation of options for alternative stochastic processes. Journal of financial economics, 3(1-2), 145-166.

Link between inflation and stock returns

In order to identify economic factors which have a significant impact on the stock prices, Chen, Roll and Ross[17] defined these last ones as the discounted dividend values:

$$p = \frac{E(c)}{k}$$

c: the continuously compounded dividend
k: the discount rate
The return is also defined as follow:

$$\frac{dp}{p} + \frac{c}{p} = \frac{d(E(c))}{E(c)} - \frac{dk}{k} + \frac{c}{p}$$

Where E(c) is the excepted cash-flow.

Therefore, the price and also the stocks return increase with the expected cash-flow and decrease with the discount rate. This last one may change with both the level of interest rates and the spreads across different maturities. The authors identified several economic factors which explain significantly the prices changes. Mostly, the industrial production and the changes in the risk premium (defined by the return on low-grade bonds minus the return on long term government bonds) explained the stocks prices. Then, the measures of unanticipated inflation and changes in expected inflation impacted the dependant variable more weakly, during the studied periods. Neither the variables related to the consumption nor the oil price changes were identified as major stock prices components. This study was conducted in 1986, so the significance of variables may have changed. The announcements made by the FED about raising interest rates in response to inflation in 2022 had a significant impact on the markets.

The study of the link between inflation and stock returns has conflicting results in the previous research. Fama (1981)[18] developed the " proxy hypothesis " according to which a rise in inflation predicts a decline in real economic activity and the stock market anticipates the decline in corporate profits associated with this slowdown. Thus, expected inflation in Fama's formulation simply acts as a proxy for the true fundamentals, the expected real economic activity. Geske and Roll (1983) and Kaul (1987)[19] analysed the negative relationship between expected inflation and stock returns, developing into the underlying link between expected inflation and expected real activity. Overall, they support the basic idea that one should not reject the traditional view that it is not expected inflation, or increases in expected inflation, per se, that cause real stock returns to fall, once the control for the link between expected inflation and expected real activity is done. Boudoukh and Richardson (1993)[20] study of about one hundred years of data, where expected inflation is found to have a positive and nearly one-for-one effect on five-year nominal stock returns. Despite conflicting results, most research identifies a negative covariance between inflation and stock returns. This is for example the case of a researcher of the Federal reserve bank of New-York in 2013: “*Stocks whose returns covary negatively with inflation shocks have unconditionally higher returns. This implies that the average market price for the risk of inflation shocks is negative: periods with positive inflation shocks tend to be bad states of nature, and investors are willing to pay insurance in the form of lower*

[17] Chen, N. F., Roll, R., & Ross, S. A. (1986). Economic forces and the stock market. Journal of business, 383-403.
[18] Fama, E., 1981, “Stock Returns, real activity, inflation, and money,” American Economic Review 71, 545-565.
[19] Geske, Robert, and Richard Roll, 1983, “The fiscal and monetary linkage between stock returns and inflation,” Journal of Finance 38, 1-33
[20] Boudoukh, Jacob, M. Richardson, 1993, “Stock returns and inflation: A long horizon perspective,” American Economic Review, 83, 1346-1355.

average returns when holding an inflation-mimicking portfolio. I estimate that holding such a portfolio gives the agent a Sharpe ratio of -0.33."[21]

In 2002, Sharpe[22] demonstrated that equity valuations (measured by price-earnings ratios) have exhibited a negative relation with measures of inflation. The relation between expected inflation and expected long-term earnings growth is negative. Indeed, when the inflation level is high the required real returns are higher. Even though these studies were conducted several years ago and not during the Ukrainian crisis, inflation seems to have a significant impact on the expected returns. Moreover, the market movements that followed the central bank announcements around the world seem to confirm this trend. As a result, the VaR of the assets may have been exceeded for some funds, making the models non-conforming after backtesting analysis.

Value at risk (VaR) models and time series

There are several VaR models. There is the parametric VaR, the historical VaR and the Monte Carlo VaR. The first one allows to define the returns distribution and the model used to forecast the variance of the assets. The forecasting models assume that the future returns can be identify using the past ones. In this manner, they can be perceived as a time series. Despite the efficient market hypothesis developed by Fama[23], it appeared that financial returns can be estimated using their past values. Most of them have a trend, a seasonality and a disturbance term that can be isolate to build forecasting models. Time series can be defined as a sequence of random variables indexed by time.

When the mean and variance of economic time series, yt, change over time, this one is called nonstationary. It is very common for economic time series to be nonstationary, but the series of the changes from one period to the next, such as financial returns (Δyt), are likely to have a mean and variance that do not change over time. Otherwise, specific models can be applied. Asset's returns are commonly assumed to be weakly stationary. This means that this time series (y_t) is time invariant, so:

1. The mean of yt is constant over time: $E(y_t) = \mu$
2. The variance of yt is constant over time: $Variance(y_t) \ = \sigma^2 < \infty$
3. The covariance of yt and yt – l does not vary over time (but it could depend on the lag l): $Covariance(y_t, y_{t-l}) = \gamma_l$

A time series is strictly stationary if for any point in time, the joint distribution of $y_1, y_2, \dots, y_m$ is the same as the joint distribution of $y_{1+k}, y_{2+k}, \dots, y_{m+k}$. The problem with this theorem is that it is complicated to prove it. For this research paper, only the weak stationarity will be examine. To check it, the Dickey Fuller test is performed. If the ETF's prices are not weakly stationary, they will be transformed in returns or log(returns) with these formulas:

$Return = \frac{P_t - P_{t-1}}{P_{t-1}}$; with P_t the current asset price and P_{t-1} the previous asset price

$$ln(Return) = \ ln(P_t) \ - \ ln(P_{t-1}) \ = \ ln\left(\frac{P_t}{P_{t-1}}\right)$$

[21] Duarte, Fernando M. (2013) : Inflation risk and the cross section of stock returns, Staff Report, No. 621, Federal Reserve Bank of New York, New York, NY

[22] Sharpe, S. A. (2002). Reexamining stock valuation and inflation: The implications of analysts' earnings forecasts. Review of Economics and Statistics, 84(4), 632-648.

[23] Eugene F. Fama, Efficient Capital Markets: A Review of Theory and Empirical Work, The Journal of Finance, Vol. 25, No. 2, Papers and Proceedings of the Twenty-Eighth Annual Meeting of the American Finance Association New York, N.Y. December, 28-30, 1969 (May, 1970)

Identifying the corresponding order of a time series is an important step to check whether it can be studied or not. In order to do it, three methods can be used: the autocorrelation function (ACF), the partial autocorrelation function (PACF) and the Akaike Information Criterion (AIC). They allows to check for randomness, because if there is no correlation between the current value and the past one(s), the time series follows a random walk and the efficient market hypothesis can be validate. It is impossible, in this case, to study or forecast the asset prices or returns.

A time series is considered as a white noise if the variables are independent and identically distributed with a mean equal to zero. So each values has a zero correlation with the other variable and the same variance (σ^2). It is important for the predictability, because it is impossible be reasonably modelled. Moreover the series of errors terms (the deviation from the observed value to the fitted one) from a time series forecast model should ideally be white noise.

The ACF and PACF plots are used to figure out the order of Auto-Regressive (AR) and Moving-Average (MA) model. First, the models parameters *p* and *q* are unknown for the AR(*p*) and MA(*q*) models. Both parameters can be estimated by inspecting the sample autocorrelation function and/or the partial autocorrelation. The infinite sequence of the autocovariance is called the autocovariance function (AcovF).

$\gamma_l = Cov(y_t, y_{t-l})\ ; with\ l = 0,1,2, \ldots$

From the weak stationarity assumption: $Var(y_t) = Var(y_{t-l}) = \gamma_0$

The autocorrelation function is obtained with the following formula:

$$\rho_l = \frac{Cov(y_t, y_{t-l})}{Var(y_t)} = \frac{\gamma_l}{\gamma_0}\ ; with\ l = 0,1,2, \ldots$$

From the weak stationarity assumption: $\gamma_l = \gamma_{-l}$ and $\rho_l = \rho_{-l}$.

Procedures for identifying *p* and *q* parameters make use of the empirically estimated *sample autocovariance function* (SACovF) or *sample autocorrelation function* (SACF).
The l^th SACF :

$$\widehat{\rho_l} = \frac{Cov\widehat{(y_t, y_{t-l})}}{Va\widehat{r(y_t)}}\ ; with\ l = 0,1,2, \ldots$$

If all SACF, except for the order 0, are closed to 0, the series is considered as white noise.

The *partial autocorrelation function* (PACF) represents another tool for identifying the order of an AR or MA. A partial correlation coefficient adjusts the correlation between two random variables at different lags for the correlation this pair may have with the other orders. The ACF ρ_l, *l* = 0, 1, 2, …, represents the *unconditional correlation* between *yt* and *yt – l* . The PACF, denoted by α_l, *l* = 1, 2, …, is the sequence of conditional correlations:

$\alpha_l = Corr(y_t, y_{t-l} | y_{t-1}, \ldots, y_{t-l+1}); with\ l = 0,1,2, \ldots$

Just as the SACF, the PACF should be plotted and the significant spikes have to be counted to determine the model parameters.

The main problem with both methods is that it does not really allow to identify which model is the best among those which are studying. More sophisticated models such as ARMA, ARCH or GARCH models may not be estimated using these methods.

The AIC model allow to identify the right order independently from the model tested. It uses the maximum likelihood function to check which model is the most suitable and penalize for adding parameters. The definitions from Stock and Watson[24] are :

"The likelihood function: the joint probability distribution of the data, treated as a function of the unknown coefficients.
The maximum likelihood estimator (MLE) of the unknown coefficients consists of the values of the coefficients that maximize the likelihood function."

For a sample {y1, y2, ..., yn}, we denote f (y; θ) the probability density function (pdf) for random variable y, conditional on the set of parameters, θ. For n independent and identically distributed observations, the likelihood function of the sample:

$$L(\theta) = f(y_1;\theta) \times f(y_2;\theta) \times \ldots \times f(y_n;\theta) = \prod_{i=1}^{n} f(y_i;\theta)$$

The AIC is computed as follow:

$$AIC = -\frac{2}{T} ln(MLE) + \frac{2}{T} \times number\ of\ parameters\ ;\ with\ T\ the\ sample\ size$$

The most suitable model is the one which minimize the AIC.

Auto-Regressive model (AR(p))

The AR model assumes that the current value (y_t) is dependent on previous values. It can be seen as a linear regression model. Consider a situation where the value of a time series at time t, y_t, is a linear function of the last p values of y and an exogenous terms, denoted by u_t. This last variable is a white noise with a mean equal to 0 and a variance σ_u^2.

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + u_t$$

The lag operator denoted l, also called backward shift operator, is an operator that shifts the time index backward by one unit. The corresponding one is identified with the AIC formula.

This model has characteristics on its expectation, variance, covariance and auto-correlation.

1. Expectation:

$$E(y_t | y_{t-1}, \ldots, y_{t-p}) = \alpha_0 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p}$$
$$E(y_t) = \alpha_0 + E\left(\sum_{i=1}^{p} \alpha_i y_{t-i}\right)$$

With $E(y_{t-i}) = \mu :\ \mu = \alpha_0 + \sum_{i=1}^{p} \alpha_i \mu$
Or :$\mu = \frac{\alpha_0}{1-\sum_{i=1}^{p} \alpha_1}$

2. Variance

$$Var(y_t) = Var(y_{t-1}) = \ldots = Var(y_{t-p}) = \sigma^2$$
$$Cov(y_{t-i}; u_t) = 0$$
$$Var(y_t) = \alpha_1^2 Var(y_{t-1}) + \ldots + \alpha_p Var(y_{t-p}) + Var(u_t) + 2Cov(y_{t-1}; u_t)$$
$$Var(y_t) = \sum_{i=1}^{p} \alpha_i^2 \sigma^2 + \sigma_u^2 = \frac{\sigma_u^2}{1 - \sum_{i=1}^{p} \alpha_i}$$

[24] Stock James H. and Watson Mark W., Introduction to econometrics, Pearson

3. Autocovariance

$Cov(y_t; y_t) = \gamma_l = Var(y_t)\ if\ L = 0$
$Cov(y_t; y_{t-1}) = \gamma_l = \alpha_1 \gamma_{l-1}$ if L≥1

4. Autocorrelation

$\rho_l = \alpha_1 \rho_{l-1} = \alpha_1^2 \rho_{l-2} = \ldots = \alpha_1^l \rho_0$

The autocorrelation function of a weakly stationary AR(1) model decays exponentially with rate α_1.

Moving average model (MA(q))

The MA model assumes that the current value (y_t) is dependent on the current and past error terms u_t.

$y_t = \mu + \beta_1 u_{t-1} + \ldots + \beta_p u_{t-p}$

μ is a constant.

1. Expectation

$E(y_t) = \mu$

2. Variance

$Var(y_t) = \left(1 + \sum_{i=1}^{p} \beta_i\right)^2 \sigma_u^2$; because white noises are uncorrelated with each other.

3. Autocovariance

$$\gamma_l = \left(1 + \sum_{i=1}^{p} \beta_i^2\right)^2 \sigma_u^2\ if\ L = 0$$

$$\gamma_l = \sum_{i=1}^{p} \beta_i^2\ \sigma_u^2\ if\ L = 1$$

Autoregressive moving average (ARMA)

The ARMA models assumes that the current variable value can be determined studying the past values and the past error terms (white noises). It combines the AR and the MA model. The ARMA (1,1) can be written:

$y_t = \alpha_0 + \alpha_1 y_{t-1} + u_t + \beta_p u_{t-p}$

Autoregressive Conditional Heteroskedastic Models (ARCH) and Generalized Autoregressive Conditional Heteroskedastic Models (GARCH)

In linear regression analysis, a standard assumption used to build the previous models is that the variance of all squared error terms is the same. This assumption is called homoskedasticity. However, many time series data exhibit heteroskedasticity. In the case of financial returns, this can be explain by particular market events where the volatility increase before recovering a level closed to the previous period. ARCH have proven to be very useful in finance to model return variance or volatility of major asset classes including equity and fixed income. These assets are subject to an observed phenomenon in finance: the volatility clustering. This refers to the tendency of large changes in asset prices (either positive or negative) to be followed by large changes to be followed by small changes. There is temporal dependence in asset returns.[25] This implies that return are not independently

[25] Benoit B. Mandelbrot, "The Variation of Certain Speculative Prices," Journal of Business 36 (1963)

identically distributed. This time series can be expressed as the multiplication of the standard deviation by the error term u_t.

$$y_t = u_t \sqrt{\sigma_t^2}$$

The variance (the squared volatility) is time dependant and can be expressed as a function of the past returns y_{t-i}.

$$\sigma_t^2 = a + \sum_{i=1}^{q} b_i y_{t-i}^2$$

The GARCH model differs from the ARCH one because the volatility take into account the past returns and the pas volatilities. This model was proposed by Bollerslev[26]. It allows to account for the fact that empirically, large change in asset returns are generally followed by large changes and small changes by small ones (Mandelbrot[27]).

$$y_t = u_t \sqrt{\sigma_t^2}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} b_i y_{t-i}^2 + \sum_{j=1}^{p} c_j \sigma_{t-j}^2$$

The ARCH model and GARCH model, provide a convenient framework to study the problem of modelling volatility clustering. They allow for both volatility clustering and unconditional heavy tails. Engle[28] was the first author to design this GARCH function to model inflation rates. A large number of variants of the initial ARCH and GARCH models have been developed and they allow extensions to capture more detailed features of financial time series.

Following the increase of financial uncertainty in the 90's, the measure of risk created and commonly used since this time is the VaR, which refers to the tail risk. Many application presumes that the asset returns are normally distributed, but in fact the financial crises have shown a divergence between the normal distribution and the asset's one. It has been empirically proven that they exhibits a negative skewness, they are dependant of market conditions and they have a serial correlation among them. Venkataraman[29] proposed the use of a fat tailed distribution as it is able to capture extreme events which have a low probability of occurrence but which result in very big losses when they occur. Other researchers, Billio and Pelizzon[30] proposed a regime switching model to compute the VaR for 10 Italian stocks and for several portfolios. This imply that the conditional distribution of returns is always normally distributed but with either high or low volatility and a constant mean. It allows to account for different market momentum such as stressed period and normal ones. Billio and Pelizzon found out by backtesting this VaR methodology that the results were better than other ones, such as the RiskMetrics or the GARCH (1,1) models. Indeed, RiskMetrics uses the Exponentially Weighted Moving Average (EWMA) methods, which is from the GARCH models family. This volatility forecast correspond to the weighted average of the previous variance and the previous squared return:

[26] T. Bollerslev, Generalized autoregressive conditional heteroscedasticity, Journal of Econometrics, 31 (1986) 307-327.
[27] B. Mandelbrot, The Variation of certain speculative prices, Journal of Business, 36 (1963) 394-419.
[28] Robert F. Engle, "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation," Econometrica 50 (1982), pp. 987–1008.
[29] S. Venkataraman, Value at risk for a mixture of normal distributions: The use of quasi-Bayesian estimation techniques, Economic Perspectives, Federal Reserve Bank of Chicago, March/April 1997, 2-13.
[30] M. Billio, L. Pelizzon, Value-at-Risk: A multivariate switching regime approach, Journal of Empirical Finance, 7 (2000) 531-554.

$$\sigma_t^2 = a + \gamma\sigma_{t-1}^2 + (1-\gamma)y_{t-1}^2$$

According to RiskMetrics, $\gamma = 0.94$ for daily data and $\gamma = 0.97$ for monthly data.

The choice of the best performing VaR model is not the only research problem to solve for the authors. The distribution of financial returns to be used has also been studied. Giot and Laurent[31] have demonstrated that using a skewed Student t distribution, Var models performed better than the ones which used a symmetric distribution. Indeed, asymmetric models are a more accurate representation of the empirical returns distribution. These authors have tested in another research paper the RiskMetrics, skewed Student t APARCH and skewed student ARCH VaR models[32]. They have demonstrated that the most sophisticated model, the skewed Student t APARCH, produced the closest VaR results from the observed data. However, they advised to practitioners to use the skewed Student ARCH model because the results were closed the first APARCH ones and it is easier to implement. Brooks and Persand[33] argued that VaR models that do not use an asymmetric distribution, whether unconditional (when the mean and the standard deviation are constant over the time) or not, underestimate the VaR level.

Nevertheless, many researchers prefer to conduct simulation rather than using parametric VaR models. Indeed, the Committee of European securities (CESR), the European legislation, does not allow the market risk analysts to select themselves the model to forecast the volatility (ARCH, GARCH, …) for UCITS which hold financial derivatives with non-linear risk features. Therefore, a regulated fund could choose to have a parametric VaR model if the managers estimate that it is more appropriate, but they would not be allowed to buy any option. The ARCH, GARCH and EWMA models are not appropriate to modelized all derivatives payoffs. In fact, UCITS choose rather the historical simulation or the Monte Carlo one to compute their VaR level.

Jackson, Maude and Perraudin[34] have demonstrated that at higher confidence levels, historical simulation worked better. The results from this method are very dependant of the sample size chosen. Hendricks[35], Vlaar[36] and Danielson[37] supported that an increase of the sample size produced more accurate VaR results. Nevertheless, the legislation impose the number of observations to use to perform the historical or the Monte Carlo simulation. In Luxembourg, this number is 250 and this corresponds to one working year.

Backtesting

A backtest measures whether the realized loss observed ex-post are coherent with the ex-ante estimations and forecast. This framework was developed by the Basel Committee as many banks used to choose the VaR for their market risk measure. Backtesting should be considered as an integral part of VaR reporting. The regulatory sources imposing the practice of backtesting for UCITS are multiple. Based on the Luxembourg law and regulation, a formal backtesting report has to be prepared on a monthly basis. The *Commission de surveillance du secteur financier (CSSF)*, the Luxemburgish financial regulator, imposes to UCITS to report their "*number of overshoots occurred during the last 250 days at reference date based on a 99% confidence interval. Each day with an*

[31] P. Giot, S. Laurent, Value-at-Risk for long and short trading positions, Journal of Applied Econometrics, (2004) forthcoming.
[32] P. Giot, S. Laurent, Market risk in commodity markets: a VaR approach, Energy Economics, 25 (2003) 435-457.
[33] C. Brooks, G. Persand, The effect of asymmetries on stock index return Value-at-Risk estimates, The Journal of Risk Finance, Winter (2003) 29-42.
[34] P. Jackson, D.J. Maude, W. Perraudin, Testing Value-at-Risk approaches to capital adequacy, Bank of England Quarterly Bulletin, 38 (1998) 256-266.
[35] D. Hendricks, Evaluation of value-at-risk models using historical data, Federal Reserve Bank of New York, Economic Police Review, 2 (1996) 39-70.
[36] P. Vlaar, Value at Risk models for Dutch bond portfolios, Journal of Banking and Finance, 24 (2000) 131-154.
[37] J. Danielsson, The emperor has no clothes: Limits to risk modelling, Journal of Banking & Finance, 26 (2002) 1273-1296.

overshoot should be counted, even in the case of a sequence of overshoots resulting from one common specific event." [38] Moreover, it reminds to follow the ESMA regulations. Below is an extract from this guideline:

> *1. A UCITS should monitor the accuracy and performance its VaR model (i.e. prediction capacity of risk estimates), by conducting a backtesting program.*
>
> *2. The backtesting program should provide for each business day a comparison of the one-day value-at-risk measure generated by the UCITS model for the UCITS' end-of-day positions to the one-day change of the UCITS' portfolio value by the end of the subsequent business day.*
>
> *3. The UCITS should carry out the backtesting program at least on a monthly basis, subject to always performing retroactively the comparison for each business day in paragraph 2.*
>
> *4. The UCITS should determine and monitor the 'overshootings' on the basis of this backtesting program. An 'overshooting' is a one-day change in the portfolio's value that exceeds the related one day value-at-risk measure calculated by the model.*
>
> *5. If the backtesting results reveal a percentage of 'overshootings' that appears to be too high, the UCITS should review the VaR model and make appropriate adjustments.*
>
> *6. The UCITS senior management should be informed at least on a quarterly basis (and where applicable the UCITS competent authority should be informed on a semi-annual basis), if the number of overshootings for each UCITS for the most recent 250 business days exceeds 4 in the 30 case of a 99% confidence interval. This information should contain an analysis and explanation of the sources of 'overshootings' and a statement of what measures if any were taken to improve the accuracy of the model. The competent authority may take measures and apply stricter criteria to the use of VaR if the 'overshootings' exceed an unacceptable number.*

Backtesting consists of a periodic comparison of the portfolio's daily VaR value with the subsequent daily trading outcomes. This reported measure of risk should be larger than all but a defined fraction of the P&L, which is the confidence interval used. When the loss are more important than the VaR value, it is an exception of the model and these last ones are counted in order to assess the VaR model validity. The Basel Committee's traffic light test is an approach which uses exclusively the number of exceptions[39]. Indeed, it is a very simple test using the following methods. x represents the number of exceptions in the previous 250 trading days, which is approximately one trading year. Let Gx be the cumulative distribution function of a Binomial function $B(250,0.01)$. According to the Basel Committee system, if $Gx(B)<0.95$, the traffic light test is green and the model is valid. If $0.95<Gx(B)<0.99999$, the traffic light test is yellow and the practitioners have to perform some other tests to check if the model is still valid, but the committee does not formally reject the model. If $Gx(B)\geq 0.99999$, then the light is red, the model is rejected and also prompts regulatory intervention.

For the 99% confidence level, if $x \leq 4$ then the VaR model is considered accurate and is located in the "Green zone". If $5 \leq x \leq 9$ then the model may be assumed accurate or inaccurate and need to be analyzed, so it is in the "Yellow zone". Above 10 exceptions, the model has to be reviewed. However, one should keep in mind that VaR has severe problems in estimating losses at times of turbulent market. Such abnormal market conditions put under stress the models and assumptions. This can lead to the quantitative rejection of the model because of a high number of exceptions. As a consequence, they are detected in backtesting procedures, making it an essential part of risk analysis. The main problem with this test is that it is very simple and it does not allow us to evaluate the suitability of a VaR model as it does not take into account the independence of exceptions. It is used to gives some insight into the backtesting results.

[38] Guideline on the UCITS risk reporting, CSSF

[39] Basel Committee of Banking Supervision (1996). Supervisory Framework for the Use of "Backtesting" in Conjunction with the Internal Models Approach to Market Risk Capital Requirements.

P. Kupiec suggested in an article published in 1995[40] a test based on failure rates known as POF-test (proportion of failures) which measures whether the number of violations is consistent with the confidence level. Hence the null hypothesis is: $H0: p = \hat{p} = x/T$, with x the number of exceptions and T the number of observations. It tests whether the observed failure rate ($\hat{p}$) is significantly different from the failure rate (p) suggested by the confidence level. In case of a 99% one, p is equal to 1% According to Kupiec, the test is conducted as a likelihood-ratio test and takes the following form:

$$LR_{POF} = -2\ln\left(\frac{p(1-p)^{T-x}p^x}{(1-\left(\frac{x}{T}\right))^{T-x}(\frac{x}{T})^x}\right)$$

Under the null hypothesis, that the model capture the right proportion of exception, which is coherent with the defined confidence level, LR_{POF} is asymptotically chi-squared distributed with one degree of freedom. The chi-squared probability density function with one degree of freedom is:

$$f(x) = \frac{1}{\sqrt{2}\,\Gamma\left(\frac{1}{2}\right)}\sqrt{x}\,e^{-\frac{x}{2}} \quad if\ x > 0$$

Kupiec, in the same article, also suggested another type of backtest which measures the time it takes for the first exception to occur. This test is named the TUFF-test (time until first failure). It is based on similar assumptions as the POF-test with the following null hypothesis: $H_0: v = \hat{v} = 1/v$, with v the time until the first failure.

The test statistic is a likelihood-ratio and takes the following form:

$$LR_{TUFF} = -2\ln\left(\frac{p(1-p)^{v-1}}{(\frac{1}{v})(1-\left(\frac{1}{v}\right))^{v-1}}\right)$$

Again, the statistic follows a chi-squared distribution with one degree of freedom. Just as the LR_{POF}, if the chi-squared value with the corresponding degree of freedom is inferior to 5%, the null hypothesis is rejected at 95% confidence level. From the mathematical definition of the statistic, it comes that it cannot be computed if the time elapsed until the first exception is strictly inferior to 2.

Haas proposed a test in 2001 to verify that the violations are independent of each other[41]. This is the null hypothesis, the alternative one being that model's exceedances are not independent. Haas suggested the likelihood ratio statistic:

$$LR_{Hass} = \sum_{i=2}^{x}\left[-2\ln\left(\frac{p(1-p)^{vi-1}}{\hat{p}(1-\hat{p})^{vi-1}}\right)\right] - 2\ln\left(\frac{p(1-p)^{v-1}}{\hat{p}(1-\hat{p})^{v-1}}\right)$$

where v_i denotes the time gap between the i-1th violation and ith violation, and $\hat{p} = x/T$. The statistic is asymptotically chi-squared distributed with x (number of exceptions) degrees of freedom. If the LR_{Hass} value exceeds the critical value of the chi-squared distribution, we reject the null hypothesis.

[40] Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. Journal of Derivatives, 2, 73-84

[41] Haas, M. (2001). New methods in backtesting. Working Paper, Financial Engineering Research Center

An advantage of this test is that it is very robust, since it can identify both problems: the dependence and the number of exceptions.

P. F. Christoffersen suggested to compute a log-likelihood ratio to test the independence of exceptions[42]. It examines whether the probability of an exception on any day depends on the outcome of the previous day. Christoffersen's conditional coverage test aims only at assessing the independence between exceptions by detecting clusters. It does not take into account nor the time elapsed between exceptions or the time between clusters. Let I_t be an indicator variable that gets the value 1 of if the VaR level is exceeded and alternatively the value 0. n_{ij} defines the number of days when condition occurred assuming that condition occurred on the previous day. The outcome can be displayed in a 2×2 contingency table:

	$I_{t-1} = 0$	$I_{t-1} = 1$
$I_t = 0$	n_{00}	n_{10}
$I_t = 1$	n_{01}	n_{11}

Then, the probability π_i is defined as follow:

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}} \quad , \ \pi_1 = \frac{n_{11}}{n_{10} + n_{11}} \quad and \ \pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10} + n_{11}}$$

An accurate VaR model should not only produces the "expected" amount of exceptions but also exceptions that are independent of each other. If the VaR model is accurate, an exception today should not depend on whether or not an exception occurred on the previous day. The null hypothesis takes the following form:$H_0 = \pi_0 = \pi_1$. To test whether it is true, the following test should be also chi-squared distributed with one degree of freedom:

$$LR_{chris} = -2ln\left[\frac{(1-\pi)^{n_{00}+n_{01}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}{\pi_0}^{n_{01}}(1-\ \pi_1)^{n_{10}}\ {\pi_1}^{n_{11}}}\right]$$

The joint test is the addition of the Kupiec's POF test and the LR_{chris}. It tests the failure rate and the independence of violation[43].

$$LR_{joint} = LR_{POF} + LR_{chris}$$

To be accepted, the joint test have to follow a chi-squared distribution with 2 degree of freedom.

As mentioned previously, the VaR-99% is not the main risk measure under Basel III, but the 10-days ES at the 97.5% level will be for setting trading book capital. Acerbi & Tasche[44] (2002) defined it as a coherent risk measure and as the conditional expected loss given exceedance of VaR model. The ES-99% is currently the primary risk measure in the Swiss Solvency test (SST). Nevertheless, this measure is criticized, particularly because it is complicated to backtest and some researchers doubt about its feasibility. Cont et al., 2010 have shown that ES estimations generally have a lack of robustness. However, the notion of robustness has been questioned for bank and insurance. Emmer et al. (2015)[45] affirmed that it is less relevant for these actors because their data are frequently composed

[42] Christoffersen, P. (1998). Evaluating interval forecasts. International Economic Review, 39, 841-862
[43] Zhang, Y., & Nadarajah, S. (2017). A review of backtesting for value at risk. Communications in Statistics – Theory and Methods, 1-24
[44] Acerbi, C. & Tasche, D. (2002). On the coherence of expected shortfall. Journal of Banking and Finance 26, 1487–1503.
[45] Cont, R., Deguest, R. & Scandolo, G. (2010). Robustness and sensitivity analysis of risk measurement procedures. Quantitative Finance 10, 593–606.

of extreme values, which cannot be considered as outliers or measurement errors. This fact implies that statistic robustness in the context of measurement errors is less relevant than in other industry.

The complexity of backtesting the ES comes from the fact that this measure is not elicitable but conditionally elicitable. The concept of elicitability was introduced by Osband (1985)[46] and Lambert et al. (2008)[47]. An elicitable risk measure is a statistic of the profit and loss distribution that can be represented as the solution of a forecast error minimization problem. When a risk measure is elicitable, we can compare sets of forecasts obtained by different modeling approaches. Emmer et al. (2015), in the previously mentioned article, talked about conditional elicitability for the ES. This means that by backtesting some VaR levels with an appropriate function, it is possible to determine the ES backtesting values.

Despite these challenges, authors have found ways to backtest ES. Costanzino & Curran (2016)[48] extended the traffic light test analogous to the Basel II one. Kratz, M., Lok, Y. H., & McNeil, A. J. (2018)[49] considered others tests in addition to the traffic light one. Indeed the Basel III guidelines requires to consider more advanced backtesting methods. They explicitly refer to the tests based on VaR at multiple level, in particular the 97.5% and 99% confidence levels and tests based on realized p-values. The goals of the authors was to find tests that "*should be more powerful than the binominal test an better able to reject models that give poor estimates of the tail, and which would thus lead to poor estimates of the expected shortfall*". Moreover, they have shown an intuitive multinominal traffic light test. The researchers have tested the efficiency of three models for different VaR levels: the Pearson, the Naas and the likelihood-ratio test (LRT). Firstly, the VaR confidence levels $\alpha_1, \dots \alpha_j$ are defined by the following formula:

$$\alpha_j = \alpha + \frac{j-1}{N}(1-\alpha), \qquad j = 1, \dots N$$

Generally, the starting level used is $\alpha = 97.5\%$, because this corresponds to the ES level to backtest. N is the number of observations.

They define the of the level α_j at time t by $I_{t,j} = I_{\{L_t > VaR_{\alpha_{j,t}}\}}$. L_t corresponds to the loss at time t. Therefore, a violation to the model occurs when the ex-post loss is larger than the ex-ante expected one. According to Christoffersen, $I_{t,j}$ must satisfy the unconditional coverage property (the number of exception should be coherent with the confidence level used) and the independence property. The number of violation for each confidence level is defined by $O_j = \sum_{t=1}^{n} I_{\{X_t = j\}}$, with $X_t = \sum_{t=1}^{N} I_{t,j}$, so the number of VaR levels that are breached.

Let θ_j be the observed probability of an exception to occur. It corresponds to the number of exceedances divided by the number of observations.

$H_0 \colon \theta_j = \alpha_j\,, \qquad j = 1, \dots N$

$H_1 \colon \theta_j \neq \alpha_j \; for\ at\ least\ one\ j$

[46] Osband, K. H. (1985). Providing Incentives for Better Cost Forecasting. Ph.D. thesis, University of California, Berkeley.
[47] Lambert, N., Pennock, D. & Shoham, Y. (2008). Eliciting properties of probability distributions. In Proceedings of the 9th ACM Conference on Electronic Commerce. EC'08, ACM, New York.
[48] Costanzino, N. & Curran, M. (2016). A simple traffic light approach to backtesting expected shortfall.
[49] Kratz, M., Lok, Y. H., & McNeil, A. J. (2018). Multinomial VaR backtests: A simple implicit approach to backtesting expected shortfall. Journal of Banking & Finance, 88, 393-407.

Indeed the goal of the tests developed is to observe whether the observed exception probability is coherent with the VaR level, for multiple confidence intervals. Kratz, M., Lok, Y. H., & McNeil, A. J. compare the Pearson chi-squared test, the Naas test and the LRT ones:

1. Pearson chi-squared test[50]:

$$S_N = \sum_{j=0}^{N} \frac{(O_{j+1} - n(\alpha_{j+1} - \alpha_j))^2}{n(\alpha_{j+1} - \alpha_j)}$$

This result must follow a chi-squared distribution with N degree of freedom to not reject H_0.

2. Naas test[51]

Naas mainly improved the Pearson test in the following way:

$$cS_N \text{, } with\ c = \frac{2E(S_N)}{var(S_N)} \text{, } E(S_N) = N\ and\ var(S_N) = 2N - \frac{N^2 + 4N + 1}{n} + \frac{1}{n}\sum_{j=0}^{N} \frac{1}{\alpha_{j+1} - \alpha_j}$$

cS_N must be chi-squared distributed with $v = cE(S_N)$ degree of freedom. For the authors, this offers an improvement when the probabilities are small and so the confidence levels high.

3. LRT

The LRT allows to compute the maximum likelihood to estimate whether $\theta_j = \alpha_j$ under the alternative hypothesis H1: $\theta_j \neq \alpha_j\ for\ at\ least\ one\ j$.

$$G_N = 2\sum_{j=0}^{N} O_j ln\left(\frac{\theta_{j+1} - \theta_j}{\alpha_{j+1} - \alpha_j}\right)$$

Nevertheless, thus formula leads to undefined test statistic when O_j is equal to 0, because

$\theta_{j+1} - \theta_j = O_j/n$. For this reason, we can use the estimates of $\theta_{j+1} - \theta_j$:

$$\hat{\theta}_{j+1} - \hat{\theta}_j = \Phi\left(\frac{\Phi^{-1}(\alpha_{j+1}) - \hat{\mu}}{\hat{\sigma}}\right) - \Phi\left(\frac{\Phi^{-1}(\alpha_j) - \hat{\mu}}{\hat{\sigma}}\right)$$

Φ is the standard normal distribution function. The hypothesis to test becomes:

H0: $\hat{\mu} = 0\ and\ \hat{\sigma} = 1$

H1: $\hat{\mu} \neq 0\ and\ \hat{\sigma} \neq 1$

$\hat{\mu}\ and\ \hat{\sigma}$ are the maximum likelihood estimators under H1. The test statistic G_N must be asymptotically chi-squared distributed with N-1 degrees of freedom and the null hypothesis is rejected if $G_N > \boldsymbol{\chi^2_{N-1}}$ or $P(\boldsymbol{\chi^2_{N-1}} \geq G_N | H0) \leq \alpha$, with α the confidence level used to accept the model.

Testing these statistical tests on several VaR models and several US indexes, they concluded that:

[50] Pearson, K. (1900). On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can reasonably be supposed to have arisen from random sampling. Philosophical Magazine Series 5 50, 157{175.

[51] Nass, C. (1959). A chi-square-test for small expectations in contingency tables, with special reference to accidents and absenteeism. Biometrika, 1959, vol. 46, no 3/4, p. 365-385.

- The Pearson test is the most simple to use for practitioners and it allows to display easily the results to the management and the regulators. They advise to use N=4 for this test and they consider that it is an improvement compare to the binomial one. But it is the one which offers the best results.
- The Naas test is more robust, particularly for N=8. If the users chooses N=4, the results are similar to the Pearson test.
- The LRT is the most powerful test, because the results are more stable for N≥4. But it has the disadvantage of being more difficult to set up and requires larger samples to work properly.

Methodology

Data

The ESMA distinguishes two kinds of Exchange Traded Funds (ETFs): Actively Managed UCITS ETFs and UCITS ETFs. The definitions of both are given in the 2014 guidelines. The first one is defined as a traded fund that generally aims to outperform an index. The manager can choose the composition of its portfolio. The UCITS ETF is defined by the ESMA as follows: "A UCITS ETF is a UCITS, at least one unit or share class of which is traded throughout the day on at least one regulated market or Multilateral Trading Facility with at least one market maker that takes action to ensure that the stock exchange value of its units or shares does not significantly vary from its net asset value and, where applicable, its Indicative Net Asset Value." The difference between the annual return of the tracking index ETF and the annual return of the tracked index is supposed to be close to 0 for passively managed ETFs.

The selection of ETFs should allow the comparison of their VaR levels according to their ESG scores. Several biases could distort the analysis. For example, some specific risks could overlap with the ESG risk, such as currency risk. Moreover, Calice, G., & Lin, M. T.[52], explored the risk premium factors for country equity returns. They found out that the default risk is included in the country risk premium, which means that depending on the country in which the companies are located, their shares are more or less risky. This study has been conducted using Giglio, S., & Xiu, D.'s model[53], which accounts for the omitted variables bias. Therefore, the geographical location of the fund may affect its risk. The ETFs' VaR level must also vary according to the country in which the index they track is located. To avoid this bias, all ETFs selected in this study will be indexed in the same country, the United States. Nevertheless, if all selected ETFs passively replicate the same single index, the comparison of the robustness of the VaR models will not be possible because they should all have the same number of exceptions, as they have the same parameters. With the Monte Carlo method, one part is randomly generated, so the VaR level should differ, but it will be impossible to compare the historical VaR backtesting results with each other. ETFs will therefore be listed in the same currency and in the same country in order to observe only the ESG risk, but they have to track different markets, either passively or actively (the ESG scores provided by Thompson Reuters should therefore differ). To study VaR levels in response to Fed announcements on inflation or interest rate increases, US dollar-denominated ETFs will be selected.

Finally, the method used to select the ETFs to be analysed will be stratified sampling. That is, the US ETFs listed in dollars will be divided into subpopulations, each corresponding to the stock index being

[52] Calice, G., & Lin, M. T. (2021). Exploring risk premium factors for country equity returns. Journal of Empirical Finance, 63, 294-322.
[53] Giglio, S., & Xiu, D. (2021). Asset pricing with omitted factors. Journal of Political Economy, 129(7), 1947-1990.

replicated. This will allow not to have the same stock index represented for all funds and thus to have a more global representation of the US equity market.

Table 2: Selection of the studied ETFs

Name	RICS	Underlying index
Vanguard S&P 500 Index Fund	VOOIV.P	S&P 500
Invesco QQQ Trust Series 1	QQQ.O	Nasdaq-100 Index
Vanguard Russell 2000 Index Fund	VTWO.O	Russell 2000 Index
SPDR Dow Jones Industrial Average ETF Trust	DIA	Dow Jones Industrial Average
Vanguard Mid-Cap Index Fund	VO	CRSP US Mid Cap
Vanguard Growth Index Fund	VUG	MSCI US Prime Market Growth Index
iShares Russell 1000 Growth ETF	IWF	Russell 1000 Growth Index

The VaR models

The first step to compute the VaR level for all models is to determine the risk factors. To identify them, it is essential to determine to which variables stock prices are sensitive to. The first risk-return framework which was modelled to give an insight about investing decisions was the Markowitz model in 1950. Following this framework, the capital asset pricing (CAPM) theory was created. The CAPM[54] supposed the existence of a market portfolio, a stock index is often used to modelized it, and the risk free asset. An asset's expected return can be computed as follows:

$$E(r_a) = r_f + \beta_a(r_m - r_f)$$

With r_a the return of asset a, r_f the risk free rate, $r_m$the market portfolio return and $\beta_a = \frac{cov(r_a;r_m)}{\sigma_m}$. In fact, this theory requires the perfect market hypothesis: there are no transaction costs or taxes, short-selling does not have an impact on the asset price, investors are rational and risk averse, all market information is available, there is no illiquidity issue and any amount of money can be borrowed to invest in assets. In practice, it exists some transaction costs, taxes, illiquid assets and the behavioural finance has shown that investors are not rational in their decisions. The Allais paradox[55] shows that investors are loss-averse rather than risk-averse. The international capital asset pricing model (ICAPM), developed by Singer and Terhaar[56], allows to account for few market imperfections that are not considered by the CAPM: illiquidity and market segmentation.

Nevertheless, the CAPM and ICAPM empirical results are challenging by the Fama and French[57] model. The Three Factor Asset Pricing Model (APT) demonstrated that firm size and the book-to-market ratio are the dominant factors in explaining the returns of nonfinancial firms. They tested this time series:

$$R_a - R_f = \alpha_a + b_a(R_m - R_f) + s_a SMB + h_a HLM + u_a$$

$R_a - R_f$: the excess return on the portfolio, the asset return minus the risk free rate

[54] Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.
[55] Allais, M. (1990). Allais paradox. In *Utility and probability* (pp. 3-9). Palgrave Macmillan, London.
[56] Terhaar, K., Staub, R., & Singer, B. D. (2003). Appropriate policy allocation for alternative investments. *The Journal of Portfolio Management*, *29*(3), 101-110.
[57] Fama, Eugene, and Kenneth French, 1992, The cross-section of expected returns, *Journal of Finance*, 47, 427-465.
Fama, Eugene, and Kenneth French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56.

$R_m - R_f$: the excess return on a broad market portfolio
SMB: (small minus big) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks
HML: (high minus low) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks

While this model has proven useful in explaining U.S. stock returns, other researchers fail to demonstrate the significance of these factors in assessing bank stock returns.[58] The main problem with this model is that it identifies firm specificities as risk factors which explain the excess return. This method may not be applied for ETF because their prices movements follow (more or less) a financial index. Therefore, studying the book-to-market ratio or the size of the companies which composed the index is pointless, because only big firms with similar book-to-market ratio will be represented in the selected sample. Indeed, big and mature firms are supposed to have less growth opportunities than young ones.

Using the CAPM model, the correlation between the ETF price is estimated to be 1 (tracking error being very low for the selected assets). So the beta is equal to 1 and the asset risk is equal to the market one:

$$E(r_a) = r_f + \beta_a(r_m - r_f) = r_f + (r_m - r_f) = r_m$$

Furthermore, not all research paper agrees on the risk factors to be taken into consideration when evaluating the price of US stocks. For these reasons, only the asset price, which corresponds to the underlying index, will be used as a risk factor and the VaR level will be calculated on it.

The Value at Risk can be mathematically defined as:

$$Probability(Loss_t \geq VaR_{1-\alpha}V(t)) = \alpha$$

With 1-α the confidence level, which is more often 99%, essentially because of the European regulation and V(t) the value of the portfolio at time t. VaR corresponds to a percentile of the distribution of portfolio Profit and Loss (P&L). It is expressed as a potential loss from the current value of the portfolio. In case a 99% confidence level, the VaR is the amount that one should not lose more than except in 1% of the cases. VaR is probably the most widely used statistic measure for portfolio managers because that measure the potential risk of financial losses. It provides the likelihood that a loss greater than a certain amount would be realized. This risk tool is usually reported on a daily basis as the regulation requires it, but it could be computed for a different given horizon. The VaR is an attractive measure because it is easy to backtest as well, but it does not describe what happens in the tail or what is the worst possible loss. The notion of Expected Shortfall (ES), was created.

$$ES_{1-\alpha}(V_t) = \mathrm{E}(\mathrm{Loss}_t | \mathrm{Loss}_t \geq VaR_{1-\alpha}(V_t))$$

This value will be the primary risk measure under Basel III and will be backtested as well with a multinominal method.

As mentioned previously, there are three methods to compute the VaR level: historical, parametric and Monte Carlo.

[58] Viale, A. M., Kolari, J. W., & Fraser, D. R. (2009). Common risk factors in bank stocks. Journal of Banking & Finance, 33(3), 464-472.

Historical VaR valuation

The Historical VaR (HVaR) is a non-parametric full valuation technique. One need to identify the *n* risk factors that affect the portfolio (exchange rates, equity prices, interest rates,…)[59], if any; if not, directly use the historical values of the asset. Then, sample the historical data on these risk factors with *T* observations (daily returns) at past dates. A $r \times m$ matrix of historical returns is obtained. One can think of a specific scenario r as a row of *R*[60].

$$R = \begin{bmatrix} r_t^{(1)} & \cdots & r_t^n \\ \vdots & \ddots & \vdots \\ r_{t-m}^{(1)} & \cdots & r_{t-m}^n \end{bmatrix}$$

A row r from *R* corresponds to a return scenario for each risk factor. Then, we obtain the price of each risk factor *T* days from now using the formula : $P_T = P_0 e^{r\sqrt{T}}$. So each instrument is priced for the desired time horizon T. In this way, we obtain the portfolio P&L as $\sum_j (V_j(P_T) - V_j(P_0))$, with $V_j(P)$ a function of j assets corresponding to the portfolio value. To end, the changes are sorted by increasing order and the one corresponding to the desired quantile is taken to obtain the HVaR. This is the usual way to compute this risk measure. Its main problem is that it often fail to capture the risk during exceptional market events, as the results are very dependant of the sample size, and it leads to exceptions because the observed loss is more important than the HVaR.

The risk factor models seen previously are not applicable to ETFs. Moreover, their specifications are reduced because they only cover the US market and are listed in dollars. Only the market risk is really significant for the selected ETFs. So the asset price will be taken as the only risk factor.

Monte Carlo VaR valuation

The Monte Carlo VaR is quite similar to the historical one, but it includes a random part. One has to simulate scenarios and revaluing positions in the portfolio. The main advantage is that it is an accurate risk measure for all instruments and allowed the user to account for a stochastic part which corresponds to the uncertainty of the market evolution.

In order to implement the Monte Carlo VaR , it is necessary to assume a structure for the random evolution of the risk factors. The Itô's Lemma equation is used to determine the future asset's prices:

$$S_{t+1} = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W(T)}$$

Where: $W(T) = \int_0^T dW(s) = \tilde{Z}\sqrt{T}, with\ \tilde{Z} \sim N(0,1)$

$\tilde{Z}$ is the random part which required an accurate random number generator to obtain a lot of possible outcomes. An efficient way of doing with any kind of underlying probability distribution can be performed in two steps:

1. Draw a random number of the Uniform distribution (between 0 and 1)

2. Use the inverse of the cumulative density function of the underlying distribution to get the desired random draw.

μ, the drift of the process, and σ, the diffusion process, can be identified using the past returns. This holds for the non-parametric approach. These variables can be forecasted using the AR, ARCH, GARCH or EWMA models.

[59] Thomas J. Linsmeier & Neil D. Pearson (2000) Value at Risk, Financial Analysts Journal, 56:2, 47-67

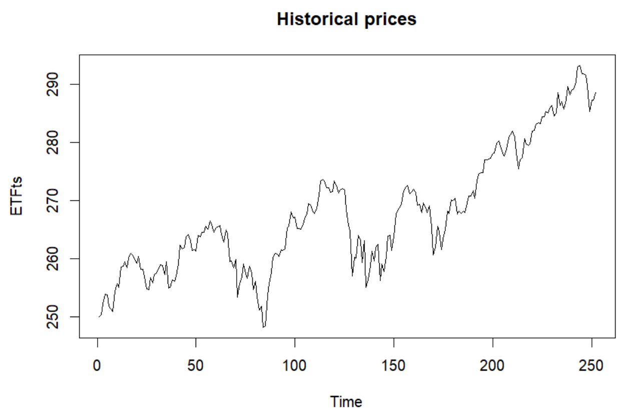
[60] Jorge Mina and JerryYi Xiao, Return to Riskmetrics: Evolution of a standard, Riskmetrics Group

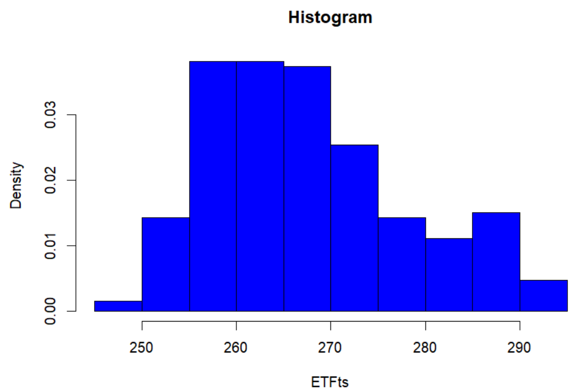
The Itô's Lemma computation is performed after having identified the drift and diffusion processes, with all of the random number generator's inverse of the cumulative density function. In the scope of this study, 5000 numbers are generated and 5000 future possible outcomes are computed. The results are sorted in the increasing order the one corresponding to the desired quantile is taken to obtain the Monte Carlo VaR result, just as the historical VaR.

Parametric VaR valuation

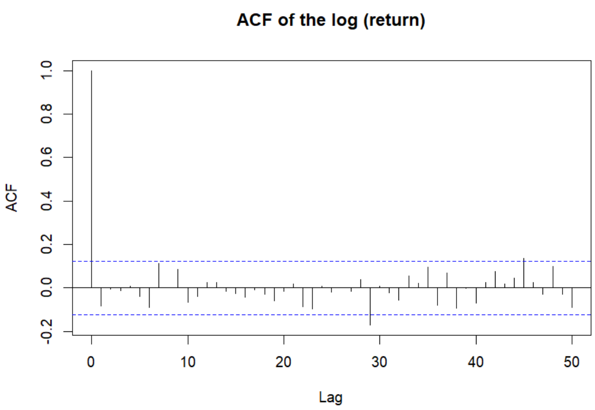
Firstly the GARCH(1,1) model will be applied for all assets, but with different return distribution: normal, student t (std) and skewed student t (sstd). The model is modified every 20 observations to have the parameters that best fit the selected data set. So, every 20 days, a new model is created and the next 20 standard deviations (volatility) are forecasted. This corresponds approximately to one working month. Then, the Var levels are computed and the Basel traffic light test is performed to know which model perform the best with this criteria. The one which produce the lower number of exceptions will be considered as the distribution which fit the best the ETFs return distribution to compute VaR levels. The best distribution will be selected to find the best model.

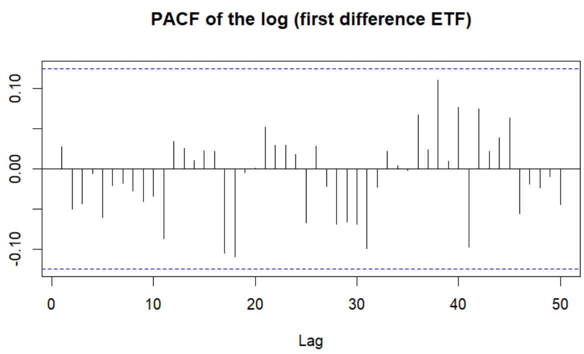
To compute a more optimal model, ARMA, ARCH and GARCH model will be applied to the assets, with the chosen distribution. This steps will allow to obtain a model with the lag(s) which is more appropriate for the volatility forecast. First, the prices of the ETF and its histogram are obtained in order to visualize the evolution of the year under study.



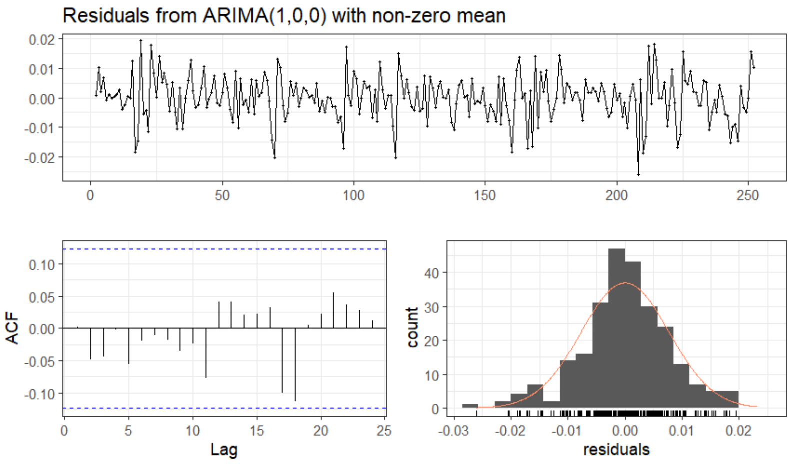


Then, the augmented Dickey Fuller test is applied to study the stationarity. As H0 (prices are stationary) was often rejected, the prices were transformed in log(return): log(P1/P0), with P1 the current price and P0 the previous one, as it is computed on a daily basis. In order to identify which lags are significant for the ARMA model, the ACF and PACF tests are performed.





These steps allow to check whether the time series has coherent autocorrelation function (decaying) and patrial ones. But the ARMA model is formally identified with the one minimizing the AIC criterion. The model's residuals are verified to ensure that there is no correlation among them, that their volatility is constant and that they are normally distributed.



The ARMA model selected will become the "mean model" to estimate the ARCH or GARCH model. Similarly to the ARMA model selection, to know which model fits the best for the selected sample of returns, the AIC criterion is observed. Then, the standard deviation of the return distribution is forecasted with the selected model and the selected lags to maximise the likelihood function. Few UCITS funds use a parametric VaR model, but the CSSF does not accept regular changes in the VaR model used. Moreover, to change it, it must be consistent with the management of the fund. For stability reasons, the model that minimizes the AIC will be recalculated every 250 observations, i.e. every working year. The results could be better by rebalancing the model more regularly, but this choice comes from the fact that the CSSF could reject a VaR model which is not easy to understand and intuitive for the investors.

The parameters μ *and* σ will be determined based on the previous year's data. Then, the Monte Carlo simulation steps are applied with the forecasted standard deviation and the forecasted drift process. Therefore, the VaR level are obtained for the confidence levels selected.

Backtesting VaR-99%and ES-97.5%

1. Traffic light test

This test is performed to count the number of exception for each backtested month. This is a simple way to determine whether the model is accepted or not. Indeed, if the traffic light is in the "Green zone", the proportion of failure test developed by Kupiec will normally accept the model. If the traffic light is in the "red zone", the Kupiec test will be rejected. Nevertheless, when this results is in the "Yellow zone", the other tests are useful to determine if the model should be accepted or rejected. This does not test the independence of exceptions.

Traffic Light test	
1	Green
2	Green
3	Green
4	Green
5	Yellow
6	Yellow
7	Yellow
8	Yellow
9	Yellow
10	Red
>10	Red

2. Proportion of failure test (POF)

As described previously in the literature review, the POF test is performed to check the coherence between the number of exceptions and the confidence level. Here, it verifies if the VaR-99% model is accepted or rejected with the numbers of exceedance which occur.

3. Time until the first failure(TUFF)/Haas test

The TUFF and the Haas test are performed to check whether the exceptions are independent from each other. Indeed, when the model is rejected this means that the exceptions are too close from each other.

4. Christoffersen test

This test check the independence of exception as well, but it accounts only for the exceptions which are consecutives and not for the spacing among them.

5. Multinominal test

The model described as the best performing in the previous paper is used: the LRT with N=4 confidence levels.

Table 3: Confidence levels, multinominal backtest

N	
1	97.5%
2	98.125%
3	98.75%
4	99.375%

Cross sectional analysis, ESG scores

As mentioned in the introduction, the dataset for the ESG data is not sufficient to allow a linear regression and establish the precise impact of the ESG score and the sub-pillars on the number of exceptions to the VaR models or the likelihood ratio POF. The correlation between the ESG scores of each ETF and their sub-pillars with the number of exceptions and the backtesting results for all VaR models is studied. The following table is obtained:

Correlations	*Historical VaR*		*Parametric VaR*		*Monte Carlo VaR*	
	VaR levels exceedances	*Backtesting results*	*VaR levels exceedances*	*Backtesting results*	*VaR levels exceedances*	*Backtesting results*
ESG score						
Environmental Pillar						
Social Pillar						
Governance Pillar						

The VaR level exceedances represents the number of exceptions of the historical, parametric and Monte-Carlo Var models for the N confidence level (Table 3). The backtesting results correspond to the POF results. This table will allow to identify whether there is a downward effect between the ESG scores and the number of exceptions/ backtests results on the studied period.

Next, the evolution of these variables during the crisis will be studied. The funds with the lowest ESG scores will have their backtesting results compared to those with higher scores, to see if the ESG score influences the robustness of the VaR level during the Ukrainian crise.

Development and results

Hypothesis tested

One of the first issues identified in the literature review in relation to the calculation of VaR levels is the distribution to be chosen to configure the variance forecasting model. Indeed, Giot and Laurent and Brooks and Persand have shown that using asymmetric distribution models, the VaR model tend to perform better than the ones which use a normal distribution This is the first hypothesis to be tested.

Hyp1: The asymmetric distribution is more appropriate for VaR models than the normal distribution.

To verify it and choose the most appropriate distribution for the parametric VaR models, the VaR levels will be estimated using the GARCH(1,1) model with the normal, student t (std) and skewed student t (sstd) distribution over the studied period. Then, their numbers of exceptions will be counted, such as a traffic light test to see which distribution produce the most coherent results with the given VaR confidence level.

As mentioned in the limitations of the VaR models, these could fail to capture the risk in case of particular market events. This is not only because the VaR model chosen may under-estimate the risk, but because this indicators is not intended to anticipate extreme shifts in returns. It is a simulation or, in the case of the parametric method, an estimate of the distribution tail of previous returns. Therefore, one can expect the Fed's announcements and inflation's expectations to have resulted in exceptions to the VaR models. To verify this, it is necessary to backtest the models on the period before the shock to see the evolution of the number of exceptions. Furthermore, it would be unusual to observe no change between both periods, as this could mean that the VaR model is significantly overestimating the risk and therefore not capturing the confidence level assigned to it. In addition, this is the opportunity to check whether the inflation announcements had a negative impact on financial returns and created exceedances.

Hyp2: The FED announcements of interest rates increase and inflation forecasts caused by the war in Ukraine had a negative impact on stock returns and created exceptions to the VaR models.

To check this statement, the comparison between the number of exceptions in 2021 and 2022 will be done.

The third hypothesis to be tested concerns the Var model that will produce the best results in the backtest. The authors do not all agree on this subject. While some build parametric models (more or less sophisticated) with different distributions, others prefer historical or Monte Carlo simulations. It is therefore necessary to test which method produces the best backtesting results. As the period under study concerns the Ukrainian crisis, the markets have been subjected to shocks regarding inflation announcements and key interest rate increases. As a result, methods that use past values as the only indicator of future values may perform poorly under these conditions. Therefore, the historical method could have unsatisfactory results because it would underestimate the risk.

Hyp3: The parametric method including a stochastic part is more efficient than historical and Monte Carlo simulations in case of financial markets volatility to estimate the VaR levels of ETFs.

It is necessary to compute the VaR levels with the tree methodologies and backtests the results with the Kupiec, Christoffersen, Haas and the traffic light test to ensure the validity of this hypothesis.

The last assumption concerns link between the ESG scores and the VaR levels of ETFs. Previous research shows that downside risk is lower when ESG scores are high, even during the 2008 crisis. Although the link to model robustness has not been established, if risk is lower in a crisis, VaR levels should be less exceeded.

Hyp4: The ESG scores have a negative correlation with the number of VaR models exceptions.

Choice of the distribution with GARCH(1,1) model

The N levels of VaR (97.5%, 98.125%, 98.75% and 99.375%) and VaR-99% have been computed with the GARCH(1,1) model. Using the following equation, the variance has been forecasted:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} b_i y_{t-i}^2 + \sum_{j=1}^{p} c_j \sigma_{t-j}^2$$

With y_{t-i} the previous return andσ_{t-j}^2 the previous variance.

$\omega = (1 - b_i - c_j)\bar{\sigma}$, with $\bar{\sigma}^2$ the long term variance

Then, the stochastic part was added as follow:

$$S_{t+1} = S_0 e^{(\mu - \frac{1}{2}\sigma^2)T + \sigma W(T)}$$

Where: $W(T) = \int_0^T dW(s) = \tilde{Z}\sqrt{T}\ with\ \tilde{Z} \sim N(0,1)$

$S_t = the\ current\ stock\ price\ at\ time\ t$

$\mu =$ the drift of the process, which corresponds to the ω of the GARCH process

$\sigma =$ the diffusion of the process, the forecasted standard deviation

$T = time$

The table represents the number of overshoots for each VaR levels and each distribution studied for a three-year period from 2020 to 2022.

Table 4: Number of exceptions to the GARCH(1,1) model for the different funds and distributions

	Dia			IWF			QQQ.O			VO			VooiV.P			VTWO.O			VUG		
	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd
VaR-97,5%	32	33	31	23	23	24	24	24	23	30	32	29	34	33	25	31	29	28	24	25	25
VaR-98,125%	29	29	26	21	20	20	19	20	20	26	30	24	29	28	21	22	22	21	22	24	23
VaR-98,75%	26	25	25	19	18	17	18	17	16	24	26	23	25	18	15	19	18	18	17	18	20
VaR-99,375%	23	22	21	15	15	11	13	13	12	23	24	19	17	12	11	12	14	13	13	14	13
VaR-99%	26	23	22	18	17	15	18	14	14	24	25	22	24	17	15	14	14	14	17	16	16

The table below represents the average number of exceedance per year. This represents the number of exceptions over 252 observations.

Table 5: Average yearly number of exceptions to the GARCH(1,1) model for the different funds and distributions

	Dia			IWF			QQQ.O			VO			VooiV.P			VTWO.O			VUG		
	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd	norm	std	sstd
VaR-97,5%	10,67	11,00	10,33	7,67	7,67	8,00	8,00	8,00	7,67	10,00	10,67	9,67	11,33	11,00	8,33	10,33	9,67	9,33	8,00	8,33	8,33
VaR-98,125%	9,67	9,67	8,67	7,00	6,67	6,67	6,33	6,67	6,67	8,67	10,00	8,00	9,67	9,33	7,00	7,33	7,33	7,00	7,33	8,00	7,67
VaR-98,75%	8,67	8,33	8,33	6,33	6,00	5,67	6,00	5,67	5,33	8,00	8,67	7,67	8,33	6,00	5,00	6,33	6,00	6,00	5,67	6,00	6,67
VaR-99,375%	7,67	7,33	7,00	5,00	5,00	3,67	4,33	4,33	4,00	7,67	8,00	6,33	5,67	4,00	3,67	4,00	4,67	4,33	4,33	4,67	4,33
VaR-99%	8,67	7,67	7,33	6,00	5,67	5,00	6,00	4,67	4,67	8,00	8,33	7,33	8,00	5,67	5,00	4,67	4,67	4,67	5,67	5,33	5,33

Overall, the VaR model with the sstd distribution exhibits a lower level of exceptions than the others. This seems consistent with the studied confidence level. The complete monthly Basel traffic light tests are in the appendix for each fund. The results are still consistent with the average value of the number of overshoots per year. The sstd distribution displays a number more consistent with its VaR level than the two other distributions studied. This is valid even in case of unstable market like in 2022 or early 2021, when the Covid-19 related exceptions were still in the sample.

Therefore, hypothesis 1 is validated and the sstd distribution is retained to calculate the parametric VaR level. However, instead of taking the GARCH (1,1) model systematically, it is the model that minimizes the AIC for each fund that will be selected and recalibrated each year. The optimal parameters are the followings:

Dia 2020:

```
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.000810    0.091119    1.56401 0.117814
ar1     -0.100474    0.058716 -1.71119 0.087046
ma1    -1.054181    0.000391    3.61959 0.000295
ma2     0.031283    0.000066   473.0378 0.000000
omega   0.000005    0.000003     1.8689 0.061641
alpha1  0.108283    0.039126     2.7675 0.005648
beta1   0.824836    0.000582    1.66029 0.096855
```

Dia 2021:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000773    0.000753  1.02659 0.304612
ma1    -0.059975    0.066023 -0.90839 0.363669
ma2    -0.141776    0.066295  -2.1386 0.032470
omega   0.000009    0.000007  1.34012 0.180205
alpha1  0.138382    0.092408  1.49751 0.134260
alpha2  0.160887    0.117977  1.36371 0.172658
beta1   0.699731    0.128632  4.20685 0.000026
```

Dia 2022:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000644    0.000405  1.58957 0.111931
ar1    -0.023603    0.072160 -0.32709 0.743601
omega   0.000011    0.000001 14.70898 0.000000
alpha1  0.284754    0.073389  3.88004 0.000104
beta1   0.545728    0.072160 -0.32709 0.743601
```

IWF 2020:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001154    0.000314   3.6812 0.000232
ma1    -0.143273    0.058782  -2.4374 0.014795
ma2    -0.141776    0.066295  -2.1386 0.032470
omega   0.000008    0.000002  3.58796 0.000333
alpha1  0.000000    0.081218   0.0000 1.000000
alpha2  0.203432    0.120724   1.6851 0.091969
beta1   0.683053    0.382415   1.7862 0.074074
```

IWF 2021:

```
        Estimate  Std. Error    t value Pr(>|t|)
mu      0.001883    0.000751    2.50913 0.012103
ar1    -1.950221    0.000753    1.02659 0.304612
ar2    -1.095422    0.064286   -1.94182 0.052159
ar3    -0.061898    0.013601   -4.55091 0.000005
omega   0.000029    0.000016    1.80511 0.071057
alpha1  0.081067    0.085661    0.94636 0.343964
alpha2  0.376796    0.161478    2.33342 0.019626
beta1   0.541137    0.128632    4.20685 0.000026
```

IWF 2022:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000966    0.000582  1.66029 0.096855
omega   0.000006    0.000008  0.76117 0.446554
alpha1  0.060979    0.056770  1.07415 0.282756
alpha2  0.142512    0.091119  1.56401 0.117814
beta1   0.763836    0.028022 27.25877 0.000000
```

QQQ.O 2020:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.001416    0.000391  3.61959 0.000295
ar1    -0.100474    0.058716 -1.71119 0.087046
ar2    -0.124832    0.064286 -1.94182 0.052159
omega   0.000008    0.000002  3.58796 0.000333
alpha1  0.059270    0.076861  0.77113 0.440629
alpha2  0.156685    0.098379  1.59266 0.111236
beta1   0.699962    0.066041 10.59887 0.000000
```

QQQ.O 2021:

```
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001740    0.000862    2.0188 0.043504
alpha1  0.253932    0.095131    2.6693 0.007601
beta1   0.745068    0.069919   10.6561 0.000000
```

QQQ.O 2022:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000933    0.000598  1.56124 0.118466
omega   0.000004    0.000005  0.69821 0.485043
alpha1  0.176163    0.062118  2.83595 0.004569
beta1   0.813576    0.063153 12.88267 0.000000
```

VO 2020:

```
        Estimate  Std. Error  t value Pr(>|t|)

mu      0.000772    0.000187   4.1171 0.000038
ar1    -0.130500    0.030756  -4.2431 0.000022
ma1     0.078682    0.012388   6.3515 0.000000
ma2     0.047961    0.011503   4.1693 0.000031
omega   0.000005    0.000001   7.7597 0.000000
alpha1  0.074821    0.018081   4.1381 0.000035
beta1   0.833601    0.085926   2.8043 0.005042
```

VO 2021:

```
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001471    0.000686    2.1448 0.031967
ar1    -1.191063    0.060732  -19.6118 0.000000
ar2    -0.151878    0.089794   -1.6914 0.090760
ar3     0.115213    0.057716    1.9962 0.045912
omega   0.000006    0.000005    1.2072 0.227338
alpha1  0.258351    0.078516    3.2904 0.001000
beta1   0.740649    0.064715   11.4448 0.000000
```

VO 2022:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000761    0.000533   1.4284 0.153189
omega   0.000016    0.000005   3.1347 0.001720
alpha1  0.240964    0.085926   2.8043 0.005042
```

VooiV.P 2020:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.000993    0.000366   2.7111 0.006706
ar1    -0.056847    0.062387  -0.9112 0.362189
omega   0.000005    0.000002   2.4443 0.014512
alpha1  0.144721    0.033726   4.2910 0.000018
beta1   0.764670    0.382415   1.7862 0.074074
```

VooiV.P 2021:

```
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.001256    0.000665   1.88781 0.059052
ar1    -1.084223    0.062387  -0.9112 0.362189
ar2    -0.094333    0.089169  -1.05791 0.290096
ar3     0.070713    0.065208   1.08442 0.278177
omega   0.000017    0.000008   2.05081 0.040285
alpha1  0.045335    0.068873   0.65825 0.510380
alpha2  0.330073    0.153894   2.14481 0.031968
beta1   0.608262    0.128473   4.73456 0.000002
```

VooiV.P 2022:

```
mu      0.000957    0.000420   2.2754 0.022881
omega   0.000008    0.000002   3.6698 0.000243
alpha1  0.281154    0.079016   3.5582 0.000373
beta1   0.639245    0.068112   9.3852 0.000000
```

VTWO.O 2020:

```
mu      0.000709    0.000513    1.380899 0.167310
ma1    -0.332049    0.062387     -0.9112 0.362189
omega   0.000004    0.000008    0.521540 0.601990
alpha1  0.005764    0.078721    0.073224 0.941628
alpha2  0.167577    0.116332    1.440511 0.149723
beta1   0.796513    0.118157    6.741135 0.000000
```

VTWO 2021:

```
mu      0.001720    0.001054   1.6312 0.102844
ar1    -1.377411    0.098157 -14.0327 0.000000
ar2    -0.381588    0.130590  -2.9220 0.003478
ar3     0.143223    0.066733   2.1462 0.031858
ma1     1.330491    0.088099  15.1022 0.000000
ma2     0.420368    0.086917   4.8364 0.000001
omega   0.000011    0.000010   1.1115 0.266354
alpha1  0.177438    0.068404   2.5940 0.009487
beta1   0.814787    0.051574  15.7984 0.000000
```

VTWO.O 2022:

```
        Estimate  Std. Error   t value Pr(>|t|)
```

```
mu      -0.000069    0.000458  -0.15080 0.880137
omega    0.000047    0.000019   2.41820 0.015598
alpha1   0.203167    0.086963   2.33625 0.019478
beta1    0.558458    0.137944   4.04845 0.000052
```

VUG 2020:

```
        Estimate  Std. Error    t value Pr(>|t|)
mu       0.001170    0.000322  3.632495 0.000281
ma1     -0.133430    0.061262 -2.178037 0.029403
ma2     -0.151689    0.063877 -2.374727 0.017562
omega    0.000008    0.000001 10.229327 0.000000
alpha1   0.000000    0.075317  0.000002 0.999998
alpha2   0.185790    0.094794  1.959930 0.050004
beta1    0.697350    0.061845 11.275713 0.000000
```

VUG 2021:

```
        Estimate  Std. Error   t value Pr(>|t|)
mu       0.001853    0.000749   2.47565 0.013299
ar1     -1.960835    0.062387  -0.9112 0.362189
omega    0.000028    0.000015   1.92166 0.054649
alpha1   0.064582    0.090916   0.71035 0.477490
alpha2   0.364145    0.164854   2.20889 0.027182
beta1    0.570273    0.113225   5.03663 0.000000
```

VUG 2022:

```
        Estimate  Std. Error  t value Pr(>|t|)
mu       0.001084    0.000586  1.84985 0.064336
ar1      0.014986    0.068311  0.21938 0.826356
omega    0.000004    0.000006  0.72372 0.469236
alpha1   0.195090    0.063518  3.07142 0.002130
beta1    0.791015    0.382415   1.7862 0.074074
```

Backtests results VaR-99%

1. Traffic light test by fund

The traffic light test was conducted on the seven funds in the scope of the study for all VaR models previously mentioned. For a 99% confidence level, the model is accepted if the number of exception is less or equal to 4, need more investigations if the number of exceedance is between 5 and 9, and rejected if it is equal or above 10.

Table 6:Basel traffic light, historical VaR 99%

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	8	8	6	8	8	6	6
01/03/2021	6	5	4	5	5	4	4
01/04/2021	0	0	0	0	0	0	0
03/05/2021	0	0	0	0	0	0	0
01/06/2021	0	0	0	0	0	0	0
01/07/2021	0	0	0	0	0	0	0
02/08/2021	0	0	0	0	0	0	0
01/09/2021	0	0	0	0	0	0	0
01/10/2021	0	1	0	0	0	0	1
01/11/2021	0	1	0	0	0	0	1
01/12/2021	1	1	0	1	1	1	1
03/01/2022	1	1	0	1	1	1	1
01/02/2022	1	2	1	1	1	1	2
01/03/2022	1	3	3	1	2	1	3
01/04/2022	2	4	4	2	3	1	4
02/05/2022	3	6	6	4	6	2	6
01/06/2022	6	9	8	7	9	4	9
01/07/2022	6	10	9	9	10	6	10
01/08/2022	6	10	9	9	10	6	10
01/09/2022	7	10	9	9	10	6	10
03/10/2022	8	10	10	9	11	6	10
01/11/2022	8	10	10	9	11	6	10
01/12/2022	7	10	10	8	10	5	10
03/01/2023	7	10	10	8	10	5	10

Table 7: Monthly changes in the number of exceptions in the historical VaR model 99%

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/03/2021	-2	-3	-2	-3	-3	-2	-2
01/04/2021	-6	-5	-4	-5	-5	-4	-4
03/05/2021	0	0	0	0	0	0	0
01/06/2021	0	0	0	0	0	0	0

01/07/2021	0	0	0	0	0	0	0
02/08/2021	0	0	0	0	0	0	0
01/09/2021	0	0	0	0	0	0	0
01/10/2021	0	1	0	0	0	0	1
01/11/2021	0	0	0	0	0	0	0
01/12/2021	1	0	0	1	1	1	0
03/01/2022	0	0	0	0	0	0	0
01/02/2022	0	1	1	0	0	0	1
01/03/2022	0	1	2	0	1	0	1
01/04/2022	1	1	1	1	1	0	1
02/05/2022	1	2	2	2	3	1	2
01/06/2022	3	3	2	3	3	2	3
01/07/2022	0	1	1	2	1	2	1
01/08/2022	0	0	0	0	0	0	0
01/09/2022	1	0	0	0	0	0	0
03/10/2022	1	0	1	0	1	0	0
01/11/2022	0	0	0	0	0	0	0
01/12/2022	-1	0	0	-1	-1	-1	0
03/01/2023	0	0	0	0	0	0	0

The traffic light results seem to be consistent for the different funds. Overall, the models started to be in the yellow zone from May 2022. They were also in the yellow zone at the beginning of the study period due to the impact of the Covid 19 crisis in the sample. The number of exceptions started to increase from February/March 2022, which is consistent with the start of the war in Ukraine. Nevertheless, the SPDR Dow Jones Industrial Average ETF Trust, the Vanguard Mid-Cap Index Fund and the Vanguard Russell 2000 Index Fund did not experience this rise in early 2022. The effect of the conflict began in April or May. But overall, one can note that the number of exceptions has increased significantly between January 2021 and 2022.

Table 8: Basel traffic light, parametric VaR 99%

Date	**Dia**	**IWF**	**QQQ.O**	**VO**	**VooiV.P**	**VTWO.O**	**VUG**
01/02/2021	6	14	10	9	13	4	15
01/03/2021	5	11	9	9	11	2	12
01/04/2021	5	8	6	6	8	1	9
03/05/2021	5	8	6	6	8	1	9
01/06/2021	5	8	6	6	8	1	9
01/07/2021	4	6	5	5	6	0	7
02/08/2021	5	6	5	5	6	0	7
01/09/2021	5	6	5	5	6	0	7
01/10/2021	4	4	4	4	4	0	5
01/11/2021	3	3	3	2	2	0	3
01/12/2021	4	3	3	2	3	0	3
03/01/2022	4	2	2	2	2	0	2
01/02/2022	3	0	0	2	1	0	0
01/03/2022	2	0	0	1	1	0	0
01/04/2022	2	1	0	2	2	0	0
02/05/2022	3	1	0	3	3	0	0
01/06/2022	4	1	0	5	4	0	0
01/07/2022	4	2	0	7	4	2	0
01/08/2022	3	2	0	7	4	2	0
01/09/2022	4	3	1	8	5	3	1
03/10/2022	5	4	2	9	6	4	2

01/11/2022	5	4	2	9	6	4	2
01/12/2022	4	4	2	10	6	5	2
03/01/2023	4	5	3	11	7	5	3

Table 9: Monthly changes in the number of exceptions in the parametric VaR model 99%

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/03/2021	-1	-3	-1	0	-2	-2	-3
01/04/2021	0	-3	-3	-3	-3	-1	-3
03/05/2021	0	0	0	0	0	0	0
01/06/2021	0	0	0	0	0	0	0
01/07/2021	-1	-2	-1	-1	-2	-1	-2
02/08/2021	1	0	0	0	0	0	0
01/09/2021	0	0	0	0	0	0	0
01/10/2021	-1	-2	-1	-1	-2	0	-2
01/11/2021	-1	-1	-1	-2	-2	0	-2
01/12/2021	1	0	0	0	1	0	0
03/01/2022	0	-1	-1	0	-1	0	-1
01/02/2022	-1	-2	-2	0	-1	0	-2
01/03/2022	-1	0	0	-1	0	0	0
01/04/2022	0	1	0	1	1	0	0
02/05/2022	1	0	0	1	1	0	0
01/06/2022	1	0	0	2	1	0	0
01/07/2022	0	1	0	2	0	2	0
01/08/2022	-1	0	0	0	0	0	0
01/09/2022	1	1	1	1	1	1	1
03/10/2022	1	1	1	1	1	1	1
01/11/2022	0	0	0	0	0	0	0
01/12/2022	-1	0	0	1	0	1	0
03/01/2023	0	1	1	1	1	0	1

The parametric model seems to have been much less robust during the Covid crisis because it had many overshoots at the beginning of 2021, which makes the model's robustness not consistent with the VaR level interval. At the time of the Ukrainian crisis, only the Vanguard Mid-Cap Index Fund fund saw its model being in the Red zone. Despite this, the parametric VaR has been more robust in 2022 than the historical VaR, the results are fluctuating depending on the fund studied. It seems that the parameters used impact the results during the backtest. The level of VaR is therefore dependent on the forecasting model estimated. It should be noted that despite the onset of the crisis in Ukraine in February, the number of exceptions decreased for some funds before increasing starting from April/May 2022.

Table 10: Basel traffic light, Monte-Carlo VaR 99%

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	12	13	12	14	12	11	12
01/03/2021	9	10	9	11	9	8	9
01/04/2021	1	2	3	2	1	2	2
03/05/2021	1	2	3	1	1	1	2
01/06/2021	1	2	3	2	1	1	2
01/07/2021	0	1	2	1	0	0	1
02/08/2021	0	1	2	1	0	0	1
01/09/2021	0	1	2	1	0	0	1

01/10/2021	0	1	1	1	1	0	1
01/11/2021	0	1	1	1	1	0	1
01/12/2021	2	2	1	3	3	1	1
03/01/2022	2	2	2	4	3	1	1
01/02/2022	2	5	4	5	5	2	4
01/03/2022	2	8	7	6	7	2	7
01/04/2022	4	9	8	7	8	2	8
02/05/2022	7	11	10	10	11	3	10
01/06/2022	9	14	13	12	14	5	13
01/07/2022	11	16	15	14	17	7	15
01/08/2022	11	16	15	14	17	7	15
01/09/2022	12	16	16	15	18	7	15
03/10/2022	13	16	16	16	18	8	15
01/11/2022	13	16	16	16	18	8	15
01/12/2022	11	15	16	14	16	7	15
03/01/2023	11	15	15	13	16	7	15

Table 11: Monthly changes in the number of exceptions in the Monte-Cralo VaR model 99%

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/03/2021	-3	-3	-3	-3	-3	-3	-3
01/04/2021	-8	-8	-6	-9	-8	-6	-7
03/05/2021	0	0	0	-1	0	-1	0
01/06/2021	0	0	0	1	0	0	0
01/07/2021	-1	-1	-1	-1	-1	-1	-1
02/08/2021	0	0	0	0	0	0	0
01/09/2021	0	0	0	0	0	0	0
01/10/2021	0	0	-1	0	1	0	0
01/11/2021	0	0	0	0	0	0	0
01/12/2021	2	1	0	2	2	1	0
03/01/2022	0	0	1	1	0	0	0
01/02/2022	0	3	2	1	2	1	3
01/03/2022	0	3	3	1	2	0	3
01/04/2022	2	1	1	1	1	0	1
02/05/2022	3	2	2	3	3	1	2
01/06/2022	2	3	3	2	3	2	3
01/07/2022	2	2	2	2	3	2	2
01/08/2022	0	0	0	0	0	0	0
01/09/2022	1	0	1	1	1	0	0
03/10/2022	1	0	0	1	0	1	0
01/11/2022	0	0	0	0	0	0	0
01/12/2022	-2	-1	0	-2	-2	-1	0
03/01/2023	0	0	-1	-1	0	0	0

The traffic light Monte Carlo is similar to the historical one, but the number of exceptions is higher. While the number of exceptions declines in early 2021, it increases significantly from the end of that year. In addition, the majority of the studied ETFs have an increasing number of exceedances, starting in February 2022.

2. Likelihood ratio proportion of failure test (LR-POF)

The tables represent the likelihood ratio of the POF test for the different VaR models and funds. The red color is used for the rejected models, where the result is nor asymptotically chi-squared distributed with one degree of freedom and a 5% confidence interval. The validated results are colored in green. When the number of exception is equal to 0, the POF model is not applicable, so the model is neither accepted or rejected because the computation cannot be done. However, it is not uncommon that the backtests of 99% VaR models on only 250 observations have no exceptions when markets are stable. They are generally accepted by practitioners and this is tolerated by the CSSF if it does not persist over time.

Table 12: Historical POF results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	7,644	7,644	3,499	7,644	7,644	3,499	3,499
01/03/2021	3,499	1,917	0,745	1,917	1,917	0,745	0,745
01/04/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
03/05/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/06/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/07/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
02/08/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/09/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/10/2021	0 exception	1,201	0 exception	0 exception	0 exception	0 exception	1,201
01/11/2021	0 exception	1,201	0 exception	0 exception	0 exception	0 exception	1,201
01/12/2021	1,201	1,201	0 exception	1,201	1,201	1,201	1,201
03/01/2022	1,201	1,201	0 exception	1,201	1,201	1,201	1,201
01/02/2022	1,201	0,117	1,201	1,201	1,201	1,201	0,117
01/03/2022	1,201	0,087	0,087	1,201	0,117	1,201	0,087
01/04/2022	0,117	0,745	0,745	0,117	0,087	1,201	0,745
02/05/2022	0,087	3,499	3,499	0,745	3,499	0,117	3,499
01/06/2022	3,499	10,123	7,644	5,424	10,123	0,745	10,123
01/07/2022	3,499	12,833	10,123	10,123	12,833	3,499	12,833
01/08/2022	3,499	12,833	10,123	10,123	12,833	3,499	12,833
01/09/2022	5,424	12,833	10,123	10,123	12,833	3,499	12,833
03/10/2022	7,644	12,833	12,833	10,123	15,752	3,499	12,833
01/11/2022	7,644	12,833	12,833	10,123	15,752	3,499	12,833
01/12/2022	5,424	12,833	12,833	7,644	12,833	1,917	12,833
03/01/2023	5,424	12,833	12,833	7,644	12,833	1,917	12,833

The proportion of failure test validates the model or is undetermined after March 2021 and until June 2022, except for the Vanguard Russell 2000 Index Fund and SPDR Dow Jones Industrial Average ETF Trust. The second one has its historical model rejected in September. Only the Vanguard Russell 2000 Index Fund has its VaR model accepted or undefined by this test for the complete period. This one seems less impacted by the crisis than the other funds. Indeed, it has much less exceedances than the other ETFs for the historical and Monte Carlo VaR models.

Table 13: Parametric POF results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	3,499	25,591	12,833	10,123	22,144	0,745	29,189
01/03/2021	1,917	15,752	10,123	10,123	15,752	0,117	18,860
01/04/2021	1,917	7,644	3,499	3,499	7,644	1,201	10,123
03/05/2021	1,917	7,644	3,499	3,499	7,644	1,201	10,123
01/06/2021	1,917	7,644	3,499	3,499	7,644	1,201	10,123
01/07/2021	0,745	3,499	1,917	1,917	3,499	0 exception	5,424
02/08/2021	1,917	3,499	1,917	1,917	3,499	0 exception	5,424
01/09/2021	1,917	3,499	1,917	1,917	3,499	0 exception	5,424
01/10/2021	0,745	0,745	0,745	0,745	0,745	0 exception	1,917
01/11/2021	0,087	0,087	0,087	0,117	0,117	0 exception	0,087
01/12/2021	0,745	0,087	0,087	0,117	0,087	0 exception	0,087
03/01/2022	0,745	0,117	0,117	0,117	0,117	0 exception	0,117
01/02/2022	0,087	0 exception	0 exception	0,117	1,201	0 exception	0 exception
01/03/2022	0,117	0 exception	0 exception	1,201	1,201	0 exception	0 exception
01/04/2022	0,117	1,201	0 exception	0,117	0,117	0 exception	0 exception
02/05/2022	0,087	1,201	0 exception	0,087	0,087	0 exception	0 exception
01/06/2022	0,745	1,201	0 exception	1,917	0,745	0 exception	0 exception
01/07/2022	0,745	0,117	0 exception	5,424	0,745	0,117	0 exception
01/08/2022	0,087	0,117	0 exception	5,424	0,745	0,117	0 exception
01/09/2022	0,745	0,087	1,201	7,644	1,917	0,087	1,201
03/10/2022	1,917	0,745	0,117	10,123	3,499	0,745	0,117
01/11/2022	1,917	0,745	0,117	10,123	3,499	0,745	0,117
01/12/2022	0,745	0,745	0,117	12,833	3,499	1,917	0,117
03/01/2023	0,745	1,917	0,087	15,752	5,424	1,917	0,087

The results are consistent with those of the traffic light. Vanguard Mid-Cap Index Fund has its parametric model rejected from 2022 while the other funds (except in January 2023 for Vanguard S&P 500 Index Fund) have their model accepted or undefined, because the number of exceedance is

0. Vanguard Russell 2000 however has many months where it has no exceptions in its backtest sample, while the markets were very disturbed. It could be that its VaR level is underestimated and therefore its risk overestimated.

Table 14: Monte-Carlo POF results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	18,860	22,144	18,860	25,591	18,860	15,752	18,860
01/03/2021	10,123	12,833	10,123	15,752	10,123	7,644	10,123
01/04/2021	1,201	0,117	0,087	0,117	1,201	0,117	0,117
03/05/2021	1,201	0,117	0,087	1,201	1,201	1,201	0,117
01/06/2021	1,201	0,117	0,087	0,117	1,201	1,201	0,117
01/07/2021	0 exception	1,201	0,117	1,201	0 exception	0 exception	1,201
02/08/2021	0 exception	1,201	0,117	1,201	0 exception	0 exception	1,201
01/09/2021	0 exception	1,201	0,117	1,201	0 exception	0 exception	1,201
01/10/2021	0 exception	1,201	1,201	1,201	1,201	0 exception	1,201
01/11/2021	0 exception	1,201	1,201	1,201	1,201	0 exception	1,201
01/12/2021	0,117	0,117	1,201	0,087	0,087	1,201	1,201
03/01/2022	0,117	0,117	0,117	0,745	0,087	1,201	1,201
01/02/2022	0,117	1,917	0,745	1,917	1,917	0,117	0,745
01/03/2022	0,117	7,644	5,424	3,499	5,424	0,117	5,424
01/04/2022	0,745	10,123	7,644	5,424	7,644	0,117	7,644
02/05/2022	5,424	15,752	12,833	12,833	15,752	0,087	12,833
01/06/2022	10,123	25,591	22,144	18,860	25,591	1,917	22,144
01/07/2022	15,752	32,928	29,189	25,591	36,802	5,424	29,189
01/08/2022	15,752	32,928	29,189	25,591	36,802	5,424	29,189
01/09/2022	18,860	32,928	32,928	29,189	40,801	5,424	29,189
03/10/2022	22,144	32,928	32,928	32,928	40,801	7,644	29,189
01/11/2022	22,144	32,928	32,928	32,928	40,801	7,644	29,189
01/12/2022	15,752	29,189	32,928	25,591	32,928	5,424	29,189
03/01/2023	15,752	29,189	29,189	22,144	32,928	5,424	29,189

In 2022, the market risk has clearly been underestimated by this model because the POF test does not validate it for any fund over the period studied.

3. Christoffersen test (LR-Chris)

Similarly to the above POF test, the accepted Christoffersen tests are in green and the rejected ones in red. This verifies whether the exceptions are independent or not. In fact, this ensure whether overshoots are consecutive or not, because it does not take in account the elapsed time between each of them. Except in early 2021 for the historical and Monte-Carlo models, the overshoots were independent according to the test results and this requirement is satisfied.

Table 15: Historical Chris results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	5,717	5,717	8,244	5,717	5,717	8,244	8,244
01/03/2021	2,480	3,206	4,153	3,206	3,206	4,153	4,153
01/04/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
03/05/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/06/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/07/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
02/08/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/09/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/10/2021	0 exception	0,008	0 exception	0 exception	0 exception	0 exception	0,008
01/11/2021	0 exception	0,008	0 exception	0 exception	0 exception	0 exception	0,008
01/12/2021	0,008	0,008	0 exception	0,008	0,008	0,008	0,008
03/01/2022	0,008	0,008	0 exception	0,008	0,008	0,008	0,008
01/02/2022	0,008	0,032	0,008	0,008	0,008	0,008	0,032
01/03/2022	0,008	0,071	0,071	0,008	0,032	0,008	0,071
01/04/2022	0,032	0,127	0,127	0,032	0,071	0,008	0,127
02/05/2022	0,071	0,286	0,286	0,127	0,286	0,032	0,286
01/06/2022	0,286	0,643	0,508	0,389	0,643	0,127	0,643
01/07/2022	0,286	0,794	0,643	0,643	0,794	0,286	0,794
01/08/2022	0,286	0,794	0,643	0,643	0,794	0,286	0,794
01/09/2022	0,389	0,794	0,643	0,643	0,794	0,286	0,794
03/10/2022	0,508	0,794	0,794	0,643	0,961	0,286	0,794
01/11/2022	0,508	0,794	0,794	0,643	0,961	0,286	0,794
01/12/2022	0,389	0,794	0,794	0,508	0,794	0,198	0,794
03/01/2023	0,389	0,794	0,794	0,508	0,794	0,198	0,794

Table 16: Parametric Chris results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	0,286	0,093	0,767	1,070	0,194	4,153	0,000
01/03/2021	0,198	0,526	1,070	1,070	0,526	7,526	0,000
01/04/2021	0,198	0,508	0,286	0,286	0,508	0,008	0,000
03/05/2021	0,198	0,508	0,286	0,286	0,508	0,008	0,000
01/06/2021	0,198	0,508	0,286	0,286	0,508	0,008	0,000
01/07/2021	0,127	0,286	0,198	0,198	0,286	0 exception	0,000
02/08/2021	0,198	0,286	0,198	0,198	0,286	0 exception	0,000
01/09/2021	0,198	0,286	0,198	0,198	0,286	0 exception	0,000
01/10/2021	0,127	0,127	0,127	0,127	0,127	0 exception	0,000
01/11/2021	0,071	0,071	0,071	0,032	0,032	0 exception	0,000
01/12/2021	0,127	0,071	0,071	0,032	0,071	0 exception	0,000
03/01/2022	0,127	0,032	0,032	0,032	0,032	0 exception	0,000
01/02/2022	0,071	0 exception	0 exception	0,032	0,008	0 exception	0 exception
01/03/2022	0,032	0 exception	0 exception	0,008	0,008	0 exception	0 exception
01/04/2022	0,032	0,008	0 exception	0,032	0,032	0 exception	0 exception
02/05/2022	0,071	0,008	0 exception	0,071	0,071	0 exception	0 exception
01/06/2022	0,127	0,008	0 exception	0,198	0,127	0 exception	0 exception
01/07/2022	0,127	0,032	0 exception	0,389	0,127	0,032	0 exception
01/08/2022	0,071	0,032	0 exception	0,389	0,127	0,032	0 exception
01/09/2022	0,127	0,071	0,008	0,508	0,198	0,071	0,001
03/10/2022	0,198	0,127	0,032	0,643	0,286	0,127	0,000
01/11/2022	0,198	0,127	0,032	0,643	0,286	0,127	0,000
01/12/2022	0,127	0,127	0,032	0,794	0,286	0,198	0,000
03/01/2023	0,127	0,198	0,071	0,961	0,389	0,198	0,000

Table 17: Monte-Carlo Chris results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	2,659	2,144	2,659	1,702	2,659	3,256	2,659
01/03/2021	4,762	3,951	4,762	3,256	4,762	5,717	4,762
01/04/2021	0,008	0,032	0,071	0,032	0,008	0,032	0,032
03/05/2021	0,008	0,032	0,071	0,008	0,008	0,008	0,032
01/06/2021	0,008	0,032	0,071	0,032	0,008	0,008	0,032
01/07/2021	0 exception	0,008	0,032	0,008	0 exception	0 exception	0,008
02/08/2021	0 exception	0,008	0,032	0,008	0 exception	0 exception	0,008
01/09/2021	0 exception	0,008	0,032	0,008	0 exception	0 exception	0,008
01/10/2021	0 exception	0,008	0,008	0,008	0,008	0 exception	0,008
01/11/2021	0 exception	0,008	0,008	0,008	0,008	0 exception	0,008
01/12/2021	0,032	0,032	0,008	0,071	0,071	0,008	0,008
03/01/2022	0,032	0,032	0,032	0,127	0,071	0,008	0,008
01/02/2022	0,032	0,198	0,127	0,198	0,198	0,032	0,127
01/03/2022	0,032	0,508	0,389	0,286	0,389	0,032	0,389
01/04/2022	0,127	0,643	0,508	0,389	0,508	0,032	0,508
02/05/2022	0,389	0,961	0,794	0,794	0,961	0,071	0,794
01/06/2022	0,643	1,556	1,342	1,143	1,556	0,198	1,342
01/07/2022	0,526	2,033	1,787	1,556	2,295	0,389	1,787
01/08/2022	0,526	2,033	1,787	1,556	2,295	0,389	1,787
01/09/2022	0,337	2,033	2,033	1,787	2,574	0,389	1,787
03/10/2022	0,194	2,033	2,033	2,033	2,574	0,508	1,787
01/11/2022	0,194	2,033	2,033	2,033	2,574	0,508	1,787
01/12/2022	0,526	1,787	2,033	1,556	2,033	0,389	1,787
03/01/2023	0,526	1,787	1,787	1,342	2,033	0,389	1,787

4. Joint test (LR-joint)

The joint test is the addition of the Kupiec's POF test and the Christoffersen one. The fact that the violations were independent increased the success of this test, but overall, the results are consistent with those of POF. Indeed, a high number of exceptions causes this test to be rejected for some funds.

Table 18: Historical joint test

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	13,361	13,361	11,743	13,361	13,361	11,743	11,743
01/03/2021	5,979	5,122	4,898	5,122	5,122	4,898	4,898
01/04/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
03/05/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/06/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/07/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
02/08/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/09/2021	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception	0 exception
01/10/2021	0 exception	1,209	0 exception	0 exception	0 exception	0 exception	1,209
01/11/2021	0 exception	1,209	0 exception	0 exception	0 exception	0 exception	1,209
01/12/2021	1,209	1,209	0 exception	1,209	1,209	1,209	1,209
03/01/2022	1,209	1,209	0 exception	1,209	1,209	1,209	1,209
01/02/2022	1,209	0,148	1,209	1,209	1,209	1,209	0,148
01/03/2022	1,209	0,158	0,158	1,209	0,148	1,209	0,158
01/04/2022	0,148	0,872	0,872	0,148	0,158	1,209	0,872
02/05/2022	0,158	3,785	3,785	0,872	3,785	0,148	3,785
01/06/2022	3,785	10,766	8,152	5,813	10,766	0,872	10,766
01/07/2022	3,785	13,627	10,766	10,766	13,627	3,785	13,627
01/08/2022	3,785	13,627	10,766	10,766	13,627	3,785	13,627
01/09/2022	5,813	13,627	10,766	10,766	13,627	3,785	13,627
03/10/2022	8,152	13,627	13,627	10,766	16,712	3,785	13,627
01/11/2022	8,152	13,627	13,627	10,766	16,712	3,785	13,627
01/12/2022	5,813	13,627	13,627	8,152	13,627	2,115	13,627
03/01/2023	5,813	13,627	13,627	8,152	13,627	2,115	13,627

Table 19: Parametric joint results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	3,785	0,093	13,601	11,193	22,339	4,898	29,219
01/03/2021	2,115	0,526	11,193	11,193	16,277	7,642	19,197
01/04/2021	2,115	0,508	3,785	3,785	8,152	1,209	10,766
03/05/2021	2,115	0,508	3,785	3,785	8,152	1,209	10,766
01/06/2021	2,115	0,508	3,785	3,785	8,152	1,209	10,766
01/07/2021	0,872	0,286	2,115	2,115	3,785	0 exception	5,813
02/08/2021	2,115	0,286	2,115	2,115	3,785	0 exception	5,813
01/09/2021	2,115	0,286	2,115	2,115	3,785	0 exception	5,813
01/10/2021	0,872	0,127	0,872	0,872	0,872	0 exception	2,115
01/11/2021	0,158	0,071	0,158	0,148	0,148	0 exception	0,158
01/12/2021	0,872	0,071	0,158	0,148	0,158	0 exception	0,158
03/01/2022	0,872	0,032	0,148	0,148	0,148	0 exception	0,148
01/02/2022	0,158	0 exception	0 exception	0,148	1,209	0 exception	0 exception
01/03/2022	0,148	0 exception	0 exception	1,209	1,209	0 exception	0 exception
01/04/2022	0,148	0,008	0 exception	0,148	0,148	0 exception	0 exception
02/05/2022	0,158	0,008	0 exception	0,158	0,158	0 exception	0 exception
01/06/2022	0,872	0,008	0 exception	2,115	0,872	0 exception	0 exception
01/07/2022	0,872	0,032	0 exception	5,813	0,872	0,148	0 exception
01/08/2022	0,158	0,032	0 exception	5,813	0,872	0,148	0 exception
01/09/2022	0,872	0,071	1,209	8,152	2,115	0,158	1,209
03/10/2022	2,115	0,127	0,148	10,766	3,785	0,872	0,148
01/11/2022	2,115	0,127	0,148	10,766	3,785	0,872	0,148
01/12/2022	0,872	0,127	0,148	13,627	3,785	2,115	0,148
03/01/2023	0,872	0,198	0,158	16,712	5,813	2,115	0,158

Table 20: Monte-Carlo joint results

Date	Dia	IWF	QQQ.O	VO	VooiV.P	VTWO.O	VUG
01/02/2021	21,519	24,289	21,519	27,293	21,519	19,008	21,519
01/03/2021	14,885	16,784	14,885	19,008	14,885	13,361	14,885
01/04/2021	1,209	0,148	0,158	0,148	1,209	0,148	0,148
03/05/2021	1,209	0,148	0,158	1,209	1,209	1,209	0,148
01/06/2021	1,209	0,148	0,158	0,148	1,209	1,209	0,148
01/07/2021	0 exception	1,209	0,148	1,209	0 exception	0 exception	1,209
02/08/2021	0 exception	1,209	0,148	1,209	0 exception	0 exception	1,209
01/09/2021	0 exception	1,209	0,148	1,209	0 exception	0 exception	1,209
01/10/2021	0 exception	1,209	1,209	1,209	1,209	0 exception	1,209
01/11/2021	0 exception	1,209	1,209	1,209	1,209	0 exception	1,209
01/12/2021	0,148	0,148	1,209	0,158	0,158	1,209	1,209
03/01/2022	0,148	0,148	0,148	0,872	0,158	1,209	1,209
01/02/2022	0,148	2,115	0,872	2,115	2,115	0,148	0,872
01/03/2022	0,148	8,152	5,813	3,785	5,813	0,148	5,813
01/04/2022	0,872	10,766	8,152	5,813	8,152	0,148	8,152
02/05/2022	5,813	16,712	13,627	13,627	16,712	0,158	13,627
01/06/2022	10,766	27,147	23,486	20,004	27,147	2,115	23,486
01/07/2022	16,277	34,962	30,975	27,147	39,097	5,813	30,975
01/08/2022	16,277	34,962	30,975	27,147	39,097	5,813	30,975
01/09/2022	19,197	34,962	34,962	30,975	43,375	5,813	30,975
03/10/2022	22,339	34,962	34,962	34,962	43,375	8,152	30,975
01/11/2022	22,339	34,962	34,962	34,962	43,375	8,152	30,975
01/12/2022	16,277	30,975	34,962	27,147	34,962	5,813	30,975
03/01/2023	16,277	30,975	30,975	23,486	34,962	5,813	30,975

5. Haas test (LR-Haas)

This test also checks the independence of the exceptions, but it is more precise because it studies the time elapsing between each of them in days and does not simply take in account the number of consecutive exceptions. The number of overshoots also matters because the Haas test is the sum of all TUFF ratio minus the first one. As a result, only the parametric model was robust in 2022 for independence.

Table 21: Haas test results

	Historical			Parametric			Monte Carlo		
	2020	2021	2022	2020	2021	2022	2020	2021	2022
Dia	47,20243	3,708286	15,8749	6,676598	0,006935	4,660811	67,98825	6,457852	39,05092
IWF	47,20243	0,123536	24,74601	43,00394	0	3,338735	68,2264	9,952745	40,02927
QQQ.O	30,47863	4,12907	22,26727	21,06809	0	2,862156	61,71214	5,393116	42,92635
VO	47,20243	3,739195	27,354	23,37173	0,80779	14,97041	82,08198	16,4757	40,78738
VooiV.P	47,20243	3,692847	31,28311	37,44396	0,732512	7,351412	67,98825	2,300683	52,3241
VTWO.O	31,50502	3,708286	16,83743	11,78712	0	9,327755	56,96195	1,178787	17,06178
VUG	30,47863	0,123536	24,74601	49,59193	0	2,862156	61,73917	8,314364	40,02927

Multinominal Backtesting

Following the methodology described in the Kratz, M., Lok, Y. H., and McNeil, A. J. paper, the LRT was computed for all funds and all studied period. First, a multinominal traffic light test was performed with the parameters present in the table below. Then, the LRT and its p-value were computed. This last one corresponds to the chi-square distribution with N degree of freedom. The null hypothesis is rejected when this value is too small. This means that the observed probabilities are not consistent with the estimated ones. When the p-value is equal to 1, this meaned that the observed probability was lower than the expected one, so the LRT calculation returned a negative result which cannot be chi-squared distributed. This holds if the number of exception is equal to 0 for all levels as well.

Table 22: Multinominal Basel traffic light test

VaR confidence interval	0,95	0,975	0,98125	0,9875	0,99375	0,99
Probability	0,05	0,025	0,01875	0,0125	0,00625	0,01
Yellow zone	19	11	8	6	4	5
Red zone	38	22	18	15	11	13

Dia	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	16	12	10	10	7	8	0,0000
01/03/2021	13	9	7	7	4	6	0,0000
01/04/2021	3	1	1	1	0	0	0,0446
03/05/2021	2	1	1	1	0	0	0,0446
01/06/2021	3	1	1	1	0	0	0,0446
01/07/2021	3	0	0	0	0	0	1,0000
02/08/2021	4	0	0	0	0	0	1,0000
01/09/2021	4	0	0	0	0	0	1,0000
01/10/2021	7	0	0	0	0	0	1,0000
01/11/2021	6	0	0	0	0	0	1,0000
01/12/2021	8	2	1	1	1	1	0,0032
03/01/2022	9	2	1	1	1	1	0,0032
01/02/2022	10	2	1	1	1	1	0,0032
01/03/2022	12	3	1	1	1	1	0,0012
01/04/2022	15	5	3	3	2	2	0,0000
02/05/2022	18	8	6	6	4	3	0,0000
01/06/2022	20	10	8	8	6	6	0,0000
01/07/2022	23	12	10	9	6	6	0,0000
01/08/2022	22	12	10	9	6	6	0,0000
01/09/2022	24	13	11	10	7	7	0,0000
03/10/2022	22	14	12	11	8	8	0,0000
01/11/2022	23	14	12	11	8	8	0,0000
01/12/2022	21	12	11	10	7	7	0,0000
03/01/2023	21	12	11	10	7	7	0,0000

IWF	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	14	12	10	6	8	0,0000
01/03/2021	15	11	9	7	4	5	0,0000
01/04/2021	6	3	2	1	0	0	0,0057
03/05/2021	4	2	2	1	0	0	0,0132
01/06/2021	5	2	2	1	0	0	0,0132
01/07/2021	4	1	1	0	0	0	0,2666
02/08/2021	4	1	1	0	0	0	0,2666
01/09/2021	4	1	1	0	0	0	0,2666
01/10/2021	3	1	1	1	0	1	0,0787
01/11/2021	3	1	1	1	0	1	0,0787
01/12/2021	4	2	1	1	0	1	0,0350
03/01/2022	5	3	1	1	0	1	0,0153
01/02/2022	10	5	3	2	1	2	0,0000
01/03/2022	14	8	6	4	2	3	0,0000
01/04/2022	15	9	7	5	3	4	0,0000
02/05/2022	19	12	10	7	5	6	0,0000
01/06/2022	22	15	13	10	7	9	0,0000
01/07/2022	25	18	14	11	7	10	0,0000
01/08/2022	25	18	14	11	7	10	0,0000
01/09/2022	26	18	14	11	7	10	0,0000
03/10/2022	26	18	14	11	8	10	0,0000
01/11/2022	26	18	14	11	8	10	0,0000
01/12/2022	25	17	14	11	8	10	0,0000
03/01/2023	24	16	14	11	8	10	0,0000

QQQ.O	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	14	9	7	5	6	0,0000
01/03/2021	16	11	7	5	3	4	0,0000
01/04/2021	8	4	2	1	0	0	0,0054
03/05/2021	6	3	2	1	0	0	0,0097
01/06/2021	6	3	2	1	0	0	0,0097
01/07/2021	5	2	1	1	0	0	0,0492
02/08/2021	5	2	1	1	0	0	0,0492
01/09/2021	5	2	1	1	0	0	0,0492
01/10/2021	4	1	0	0	0	0	0,7398
01/11/2021	3	1	0	0	0	0	0,7398
01/12/2021	3	1	0	0	0	0	0,7398
03/01/2022	4	1	0	0	0	0	0,7398
01/02/2022	8	2	1	1	1	1	0,0083
01/03/2022	11	5	3	3	2	3	0,0000
01/04/2022	11	6	4	4	3	4	0,0000
02/05/2022	14	8	6	6	5	6	0,0000
01/06/2022	18	11	9	9	7	8	0,0000
01/07/2022	22	13	10	10	7	9	0,0000
01/08/2022	22	13	10	10	7	9	0,0000
01/09/2022	23	14	11	10	7	9	0,0000
03/10/2022	23	14	12	11	8	10	0,0000
01/11/2022	24	14	12	11	8	10	0,0000
01/12/2022	24	14	12	11	8	10	0,0000
03/01/2023	23	14	12	11	8	10	0,0000

VO	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	18	12	10	9	7	8	0,0000
01/03/2021	15	9	7	6	4	5	0,0000
01/04/2021	5	2	1	0	0	0	0,1561
03/05/2021	4	1	1	0	0	0	0,2715
01/06/2021	4	2	1	0	0	0	0,1561
01/07/2021	2	1	0	0	0	0	0,7256
02/08/2021	3	1	0	0	0	0	0,7256
01/09/2021	3	1	0	0	0	0	0,7256
01/10/2021	4	2	0	0	0	0	0,4523
01/11/2021	4	2	0	0	0	0	0,4523
01/12/2021	6	4	2	2	1	1	0,0001
03/01/2022	8	5	2	2	1	1	0,0001
01/02/2022	12	6	3	2	1	1	0,0000
01/03/2022	16	7	4	2	1	1	0,0000
01/04/2022	18	8	5	3	2	2	0,0000
02/05/2022	22	11	8	5	3	4	0,0000
01/06/2022	24	13	11	8	6	7	0,0000
01/07/2022	28	16	13	10	8	9	0,0000
01/08/2022	27	16	13	10	8	9	0,0000
01/09/2022	28	17	13	10	8	9	0,0000
03/10/2022	27	17	14	11	8	9	0,0000
01/11/2022	29	17	14	11	8	9	0,0000
01/12/2022	28	15	12	9	7	8	0,0000
03/01/2023	26	14	12	9	7	8	0,0000

VTWO.O	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	13	11	8	6	6	0,0000
01/03/2021	17	10	8	6	4	4	0,0000
01/04/2021	8	3	2	0	0	0	0,0147
03/05/2021	5	1	1	0	0	0	0,2350
01/06/2021	4	1	1	0	0	0	0,2350
01/07/2021	3	0	0	0	0	0	1,0000
02/08/2021	3	0	0	0	0	0	1
01/09/2021	3	0	0	0	0	0	1
01/10/2021	5	0	0	0	0	0	1
01/11/2021	5	0	0	0	0	0	1
01/12/2021	6	1	1	1	1	1	0,0367
03/01/2022	9	1	1	1	1	1	0,0367
01/02/2022	11	3	2	2	1	1	0,0007
01/03/2022	11	3	2	2	1	1	0,0007
01/04/2022	11	3	2	2	1	1	0,0007
02/05/2022	15	6	4	3	1	2	0,0000
01/06/2022	18	9	7	5	3	4	0,0000
01/07/2022	21	11	9	7	5	6	0,0000
01/08/2022	21	11	9	7	5	6	0,0000
01/09/2022	22	11	9	7	5	6	0,0000
03/10/2022	21	12	10	7	5	6	0,0000
01/11/2022	22	12	10	7	5	6	0,0000
01/12/2022	22	11	9	6	4	5	0,0000
03/01/2023	20	11	9	6	4	5	0,0000

VooiV.P	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	13	10	10	8	8	0,0000
01/03/2021	16	10	7	7	5	5	0,0000
01/04/2021	5	2	1	1	0	0	0,0193
03/05/2021	3	1	1	1	0	0	0,0496
01/06/2021	4	1	1	1	0	0	0,0496
01/07/2021	3	0	0	0	0	0	1,0000
02/08/2021	4	0	0	0	0	0	1,0000
01/09/2021	4	0	0	0	0	0	1,0000
01/10/2021	5	1	1	0	0	0	0,1984
01/11/2021	4	1	1	0	0	0	0,1984
01/12/2021	6	3	2	1	0	1	0,0022
03/01/2022	6	3	2	1	0	1	0,0022
01/02/2022	10	4	2	1	0	1	0,0008
01/03/2022	15	6	4	2	1	2	0,0000
01/04/2022	16	7	5	3	2	3	0,0000
02/05/2022	19	10	8	6	5	6	0,0000
01/06/2022	21	13	11	9	7	9	0,0000
01/07/2022	25	16	13	10	8	10	0,0000
01/08/2022	24	16	13	10	8	10	0,0000
01/09/2022	25	17	14	10	8	10	0,0000
03/10/2022	24	17	14	11	9	11	0,0000
01/11/2022	25	17	14	11	9	11	0,0000
01/12/2022	23	15	13	10	9	10	0,0000
03/01/2023	23	15	13	10	9	10	0,0000

VUG	Historical						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	20	14	10	8	5	6	0,0000
01/03/2021	16	11	7	6	3	4	0,0000
01/04/2021	7	3	1	1	0	0	0,0097
03/05/2021	5	2	1	1	0	0	0,0251
01/06/2021	6	2	1	1	0	0	0,0251
01/07/2021	5	1	0	0	0	0	0,5589
02/08/2021	5	1	0	0	0	0	0,5589
01/09/2021	5	1	0	0	0	0	0,5589
01/10/2021	4	1	1	1	0	1	0,0637
01/11/2021	4	1	1	1	0	1	0,0637
01/12/2021	5	1	1	1	0	1	0,0637
03/01/2022	7	2	1	1	0	1	0,0251
01/02/2022	12	5	4	2	1	2	0,0000
01/03/2022	16	8	6	4	2	3	0,0000
01/04/2022	16	9	7	5	3	4	0,0000
02/05/2022	20	12	9	7	5	6	0,0000
01/06/2022	23	15	12	10	8	9	0,0000
01/07/2022	27	18	13	11	8	10	0,0000
01/08/2022	27	18	13	11	8	10	0,0000
01/09/2022	28	19	13	11	8	10	0,0000
03/10/2022	28	19	13	11	9	10	0,0000
01/11/2022	28	19	13	11	9	10	0,0000
01/12/2022	28	19	13	11	9	10	0,0000
03/01/2023	26	18	13	11	9	10	0,0000

Dia	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	12	9	7	6	5	6	1
01/03/2021	10	7	6	5	4	5	1
01/04/2021	7	6	6	5	4	5	1
03/05/2021	7	6	6	5	4	5	1
01/06/2021	9	7	7	6	4	5	1
01/07/2021	9	7	6	5	3	4	1
02/08/2021	10	8	7	6	3	5	1
01/09/2021	10	8	7	6	3	5	1
01/10/2021	10	8	7	5	2	4	1
01/11/2021	8	6	5	4	1	3	1
01/12/2021	9	7	6	5	2	4	1
03/01/2022	10	7	6	5	2	4	1
01/02/2022	9	6	5	4	1	3	1
01/03/2022	8	5	4	3	1	2	1
01/04/2022	9	5	4	3	1	2	1
02/05/2022	11	6	5	4	2	3	1
01/06/2022	11	6	5	4	3	4	1
01/07/2022	12	6	6	4	3	4	1
01/08/2022	11	5	5	3	3	3	1
01/09/2022	13	7	7	4	4	4	1
03/10/2022	13	7	7	5	5	5	1
01/11/2022	13	7	7	5	5	5	1
01/12/2022	13	6	6	4	4	4	1
03/01/2023	13	7	6	4	4	4	1

QQQ.O	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	25	19	17	13	7	10	0,0000
01/03/2021	21	16	14	11	6	9	0,0000
01/04/2021	18	13	11	8	5	6	0,0000
03/05/2021	17	12	10	8	5	6	0,0000
01/06/2021	16	12	10	8	5	6	0,0000
01/07/2021	13	9	8	7	4	5	0,0000
02/08/2021	11	8	7	7	4	5	0,0000
01/09/2021	10	8	7	7	4	5	0,0000
01/10/2021	7	5	5	5	3	4	0,0000
01/11/2021	4	3	3	3	2	3	0,0005
01/12/2021	4	3	3	3	2	3	0,0005
03/01/2022	3	2	2	2	1	2	0,0095
01/02/2022	0	0	0	0	0	0	1,0000
01/03/2022	1	1	1	0	0	0	0,5773
01/04/2022	2	2	2	0	0	0	0,2666
02/05/2022	4	3	2	0	0	0	0,2245
01/06/2022	6	4	2	0	0	0	0,1887
01/07/2022	8	5	2	0	0	0	0,1582
01/08/2022	8	5	2	0	0	0	0,1582
01/09/2022	11	6	3	1	1	1	0,0090
03/10/2022	13	7	4	2	2	2	0,0004
01/11/2022	14	8	5	2	2	2	0,0002
01/12/2022	15	8	5	2	2	2	0,0002
03/01/2023	16	9	6	3	2	3	0,0000

IWF	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	25	19	18	15	11	14	0,0000
01/03/2021	21	15	14	12	9	11	0,0000
01/04/2021	18	12	11	9	6	8	0,0000
03/05/2021	17	11	10	9	6	8	0,0000
01/06/2021	15	11	10	9	6	8	0,0000
01/07/2021	12	9	8	7	5	6	0,0000
02/08/2021	10	7	7	7	5	6	0,0000
01/09/2021	9	7	7	7	5	6	0,0000
01/10/2021	6	5	5	5	3	4	0,0013
01/11/2021	3	3	3	3	2	3	0,0175
01/12/2021	3	3	3	3	2	3	0,0175
03/01/2022	2	2	2	2	1	2	0,1365
01/02/2022	1	1	0	0	0	0	0,1681
01/03/2022	4	2	1	0	0	0	0,3663
01/04/2022	5	3	2	1	0	1	0,1426
02/05/2022	8	6	3	1	0	1	0,6599
01/06/2022	10	8	4	1	0	1	0,8332
01/07/2022	13	10	6	2	0	2	0,8254
01/08/2022	13	10	6	2	0	2	0,8254
01/09/2022	15	12	8	4	1	3	0,9026
03/10/2022	17	13	9	5	2	4	0,1604
01/11/2022	19	14	9	5	2	4	0,2321
01/12/2022	20	15	10	5	2	4	0,3278
03/01/2023	21	16	11	6	2	5	0,2220

VO	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	16	13	9	9	8	9	0,0000
01/03/2021	14	11	9	9	9	9	0,0000
01/04/2021	10	8	6	6	6	6	0,0000
03/05/2021	10	8	6	6	6	6	0,0000
01/06/2021	11	9	7	6	6	6	0,0000
01/07/2021	9	8	6	5	5	5	0,0000
02/08/2021	9	8	6	5	5	5	0,0000
01/09/2021	9	8	6	5	5	5	0,0000
01/10/2021	7	6	5	4	4	4	0,0250
01/11/2021	5	4	3	2	2	2	0,0652
01/12/2021	7	5	4	2	2	2	0,2118
03/01/2022	7	5	4	2	2	2	0,2118
01/02/2022	6	4	4	2	1	2	0,0528
01/03/2022	7	4	4	1	0	1	0,0640
01/04/2022	8	5	5	2	1	2	0,2149
02/05/2022	12	7	7	3	1	3	0,1230
01/06/2022	14	9	9	6	2	5	0,0000
01/07/2022	18	13	12	8	2	7	0,0000
01/08/2022	18	13	12	8	2	7	0,0000
01/09/2022	21	15	13	9	3	8	0,0000
03/10/2022	21	16	14	10	4	9	0,0000
01/11/2022	23	17	15	10	4	9	0,0000
01/12/2022	24	19	17	11	5	10	0,0000
03/01/2023	27	21	19	12	5	11	0,0000

VooiV.P	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	22	19	15	13	11	13	0,0000
01/03/2021	20	17	14	11	9	11	0,0000
01/04/2021	17	14	11	8	6	8	0,0000
03/05/2021	16	14	11	8	6	8	0,0000
01/06/2021	15	12	10	8	6	8	0,0000
01/07/2021	12	9	8	6	5	6	0,0000
02/08/2021	12	9	8	6	5	6	0,0000
01/09/2021	12	9	8	6	5	6	0,0000
01/10/2021	10	6	6	4	4	4	0,0087
01/11/2021	7	3	3	2	2	2	1
01/12/2021	8	4	4	3	3	3	1
03/01/2022	7	3	3	2	2	2	1
01/02/2022	6	3	2	1	1	1	1
01/03/2022	8	4	2	1	1	1	1
01/04/2022	10	5	3	2	2	2	1
02/05/2022	15	9	6	3	2	3	0,0190
01/06/2022	17	10	7	4	3	4	0,0001
01/07/2022	21	11	8	4	3	4	0,0000
01/08/2022	20	11	8	4	3	4	0,0000
01/09/2022	23	13	10	6	4	5	0,0000
03/10/2022	24	14	11	7	5	6	0,0000
01/11/2022	25	15	11	7	5	6	0,0000
01/12/2022	27	17	12	7	4	6	0,0000
03/01/2023	28	18	13	8	5	7	0,0000

VTWO.O	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	9	6	6	6	3	4	1
01/03/2021	7	5	5	4	1	2	1
01/04/2021	5	4	4	3	1	1	1
03/05/2021	5	4	4	3	1	1	1
01/06/2021	5	4	4	3	1	1	1
01/07/2021	4	3	3	2	0	0	1
02/08/2021	4	3	3	2	0	0	1
01/09/2021	4	3	3	2	0	0	1
01/10/2021	3	2	2	1	0	0	1
01/11/2021	2	1	1	0	0	0	1
01/12/2021	3	2	2	0	0	0	1
03/01/2022	3	2	2	0	0	0	1
01/02/2022	4	3	2	0	0	0	1
01/03/2022	3	2	1	0	0	0	1
01/04/2022	3	2	1	0	0	0	1
02/05/2022	4	2	1	0	0	0	1
01/06/2022	6	3	2	1	0	0	1
01/07/2022	9	5	4	3	2	2	1
01/08/2022	9	5	4	3	2	2	1
01/09/2022	10	6	5	4	3	3	1
03/10/2022	11	7	6	5	4	4	1
01/11/2022	12	7	6	5	4	4	1
01/12/2022	14	8	6	6	4	5	1
03/01/2023	16	9	7	6	4	5	1

Dia	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	14	13	13	12	12	12	0,0000
01/03/2021	11	10	10	9	9	9	0,0000
01/04/2021	2	2	2	1	1	1	0,0032
03/05/2021	1	1	1	1	1	1	0,0164
01/06/2021	2	1	1	1	1	1	0,0164
01/07/2021	2	0	0	0	0	0	1,0000
02/08/2021	3	1	1	1	0	0	0,1132
01/09/2021	3	1	1	1	0	0	0,1132
01/10/2021	6	4	2	2	0	0	0,0020
01/11/2021	6	4	2	2	0	0	0,0020
01/12/2021	8	6	4	4	2	2	0,0000
03/01/2022	11	7	5	4	2	2	0,0000
01/02/2022	13	8	5	4	2	2	0,0000
01/03/2022	19	10	6	4	2	2	0,0000
01/04/2022	22	12	8	6	3	4	0,0000
02/05/2022	25	15	11	9	6	7	0,0000
01/06/2022	27	18	14	11	8	9	0,0000
01/07/2022	30	21	17	14	10	11	0,0000
01/08/2022	29	20	16	13	10	11	0,0000
01/09/2022	31	21	17	14	11	12	0,0000
03/10/2022	29	19	17	14	12	13	0,0000
01/11/2022	30	19	17	14	12	13	0,0000
01/12/2022	28	17	15	12	10	11	0,0000
03/01/2023	26	16	14	12	10	11	0,0000

VTWO.O	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	15	14	14	11	13	0,0000
01/03/2021	15	11	11	11	8	10	0,0000
01/04/2021	6	3	3	3	1	2	0,0000
03/05/2021	4	2	2	2	1	2	0,0002
01/06/2021	5	3	2	2	1	2	0,0001
01/07/2021	4	2	1	1	0	1	0,0250
02/08/2021	4	2	1	1	0	1	0,0250
01/09/2021	4	2	1	1	0	1	0,0250
01/10/2021	4	2	1	1	1	1	0,0037
01/11/2021	4	3	1	1	1	1	0,0016
01/12/2021	6	4	2	2	1	2	0,0000
03/01/2022	9	5	3	3	1	2	0,0000
01/02/2022	14	10	8	6	3	5	0,0000
01/03/2022	19	14	12	9	5	8	0,0000
01/04/2022	21	15	13	10	6	9	0,0000
02/05/2022	26	18	16	13	8	11	0,0000
01/06/2022	29	21	19	16	11	14	0,0000
01/07/2022	34	24	22	19	12	16	0,0000
01/08/2022	34	24	22	19	12	16	0,0000
01/09/2022	35	25	23	19	12	16	0,0000
03/10/2022	34	25	23	19	12	16	0,0000
01/11/2022	34	24	23	19	12	16	0,0000
01/12/2022	33	23	22	18	12	15	0,0000
03/01/2023	30	22	21	17	12	15	0,0000

QQQ.O	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	19	15	15	12	10	12	0,0000
01/03/2021	15	11	11	9	7	9	0,0000
01/04/2021	7	4	4	3	1	3	0,0000
03/05/2021	5	3	3	3	1	3	0,0000
01/06/2021	7	3	3	3	1	3	0,0000
01/07/2021	6	2	2	2	1	2	0,0008
02/08/2021	6	2	2	2	1	2	0,0008
01/09/2021	6	2	2	2	1	2	0,0008
01/10/2021	6	1	1	1	0	1	0,1014
01/11/2021	6	1	1	1	0	1	0,1014
01/12/2021	7	1	1	1	0	1	0,1014
03/01/2022	10	2	2	2	0	2	0,0060
01/02/2022	15	4	6	5	1	4	0,0000
01/03/2022	20	7	10	8	3	7	0,0000
01/04/2022	21	8	11	9	4	8	0,0000
02/05/2022	28	10	13	11	6	10	0,0000
01/06/2022	30	13	16	14	9	13	0,0000
01/07/2022	36	15	19	16	10	15	0,0000
01/08/2022	36	15	19	16	10	15	0,0000
01/09/2022	37	16	20	17	10	16	0,0000
03/10/2022	36	16	20	17	11	16	0,0000
01/11/2022	36	16	20	17	11	16	0,0000
01/12/2022	36	16	20	17	11	16	0,0000
03/01/2023	33	15	19	16	11	15	0,0000

VUG	Parametric						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	27	20	18	15	12	15	0,0000
01/03/2021	23	16	14	12	9	12	0,0000
01/04/2021	20	13	11	9	6	9	0,0000
03/05/2021	19	12	10	9	6	9	0,0000
01/06/2021	17	12	10	9	6	9	0,0000
01/07/2021	14	9	8	7	5	7	0,0000
02/08/2021	12	7	7	7	5	7	0,0000
01/09/2021	11	7	7	7	5	7	0,0000
01/10/2021	7	5	5	5	3	5	0,0000
01/11/2021	3	3	3	3	2	3	0,0004
01/12/2021	3	3	3	3	2	3	0,0004
03/01/2022	2	2	2	2	1	2	0,0124
01/02/2022	1	1	0	0	0	0	0,9680
01/03/2022	4	2	1	0	0	0	0,8942
01/04/2022	5	3	2	1	0	0	0,2490
02/05/2022	8	4	3	1	0	0	0,1729
01/06/2022	10	5	3	1	0	0	0,1928
01/07/2022	13	6	3	1	0	0	0,2148
01/08/2022	13	6	3	1	0	0	0,2148
01/09/2022	16	8	4	2	1	1	0,0089
03/10/2022	18	9	5	3	2	2	0,0003
01/11/2022	20	10	5	3	2	2	0,0003
01/12/2022	21	11	6	3	2	2	0,0002
03/01/2023	22	12	7	4	2	3	0,0000

VO	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	14	14	14	14	12	14	0,0000
01/03/2021	11	11	11	11	9	11	0,0000
01/04/2021	2	2	2	2	1	2	0,0008
03/05/2021	1	1	1	1	1	1	0,0154
01/06/2021	2	2	2	2	1	2	0,0008
01/07/2021	1	1	1	1	0	1	0,0992
02/08/2021	2	1	1	1	0	1	0,0992
01/09/2021	2	1	1	1	0	1	0,0992
01/10/2021	4	3	2	2	0	1	0,0026
01/11/2021	4	3	2	2	0	1	0,0026
01/12/2021	6	5	4	4	2	3	0,0000
03/01/2022	8	6	5	5	3	4	0,0000
01/02/2022	12	9	7	7	4	5	0,0000
01/03/2022	17	12	10	8	4	6	0,0000
01/04/2022	19	13	11	9	5	7	0,0000
02/05/2022	23	16	14	12	7	10	0,0000
01/06/2022	25	18	16	14	10	12	0,0000
01/07/2022	29	22	19	16	12	14	0,0000
01/08/2022	28	22	19	16	12	14	0,0000
01/09/2022	29	23	20	17	12	15	0,0000
03/10/2022	28	22	20	17	13	16	0,0000
01/11/2022	30	22	20	17	13	16	0,0000
01/12/2022	29	20	18	15	11	14	0,0000
03/01/2023	27	19	17	14	10	13	0,0000

VooiV.P	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	18	13	13	13	12	12	0,0000
01/03/2021	15	10	10	10	9	9	0,0000
01/04/2021	4	2	2	2	1	1	0,0001
03/05/2021	3	1	1	1	1	1	0,0052
01/06/2021	4	2	1	1	1	1	0,0021
01/07/2021	3	1	0	0	0	0	0,5779
02/08/2021	4	1	0	0	0	0	0,5779
01/09/2021	4	1	0	0	0	0	0,5779
01/10/2021	5	3	2	1	0	1	0,0019
01/11/2021	4	3	2	1	0	1	0,0019
01/12/2021	6	5	4	3	2	3	0,0000
03/01/2022	6	5	4	3	2	3	0,0000
01/02/2022	10	8	7	6	2	5	0,0000
01/03/2022	15	13	12	8	3	7	0,0000
01/04/2022	18	14	13	9	4	8	0,0000
02/05/2022	22	17	16	12	7	11	0,0000
01/06/2022	24	19	19	15	10	14	0,0000
01/07/2022	28	23	22	18	12	17	0,0000
01/08/2022	27	23	22	18	12	17	0,0000
01/09/2022	28	24	23	19	13	18	0,0000
03/10/2022	27	23	22	19	14	18	0,0000
01/11/2022	28	24	22	19	14	18	0,0000
01/12/2022	27	22	20	17	12	16	0,0000
03/01/2023	27	22	20	17	12	16	0,0000

VUG	Monte Carlo						
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	LRT p-value
01/02/2021	20	15	15	13	11	12	0,0000
01/03/2021	16	11	11	10	8	9	0,0000
01/04/2021	6	3	3	3	1	2	0,0000
03/05/2021	4	2	2	2	1	2	0,0002
01/06/2021	6	3	2	2	1	2	0,0001
01/07/2021	5	2	1	1	0	1	0,0237
02/08/2021	5	2	1	1	0	1	0,0237
01/09/2021	5	2	1	1	0	1	0,0237
01/10/2021	5	2	1	1	1	1	0,0033
01/11/2021	5	3	1	1	1	1	0,0014
01/12/2021	7	4	2	2	1	1	0,0000
03/01/2022	10	6	4	3	1	1	0,0000
01/02/2022	15	11	9	7	4	4	0,0000
01/03/2022	20	15	13	10	6	7	0,0000
01/04/2022	22	16	14	11	7	8	0,0000
02/05/2022	27	19	17	13	9	10	0,0000
01/06/2022	29	22	20	16	12	13	0,0000
01/07/2022	34	25	23	19	13	15	0,0000
01/08/2022	34	25	23	19	13	15	0,0000
01/09/2022	35	26	24	20	13	15	0,0000
03/10/2022	34	26	24	20	13	15	0,0000
01/11/2022	34	25	24	20	13	15	0,0000
01/12/2022	33	24	23	19	13	15	0,0000
03/01/2023	30	22	21	18	13	15	0,0000

ESG and VaR models robustness analysis

The correlation between the ESG scores and the number of exceptions and POF results was studied for each fund. The Christoffersen test results are not in the scope of this test, because they were homogenous among the ETFs during 2022. Only few exceptions were following each other. Overall, the ESG scores presents a high positive correlation with the number of exceptions. Indeed, the number of exceptions increased during the period, just as the ESG scores. Only one fund has its social pillar negatively correlated with the VaR robustness indicators. This would have been coherent with the previous researches which identified this pillar as very important in the downside risk reduction.

There is a limit to this analysis: the ETFs are tracking a financial index. Therefore, the proportion of positions increases or decreases with their value. It is possible that best in class firms are more represented if their financial performances are better than their peers ones. The underlying index which is replicated is then evolving. The number of exceptions assigned to the fund at the beginning of the period is not necessarily representative of the number that would have been estimated with the new asset mix (with a higher ESG score), although this is still very representative of the US equity market, which is itself evolving.

1. Dia

Table 23: Dia, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	Historical					
	0,95	0,975	0,98125	0,9875	0,99375	0,99
ESG score	0,651	0,750	0,784	0,768	0,789	0,806
Environemental	0,911	0,910	0,895	0,901	0,892	0,871
Social	-0,256	-0,173	-0,145	-0,126	-0,085	-0,105
Governance	0,556	0,658	0,701	0,668	0,682	0,725
Correlation	**Parametric**					
	0,95	0,975	0,98125	0,9875	0,99375	0,99
ESG score	0,817	0,641	0,807	0,614	0,869	0,757
Environemental	0,847	0,497	0,761	0,552	0,833	0,772
Social	-0,037	0,051	-0,030	0,295	0,009	0,082
Governance	0,716	0,598	0,729	0,441	0,776	0,626
Correlation	**Monte Carlo**					
	0,95	0,975	0,98125	0,98750	0,99375	0,99
ESG score	0,604	0,532	0,677	0,681	0,773	0,759
Environemental	0,905	0,913	0,933	0,921	0,907	0,911
Social	-0,227	-0,343	-0,241	-0,252	-0,166	-0,169
Governance	0,482	0,428	0,576	0,592	0,686	0,667

Correlation	Historical 99% LR pof	Parametric 99% LR pof	Monte Carlo 99% LR pof
ESG score	0,886	0,750	0,814
Environemental	0,716	0,616	0,877
Social	0,021	0,170	-0,153
Governance	0,851	0,655	0,749

Table 24: Dia, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	77,44	75,31	80,81	73,81	1	3	2
01/02/2023	78,26	76,24	80,41	76,45	7	4	11
Evolution	1,05%	1,23%	-0,50%	3,58%	6	1	9

2. IWF

Table 25: IWF, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	Historical					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,968	0,967	0,966	0,980	0,979	0,991
Environemental	0,950	0,964	0,942	0,938	0,919	0,939
Social	0,971	0,972	0,964	0,974	0,974	0,985
Governance	0,937	0,947	0,930	0,952	0,931	0,968
Correlation	**Parametric**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,914	0,933	0,890	0,778	0,635	0,803
Environemental	0,755	0,778	0,710	0,552	0,357	0,593
Social	0,893	0,913	0,864	0,745	0,597	0,772
Governance	0,879	0,899	0,862	0,749	0,591	0,777
Correlation	**Monte Carlo**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,950	0,956	0,966	0,967	0,980	0,962
Environemental	0,972	0,958	0,943	0,964	0,938	0,946
Social	0,964	0,961	0,968	0,972	0,974	0,959
Governance	0,919	0,932	0,940	0,947	0,952	0,943

Correlation	Historical 99% LR pof	Parametric 99% LR pof	Monte Carlo 99% LR pof
ESG score	0,993	0,396	0,977
Environemental	0,899	0,289	0,947
Social	0,978	0,367	0,973
Governance	0,994	0,292	0,971

Table 26: IWF, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	70,87	65,09	75,08	69,18	2	0	5
03/01/2023	73,46	66,45	76,64	73,63	10	5	15
Evolution	3,65%	2,09%	2,07%	6,43%	8	5	10

3. QQQ.O

Table 27: QQQ.O, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	Historical					
	0,950	**0,975**	**0,981**	**0,9875**	**0,99375**	**0,99**
ESG score	0,977	0,967	0,976	0,974	0,966	0,966
Environemental	0,984	0,977	0,971	0,975	0,965	0,972
Social	0,937	0,926	0,942	0,940	0,938	0,931
Governance	0,970	0,957	0,971	0,965	0,956	0,956

Correlation	Parametric					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,930	0,936	0,783	0,671	0,691	0,671
Environemental	0,889	0,900	0,718	0,585	0,626	0,585
Social	0,914	0,920	0,787	0,669	0,679	0,669
Governance	0,939	0,941	0,796	0,703	0,723	0,703

Correlation	Monte Carlo					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,938	0,962	0,956	0,962	0,974	0,962
Environemental	0,985	0,980	0,977	0,980	0,975	0,980
Social	0,884	0,918	0,910	0,918	0,940	0,918
Governance	0,918	0,951	0,945	0,951	0,965	0,951

Correlation	Historical 99% LR pof	Parametric 99% LR pof	Monte Carlo 99% LR pof
ESG score	0,985	0,280	0,974
Environemental	0,963	0,332	0,987
Social	0,956	0,233	0,931
Governance	0,989	0,272	0,969

Table 28: QQQ.O, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	72,27	66,90	75,60	71,33	1	0	4
01/02/2023	75,51	68,86	77,51	76,28	10	3	15
Evolution	4,48%	2,93%	2,53%	6,94%	9	3	11

4. VO

Table 29: VO, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	Historical					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,835	0,776	0,793	0,784	0,782	0,778
Environemental	0,830	0,763	0,782	0,765	0,755	0,758
Social	0,837	0,775	0,792	0,781	0,783	0,779
Governance	0,832	0,781	0,797	0,792	0,792	0,786

Correlation	Parametric					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,852	0,846	0,847	0,837	0,829	0,841
Environemental	0,820	0,807	0,812	0,796	0,791	0,801
Social	0,840	0,833	0,837	0,827	0,809	0,829
Governance	0,859	0,855	0,856	0,850	0,841	0,853

Correlation	Monte Carlo					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,837	0,769	0,797	0,766	0,763	0,802
Environemental	0,834	0,757	0,787	0,759	0,748	0,792
Social	0,840	0,773	0,798	0,765	0,761	0,797
Governance	0,833	0,771	0,797	0,769	0,771	0,805

Correlation	Historical 99% LR pof	Parametric 99% LR pof	Monte Carlo 99% LR pof
ESG score	0,671	0,776	0,782
Environemental	0,623	0,721	0,766
Social	0,667	0,757	0,774
Governance	0,690	0,788	0,789

Table 30: VO, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	58,15	48,96	62,35	58,75	1	2	5
01/02/2023	64,14	54,45	66,87	66,78	8	11	13
Evolution	10,30%	11,22%	7,25%	13,67%	7	9	8

5. VooiV.P

Table 31: VOOIV.P, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	**Historical**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,950	0,974	0,982	0,979	0,975	0,979
Environemental	0,963	0,952	0,946	0,938	0,912	0,938
Social	0,892	0,907	0,926	0,938	0,935	0,938
Governance	0,916	0,954	0,963	0,958	0,962	0,958

Correlation	**Parametric**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,937	0,888	0,906	0,865	0,874	0,892
Environemental	0,811	0,733	0,754	0,701	0,737	0,735
Social	0,873	0,835	0,840	0,794	0,787	0,827
Governance	0,956	0,915	0,934	0,900	0,905	0,923

Correlation	**Monte Carlo**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,948	0,962	0,948	0,974	0,979	0,974
Environemental	0,951	0,959	0,968	0,952	0,935	0,952
Social	0,899	0,896	0,892	0,907	0,918	0,907
Governance	0,914	0,934	0,909	0,954	0,965	0,954

Correlation	**Historical 99% LR pof**	**Parametric 99% LR pof**	**Monte Carlo 99% LR pof**
ESG score	0,964	0,571	0,967
Environemental	0,877	0,292	0,935
Social	0,901	0,470	0,888
Governance	0,970	0,673	0,955

Table 32: VOOIV.P, variables evolution since the beginning of the war

	ESG Score	**Environment**	**Social**	**Governance**	**Hist_99%**	**Para_99%**	**MC_99%**
01/02/2022	71,69	68,34	75,89	67,87	1	1	5
01/02/2023	73,86	69,51	76,72	72,20	10	7	16
Evolution	3,01%	1,71%	1,09%	6,38%	9	6	11

6. VTWO.O

Table 33: VTWO.O, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	**Historical**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,8537756	0,869012781	0,872441793	0,802489458	0,7786089	0,802489
Environemental	0,1917816	0,222217046	0,199636479	0,09491257	0,0238714	0,094913
Social	0,693878	0,703848089	0,709529784	0,625522745	0,6065501	0,625523
Governance	0,9094933	0,921615151	0,927970392	0,877400041	0,8623281	0,8774

Correlation	**Parametric**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,9651844	0,941925316	0,931438177	0,945250203	0,9085283	0,927626
Environemental	0,4131708	0,353531554	0,297882451	0,317605715	0,2848405	0,354975
Social	0,9253411	0,901555146	0,863740897	0,872755427	0,8224819	0,876496
Governance	0,9605241	0,94600479	0,951525877	0,960896095	0,9333742	0,932614

Correlation	**Monte Carlo**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,8375833	0,896659253	0,89355878	0,872500815	0,7786089	0,875329
Environemental	0,1981541	0,226593228	0,185170557	0,169993076	0,0238714	0,183669
Social	0,6698002	0,741246836	0,742613337	0,713249784	0,6065501	0,714984
Governance	0,890663	0,946107059	0,946765315	0,931470346	0,8623281	0,93417

Correlation	**Historical 99% LR pof**	**Parametric 99% LR pof**	**Monte Carlo 99% LR pof**
ESG score	0,5312994	0,8174886	0,850421053
Environemental	-0,203398	0,569267393	0,145023835
Social	0,3677714	0,886475008	0,695779546
Governance	0,6450367	0,748326507	0,91467603

Table 34: VTWO.O, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	42,22	24,89	45,33	51,37	1	0	2
01/02/2023	44,60	25,94	46,63	55,60	5	5	7
Evolution	5,64%	4,22%	2,87%	8,23%	4	5	5

7. VUG

Table 35: VUG, correlation between the ESG score, its sub-pillar and the backtesting results

Correlation	**Historical**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,977	0,983	0,981	0,981	0,985	0,988
Environemental	0,980	0,987	0,984	0,984	0,977	0,981
Social	0,900	0,913	0,931	0,931	0,949	0,934
Governance	0,971	0,974	0,964	0,964	0,967	0,976

Correlation	**Parametric**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,921	0,874	0,816	0,718	0,637	0,616
Environemental	0,924	0,876	0,846	0,723	0,612	0,596
Social	0,898	0,870	0,849	0,712	0,602	0,609
Governance	0,895	0,841	0,763	0,690	0,634	0,600

Correlation	**Monte Carlo**					
	0,95	**0,975**	**0,98125**	**0,9875**	**0,99375**	**0,99**
ESG score	0,931	0,938	0,959	0,972	0,981	0,967
Environemental	0,952	0,943	0,965	0,971	0,984	0,960
Social	0,840	0,835	0,868	0,878	0,931	0,883
Governance	0,925	0,942	0,957	0,974	0,964	0,966

Correlation	**Historical 99% LR pof**	**Parametric 99% LR pof**	**Monte Carlo 99% LR pof**
ESG score	0,990	0,277	0,992
Environemental	0,956	0,273	0,983
Social	0,921	0,148	0,924
Governance	0,993	0,306	0,985

Table 36: VUG, variables evolution since the beginning of the war

	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
01/02/2022	70,378272	64,37988245	74,805641	68,5005544	2	0	4
01/02/2023	73,619752	66,72013866	76,732495	73,5022784	10	3	15
Evolution	4,61%	3,64%	2,58%	7,30%	8	3	11

Table 37: Summary, 01/02/2022

Fund	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
VTWO.O	42,22	24,89	45,33	51,37	1	0	2
VO	58,15	48,96	62,35	58,75	1	2	5
VUG	70,38	64,38	74,81	68,50	2	0	4
IWF	70,87	65,09	75,08	69,18	2	0	5
VooiV.P	71,69	68,34	75,89	67,87	1	1	5
QQQ.O	72,27	66,90	75,60	71,33	1	0	4
Dia	77,44	75,31	80,81	73,81	1	3	2

Table 38: Summary, 01/01/2023

Fund	ESG Score	Environment	Social	Governance	Hist_99%	Para_99%	MC_99%
VTWO.O	44,60	25,94	46,63	55,60	5	5	7
VO	64,14	54,45	66,87	66,78	8	11	13
IWF	73,46	66,45	76,64	73,63	10	5	15
VUG	73,62	66,72	76,73	73,50	10	3	15
VooiV.P	73,86	69,51	76,72	72,20	10	7	16
QQQ.O	75,51	68,86	77,51	76,28	10	3	15
Dia	78,26	76,24	80,41	76,45	7	4	11

The summary tables show that the fund with the lowest ESG score is the one which has the lowest number of exception for the period. Then, the best in class asset has the second lowest number of exceedances, but overall, the US ETFs with a higher ESG score did nor perform better than the lowest one concerning their VaR models robustness.

Discussion

The study conducted aimed to test two hypotheses related to the validity and robustness of VaR (Value at Risk) models in forecasting financial returns. The first hypothesis focused on the appropriate distribution assumption for capturing the VaR level of assets. The results indicated that the skewed student t distribution, implemented with a similar GARCH(1,1) model, was more suitable in capturing VaR levels compared to other distributions. On average, this distribution generated fewer exceptions, particularly for high confidence levels ranging from 98.75% to 99.375%. However, for lower VaR confidence intervals, such as 97.5% and 98.125%, the results varied. Surprisingly, for the 95% confidence interval, the normal distribution performed better. Despite this, since the 95% confidence interval is not regulatory, the hypothesis regarding the skewed student t distribution remains valid. However, if there were regulatory changes in that direction, further tests would be necessary to determine the most appropriate distribution.

The statement aligns with the understanding that extreme market movements can occur in finance and should not be considered as measurement errors. Thus, incorporating such movements into calculations is crucial, especially during crisis periods. The skewed student t distribution proves particularly fitting during such crisis periods.

The second hypothesis aimed to examine whether consecutive announcements by the Federal Reserve (FED) regarding inflation, as well as the crisis in Ukraine, caused exceptions in the VaR models due to market disturbances. The launch of the war between Russia and Ukraine led to historically high levels of inflation. To assess the impact of these events on the robustness of the VaR model, the backtesting results from 2022 were compared to those of the previous period, specifically 2021. If the number of exceptions in 2022 exceeded those in 2021, it would suggest a negative impact on the robustness of the VaR models caused by the Federal announcements. Several tests were conducted, including the Basel traffic light, which counts the number of exceptions to verify their consistency with the expected VaR levels. The results indicated that the occurrence of exceedances increased in 2022 compared to 2021 for both historical and Monte Carlo VaR models. Even the parametric VaR model experienced a growth in the number of exceptions, albeit at a later stage than the other models.

This variation in the number of overshoots added to the fact that the dates corresponded mostly to dates corresponding to a change in expected inflation or to announcements on interest rate increases suggests that these events had a direct impact on the failure of the VaR model. To confirm this, one need to look at the results of the other backtests. The Proportion of failure test ensures that the number of exceptions is consistent with the confidence level. For the validation of this hypothesis, we can consider that models with 0 exceptions are accepted, even if it requires more analysis and reflection to assess and compare the robustness of the models. Here, the purpose is to demonstrate the impact of the Ukrainian crisis on the number of exceptions and its impact the model acceptance.

The provided examples demonstrate specific instances where the occurrence of exceptions in VaR models aligns with significant market events and announcements by the Federal Reserve (FED) regarding inflation and interest rates. These events had a notable impact on investor sentiment and market performance.

On March 4, 2022, U.S. stocks experienced a sharp decline due to investor concerns surrounding Russia's invasion of Ukraine. The Dow Jones, Nasdaq, S&P 500, and VIX all suffered losses on that day, reflecting the uncertainty and potential implications of the conflict.

In early May 2022, when the FED committee met and announced a 50 basis point increase in interest rates, investors were apprehensive about its adequacy in addressing inflation. They were already anticipating a further 75 basis point increase, which led to market unease.

On June 9, 2022, the release of the U.S. consumer price index (CPI) data triggered a significant decline in stock markets. Investors feared that the FED would be compelled to raise interest rates more aggressively to combat inflation. This resulted in substantial losses for leading U.S. indexes such as the Stoxx 600, S&P 500, Dow Jones, and Nasdaq Composite.

Another notable event occurred on August 26, when Federal Reserve Chairman Jerome Powell's statements dashed hopes that the FED would soon ease rate hikes to tackle inflation. This led to a sharp drop in the S&P 500, with technology stocks being particularly affected.

Furthermore, on September 13, the stock market experienced a significant decline as it became apparent that inflation was not decelerating as anticipated. The S&P 500 recorded its largest drop since June 2020, indicating the impact of inflation concerns on market performance.

These examples support the hypothesis that consecutive announcements by the FED on inflation, coupled with the crisis in Ukraine, disrupted the markets and generated exceptions in VaR models. The dates of the exceptions coincide with these events, highlighting their influence on investor behavior and market volatility.

The analysis of the VaR models reveals that the historical model was generally accepted from March 2021 until June 2022, with the exception of the Vanguard Russell 2000 Index Fund and SPDR Dow Jones Industrial Average ETF Trust. The Vanguard Russell 2000 Index Fund had its VaR model accepted for the entire period and exhibited fewer exceptions compared to other ETFs, indicating it was less impacted by the crisis. On the other hand, the SPDR Dow Jones Industrial Average ETF Trust had its model rejected in September 2022.

The historical model's acceptance aligns with market movements during the analyzed period. Exceptions related to the COVID-19 crisis were observed at the beginning of 2021. The model was accepted throughout the rest of the year but began to be rejected following key interest rate hike announcements and rising inflation. The results of the parametric VaR backtest are consistent with the traffic light test, except for the Vanguard Mid-Cap Index Fund, where the model is rejected until January 2023. However, the parametric model does not specifically highlight the impact of the war in Ukraine on the US markets.

Between March and July 2022, all funds had their Monte Carlo models rejected, and the dates of exceptions strongly corresponded to the market disruption caused by the inflation generated by the war in Ukraine. Although the Monte Carlo model is not robust, its results confirm the impact of inflation and FED announcements on the number of exceptions that cause the VaR models to fail. The parametric model also experienced an increase in exceptions in 2022, although it remained below the levels of the historical and Monte Carlo models.

To validate the third hypothesis and ensure the overall validity of the backtesting procedure, additional tests such as the Christoffersen, Haas, and joint tests should be performed. These tests assess the independence of exceedances and provide further insight into the robustness of the models.

It is important to note that models without exceptions cannot be considered accepted or rejected. This is a limitation of the backtesting methods used, as they rely on log-likelihood estimators that cannot be applied when the number of exceptions is zero. For instance, in the Kupiec test, if no exceptions occur, the observed probability ($\hat{p}$) is 0, and the test cannot be applied. This issue extends to other tests like TUFF, Haas, Christoffersen, and joint ratios. Similarly, when the observed probabilities are lower than the expected ones in multinomial backtesting, the results become undefined as the logarithm of a negative number is not valid for chi-squared distribution.

According to the hypothesis, a parametric model incorporating a forecasting model and a stochastic component should outperform simple historical or Monte Carlo simulations. It has been established that the skewed distribution used in the parametric model is more effective than the normal distribution for higher confidence levels, and parametric methods can account for this. The number of exceptions recorded during the Basel traffic light test was lower for the parametric model in 2022. Except for the Vanguard Mid-Cap Index Fund, the Kupiec test was accepted or undefined for this same year. The parametric model also had fewer undefined results compared to the historical model on average.

The Vanguard Russell 2000 Index Fund, however, had numerous undefined results with 0 exceptions. While the Monte Carlo model exhibited significantly more exceptions than expected confidence intervals, indicating an underestimation of risk, it is still preferable to compare it with the other two models. Unlike the historical model, the results of the parametric VaR model varied more across different months and funds, leading to varying outcomes in the Kupiec test. In 2021, the proportion of failure test was more frequently rejected for the parametric model. Overall, the exceptions were not consistently consecutive, except for the historical model in early 2021. Hence, the Christoffersen test was chi-squared distributed for almost the entire period.

Performing the additional tests and analyzing the results will help further validate the third hypothesis and provide a comprehensive evaluation of the backtesting procedure, taking into account the independence of exceedances and the requirements of Basel regulations.

It is the joint test which combines the test of independence of the exceptions with their proportion compared to the confidence interval which allows to compare the robustness for these two properties. The accepting results of the Christoffersen model influenced the results of the joint test upwards. They were more chi-squared distributed. Only one fund had its parametric model rejected in 2022 and fewer funds had their models undefined compared to the historical one. Nevertheless, the beginning of 2021 did not see the joint test accepted for four funds. Like the POF test, the results are less uniform for the parametric VaR model, depending on the fund studied, than the other models.

The Haas test accepted the model only for the parametric model in 2022. The study therefore concludes that, overall, the parametric VaR model with a skewed student t distribution was the most robust in 2022. However, it has the problem of not producing uniform results for all funds and is therefore very dependent on the forecasting model used. The hypothesis is therefore validated under the Basel II regulation, as it better captures risk in a volatile market in 2022.

The multinominal results are less consistent. For SPDR Dow Jones Industrial Average ETF Trust and Vanguard Russell 2000 Index Fund, the risk is underestimate for the all period. The observed probability of failure is less than the one expected, even during the crisis. The parametric model is still rejected at the end of 2022, except for iShares Russell 1000 Growth ETF. Therefore, three funds out of seven did not see their VaR model rejected, that is enough to confirm that it is less reject than the historic or the Monte Carlo ones, but not to conclude that the model is more robust to capture the right ES-97.5%. To conclude on this hypothesis, the parametric method including a stochastic part is more efficient than historical and Monte Carlo simulations during the extreme market movements caused by the Ukrainian crisis for the VaR-99%. Its efficiency concerning the new regulation remains to be proven. There is a limit to this conclusion: it is the fact that outside of crisis periods, this model can overestimate the risk of funds and remains very dependent on the model used to forecast the volatility. Moreover, this methods is not commonly used by the practitioners, because of regulations standards.

The last hypothesis regarding the negative link between ESG scores and VaR model exceptions is rejected. In 2022, there was a positive correlation between these two variables. The number of exceptions increased along with the ESG scores of the funds. Moreover, the fund with the lowest overall ESG score had the fewest exceptions and the model was not rejected by the backtests. Vanguard Russell 2000 Index Fund, with the lowest overall score, did not fall into the "red zone" of the Basel traffic light test regardless of the model used.

Regarding the proportion of failure and the independence of exceptions tested by the Haas test, the parametric method outperformed the others. It was the most efficient during the Ukrainian crisis. However, extending this result to ES-97.5% is challenging. Overall, the rejection of multinomial VaR models was caused by market disruptions.

Contrary to expectations, the ESG scores did not negatively influence the rejection of VaR models for US ETFs. Although the limited data makes it difficult to determine the precise relationship, there was no negative correlation between the different ESG scores and the number of exceptions. The appendix graphs do not show a clear link between the evolution of scores and the number of exceptions, but there is no detrimental effect.

Conclusion

In this paper, three VaR models were backtested during the Ukrainian war and the preceding period. Firstly, it was demonstrated that the skewed student-t distribution, an asymmetric distribution, better captured tail risk for high confidence intervals compared to the normal distribution. This finding is consistent with previous studies that recommend using the skewed student-t distribution for parametric VaR models. The parametric VaR models outperformed the historical and Monte Carlo VaR models during extreme market movements caused by inflation and the FED announcements, specifically under the Basel II regulation. In 2022, the number of exceptions was lower and they were found to be independent for all models.

For all models, there were only a few consecutive exceptions, leading to the acceptance of the Christoffersen test for all funds when exceptions occurred. However, the Haas test, which analyzes independence by considering the number of days between each overshoot, was only accepted for the parametric method in 2022.

The limitations of this parametric model include the overestimation of downside risk when there is no financial crisis or extreme market movements. This was evident from the undefined proportion of failure test results for some American ETFs during a significant period. If there are no exceedances at all, it is not possible to determine whether the VaR model is robust or not, as all tests become undefined. This is the reason why the multinomial backtest results for three funds yielded undefined results for ES-97.5%. The estimated probability of an exception occurring was lower than the expected probability. Therefore, it is not possible to confirm that the parametric method is more robust under the Basel III regulation.

In conclusion, the study highlights the effectiveness of the skewed student-t distribution in capturing tail risk for high confidence intervals. The parametric VaR models outperformed other models during extreme market movements under the Basel II regulation. However, the limitations of the parametric model, such as overestimation of downside risk in non-crisis periods and undefined results when there are no exceptions, need to be considered. The validity of the parametric method under the Basel III regulation would requires further investigation.

The study found that the ESG scores did not have a downward influence on the number of exceptions in the VaR models. In fact, the global ESG score and its sub-pillars were positively correlated with the number of exceedances. Although this correlation was less pronounced for the parametric VaR model, overall, all variables increased during the period. These results do not provide a clear assessment of the impact of ESG scores on the robustness of VaR-99% models but confirm that the scores did not have a downward impact. Contrary to expectations, the "best in class" American ETFs did not have their VaR models more accepted than others. Interestingly, the fund with the lowest ESG score had its VaR models accepted the most. This suggests that the ESG score did not have an upward impact on returns during the extreme market movements caused by inflation and the Ukrainian crisis.

These findings imply that the parametric VaR model is more suitable for capturing tail risk during extreme market movements, even though it may overestimate risk during non-disrupted periods due to the chosen distribution. This emphasizes the importance of stress tests conducted by market risk agencies and required by market authorities for UCITS funds. Stress tests describe what could happen during extreme market movements, both historically and under specific extreme scenarios, such as a significant increase in inflation. While these tests are not designed to describe downside risk with a specific confidence level, they assess the worst potential loss a fund could experience under certain market conditions. Similarly, some parametric VaR models could be included in risk reports to estimate potential losses for investors during extreme market events. It should be noted that parametric VaR is not an accepted risk measure by regulators for funds holding assets with non-linear payoffs. However, it is the only model that did not underestimate risk during the Ukrainian war. This method may be suitable for internally used by fund managers to establish a more realistic VaR value during extreme market conditions, particularly in the short term. For daily VaR calculations, the forecasted variance could be computed using past option prices. Thus, this method could provide a more accurate estimate of VaR during extreme market conditions.

The study acknowledges several limitations that should be considered. Firstly, the lack of ESG data for the studied ETFs hindered the formal establishment of the relationship between ESG scores and backtesting results, particularly the number of exceptions. This limited the ability to draw definitive conclusions about the impact of ESG scores on VaR model robustness during the Ukrainian crisis.

Another limitation is related to the selection of parameters for the skewed student-t parametric model, where the only criterion used was the AIC (Akaike information criterion). The significance of each parameter was not carefully examined, indicating that there is room for improvement in determining the optimal parameters. It is important to search for the optimal forecasting GARCH model that can closely match the observed variance. Additionally, recalculating the parameters more frequently to capture the evolving volatility is essential. However, this presents computational challenges and feasibility concerns, especially for daily calculations. Analyzing the p-values of each parameter and retaining only significant lags is also crucial.

The study's scope is limited to American ETFs, and it may not be generalizable to European equities or bonds. Further investigations are needed to explore the impact of the crisis on VaR models of other asset classes and to determine whether the influence of ESG scores differs across regions. For example, the impact on bond VaR models might be significant following central banks' announcements regarding interest rate increases.

Regulatory standards restrict managers from choosing parametric methods to assess VaR levels for funds. This limitation, imposed by ESMA, is exacerbated by the requirement to perform backtests using the last 250 observations, equivalent to one working year. This sample size sometimes results in zero model exceedances, particularly for high confidence intervals, leading to undefined backtesting results for the Kupiec and Christoffersen tests. To mitigate this issue, a larger sample size should be considered to ensure the presence of overshoots. If there are no overshoots over a larger sample, it would be more appropriate to conclude that the model overestimates tail risk rather than obtaining undefined results. Some authors suggest using a sample size of 500 observations to obtain consistent backtests.

Future studies should address these limitations by employing a more extensive and accurate backtest sample, even if it deviates from European legislation requirements. Additionally, considering different asset classes, currencies, and locations would allow for a more comprehensive examination of the potential impact of the crisis on each asset class. Furthermore, with the availability of more extensive ESG data, it would be valuable to establish a clearer relationship between ESG scores and VaR model robustness.

References

Acerbi, C. & Tasche, D. (2002). On the coherence of expected shortfall. Journal of Banking and Finance 26, 1487–1503.

Ahmed, M. F., Gao, Y., & Satchell, S. (2021). Modeling demand for ESG. The European Journal of Finance, 27(16), 1669-1683.

Albuquerque, R., Koskinen, Y., & Zhang, C. (2019). Corporate social responsibility and firm risk: Theory and empirical evidence. Management Science, 65(10), 4451-4469.

Allais, M. (1990). Allais paradox. In Utility and probability. Palgrave Macmillan, London, 3-9.

Bax, K., Sahin, Ö., Czado, C., & Paterlini, S. (2023). ESG, Risk, and (tail) dependence. International Review of Financial Analysis, 102513.

Billio M., Pelizzon L. (2000). Value-at-Risk: A multivariate switching regime approach. Journal of Empirical Finance, 531-554.

Blackrock and Ceres (2015). 21st Century Engagement, Investor Strategies for Incorporating ESG Considerations into Corporate Interactions.

Bollerslev T. (1986). Generalized autoregressive conditional heteroscedasticity, Journal of Econometrics, 307-327.

Boudoukh, Jacob, M. Richardson, 1993, "Stock returns and inflation: A long horizon perspective," American Economic Review, 83, 1346-1355.

Brooks C., Persand G. (2003). The effect of asymmetries on stock index return Value-at-Risk estimates. The Journal of Risk Finance, 29-42.

Calice, G., & Lin, M. T. (2021). Exploring risk premium factors for country equity returns. Journal of Empirical Finance, 63, 294-322.

Chatterji, A. K., Durand, R., Levine, D. I., & Touboul, S. (2016). Do ratings of firms converge? Implications for managers, investors and strategy researchers. Strategic Management Journal, 37(8), 1597-1614.

Chen, N. F., Roll, R., & Ross, S. A. (1986). Economic forces and the stock market. Journal of business, 383-403.

Chiaramonte, L., Dreassi, A., Girardone, C., & Piserà, S. (2022). Do ESG strategies enhance bank stability during financial turmoil? Evidence from Europe. The European Journal of Finance, 28(12), 1173-1211.

Christoffersen, P. (1998). Evaluating interval forecasts. International Economic Review, 39, 841-862.

Cont, R., Deguest, R. & Scandolo, G. (2010). Robustness and sensitivity analysis of risk measurement procedures. Quantitative Finance 10, 593–606.

Costanzino, N. & Curran, M. (2016). A simple traffic light approach to backtesting expected shortfall.

Cox, J. C., & Ross, S. A. (1976). The valuation of options for alternative stochastic processes. Journal of financial economics, 3(1-2), 145-166.

Danielsson J. (2002). The emperor has no clothes: Limits to risk modelling. Journal of Banking and Finance, 1273-1296.

Delmas, M. A., Etzion, D., & Nairn-Birch, N. (2013). Triangulating environmental performance: What do corporate social responsibility ratings really capture?. Academy of Management Perspectives, 27(3), 255-267.

Duarte, Fernando M. (2013) : Inflation risk and the cross section of stock returns, Staff Report, No. 621, Federal Reserve Bank of New York, New York.

Eccles, R. G., Lee, L. E., & Stroehle, J. C. (2020). The social origins of ESG: An analysis of Innovest and KLD. Organization & Environment, 33(4), 575-596.

Engle Robert F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation. Econometrica, 987–1008.

Fama Eugene F., Efficient Capital Markets. (1970). A Review of Theory and Empirical Work, The Journal of Finance, Vol. 25, No. 2, Papers and Proceedings of the Twenty-Eighth Annual Meeting of the American Finance Association New York.

Fama, Eugene, and Kenneth French, (1993), Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33, 3-56.

Fama, E.. (1981). Stock Returns, real activity, inflation, and money. American Economic Review 71, 545-565.

Fama, Eugene, and Kenneth French, (1992), The cross-section of expected returns, Journal of Finance, 47, 427-465.

Friedman M., A Friedman Doctrine (1970). The Social Responsibility of Business is to Increase Its Profits. New York Times.

Geske, Robert, and Richard Roll. (1983). The fiscal and monetary linkage between stock returns and inflation. Journal of Finance 38, 1-33.

Giese, G., Lee, L. E., Melas, D., Nagy, Z., & Nishikawa, L. (2019). Foundations of ESG investing: How ESG affects equity valuation, risk, and performance. The Journal of Portfolio Management, 45(5), 69-83.

Giglio, S., & Xiu, D. (2021). Asset pricing with omitted factors. Journal of Political Economy, 129(7).

Giot P., Laurent S. (2004). Value-at-Risk for long and short trading positions. Journal of Applied Econometrics.

Giot P., Laurent S. (2003). Market risk in commodity markets: a VaR approach. Energy Economics, 435-457.

Haas, M. (2001). New methods in backtesting. Working Paper, Financial Engineering Research Center.

Hendricks D. (1996). Evaluation of value-at-risk models using historical data. Federal Reserve Bank of New York, Economic Police Review, 39-70.

Hoepner Andreas G.F., Oikonomou Ioannis, Sautner Zacharias, Starks Laura T. and Zhou Xiao Y., 2018, ESG Shareholder Engagement and Downside Risk, American Finance Association, Philadelphia.

Jackson P., Maude D.J., Perraudin W. (1998). Testing Value-at-Risk approaches to capital adequacy. Bank of England Quarterly Bulletin, 256-266.

Jorge Mina and JerryYi Xiao, Return to Riskmetrics: Evolution of a standard, Riskmetrics Group.

Kratz, M., Lok, Y. H., & McNeil, A. J. (2018). Multinomial VaR backtests: A simple implicit approach to backtesting expected shortfall. Journal of Banking & Finance, 88, 393-407.

Kupiec, P. (1995). Techniques for verifying the accuracy of risk measurement models. Journal of Derivatives, 73-84.

Lambert, N., Pennock, D. & Shoham, Y. (2008). Eliciting properties of probability distributions. In Proceedings of the 9th ACM Conference on Electronic Commerce. EC'08, ACM, New York.

Luo, H. A., & Balvers, R. J. (2014). Social screens and systematic boycott risk. Forthcoming, Journal of Financial and Quantitative Analysis.

Mandelbrot B. (1963). The Variation of certain speculative prices, Journal of Business, 394-419.

Markowitz M. Harry. (1952). Portfolio Selection, Journal of Finance, 7 (1), 77-91.

Nass, C. (1959). A chi-square-test for small expectations in contingency tables, with special reference to accidents and absenteeism. Biometrika, 1959, vol. 46, no 3/4, p. 365-385.

Osband, K. H. (1985). Providing Incentives for Better Cost Forecasting. Ph.D. thesis, University of California, Berkeley.

Pearson, K. (1900). On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can reasonably be supposed to have arisen from random sampling. Philosophical Magazine Series 5 50, 157{175.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 19(3), 425-442.

Sharpe, S. A. (2002). Reexamining stock valuation and inflation: The implications of analysts' earnings forecasts. Review of Economics and Statistics, 84(4), 632-648.

Sherwood, M. W., & Pollard, J. L. (2018). The risk-adjusted return potential of integrating ESG strategies into emerging market equities. Journal of Sustainable Finance & Investment, 8(1), 26-44.

Stock James H. and Watson Mark W., Introduction to econometrics, Pearson.

Terhaar, K., Staub, R., & Singer, B. D. (2003). Appropriate policy allocation for alternative investments. The Journal of Portfolio Management, 29(3), 101-110.

Thomas J. Linsmeier & Neil D. Pearson (2000) Value at Risk, Financial Analysts Journal, 56:2, 47-67.

Venkataraman S. (1997). Value at risk for a mixture of normal distributions: The use of quasi-Bayesian estimation techniques. Economic Perspectives, Federal Reserve Bank of Chicago, 2-13.

Viale, A. M., Kolari, J. W., & Fraser, D. R. (2009). Common risk factors in bank stocks. Journal of Banking & Finance, 33(3), 464-472.

Vlaar P. (2000). Value at Risk models for Dutch bond portfolios. Journal of Banking and Finance, 131-154.

Zhang, Y., & Nadarajah, S. (2017). A review of backtesting for value at risk. Communications in Statistics – Theory and Methods, 1-24.

Appendix

GARCH (1,1) traffic light test results

Table 39: Dia, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	21	13	13	12	12	12	20	13	12	12	11	12	20	12	11	11	10	10
01/03/2021	18	10	10	9	9	9	17	10	9	9	8	9	17	9	8	8	7	7
01/04/2021	11	5	5	5	5	5	11	5	5	5	5	5	12	6	5	5	5	5
03/05/2021	11	5	5	5	5	5	11	5	5	5	5	5	12	6	5	5	5	5
01/06/2021	13	6	6	6	5	6	12	6	6	6	5	5	12	7	6	5	5	5
01/07/2021	12	5	5	5	4	5	11	5	5	5	4	4	11	6	5	4	4	4
02/08/2021	13	6	5	5	4	5	12	6	5	5	4	4	12	7	6	4	4	4
01/09/2021	13	6	5	5	4	5	12	6	5	5	4	4	12	7	6	4	4	4
01/10/2021	11	4	3	3	2	3	10	4	3	3	2	2	10	5	4	2	2	2
01/11/2021	8	2	1	1	0	1	7	2	1	1	0	0	7	2	2	0	0	0
01/12/2021	10	4	3	3	2	3	9	4	3	3	1	2	9	4	4	2	2	2
03/01/2022	12	5	3	3	2	3	11	5	3	3	1	2	12	6	5	2	2	2
01/02/2022	12	6	4	3	2	3	11	6	3	3	1	2	12	7	5	2	2	2
01/03/2022	14	6	4	3	2	3	13	6	3	3	1	2	18	8	6	2	2	2
01/04/2022	16	8	6	4	3	4	16	8	4	4	2	3	21	10	8	4	3	3
02/05/2022	19	11	9	7	6	7	19	11	7	7	5	6	24	13	11	7	6	6
01/06/2022	19	12	10	8	7	8	20	12	8	7	6	7	25	14	12	9	8	8
01/07/2022	23	16	13	11	9	11	24	15	11	10	8	10	29	18	15	12	10	11
01/08/2022	22	15	13	11	9	11	23	14	11	10	8	10	28	17	14	12	10	11
01/09/2022	24	16	14	12	10	12	25	15	12	11	9	11	30	18	15	13	11	12
03/10/2022	22	17	15	13	11	13	23	16	13	12	10	12	28	19	16	14	12	13
01/11/2022	22	17	15	13	11	13	23	16	13	12	10	12	28	19	16	14	12	13
01/12/2022	20	15	13	11	9	11	21	14	11	10	9	10	26	17	14	12	10	11
03/01/2023	19	14	13	11	9	11	20	13	11	10	9	10	24	15	13	12	10	11

Table 40: IWF, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	18	12	11	11	10	10	13	11	10	10	9	9	14	11	10	9	8	8
01/03/2021	14	9	8	8	7	7	10	8	7	7	6	6	11	8	7	6	6	6
01/04/2021	7	5	5	5	4	4	7	5	4	4	4	4	8	5	4	4	4	4
03/05/2021	7	5	5	5	4	4	7	5	4	4	4	4	8	5	4	4	4	4
01/06/2021	8	5	5	5	4	4	8	6	4	4	4	4	9	6	5	4	4	4
01/07/2021	6	3	3	3	3	3	6	4	3	3	3	3	7	4	4	3	3	3
02/08/2021	6	3	3	3	3	3	6	4	3	3	3	3	7	4	4	3	3	3
01/09/2021	6	3	3	3	3	3	6	4	3	3	3	3	7	4	4	3	3	3
01/10/2021	4	2	2	2	1	2	5	3	2	2	2	2	5	4	3	2	2	2
01/11/2021	4	1	1	1	0	1	5	2	1	1	1	1	5	4	2	1	1	1
01/12/2021	5	2	2	2	0	2	6	3	2	2	1	2	6	4	2	1	1	1
03/01/2022	8	3	3	2	0	2	9	4	3	2	1	2	6	4	2	1	1	1
01/02/2022	10	4	4	3	1	3	12	5	4	3	2	3	7	5	3	2	1	2
01/03/2022	11	5	5	4	2	4	13	6	5	4	3	4	8	5	3	2	1	2
01/04/2022	12	6	5	4	2	4	14	7	5	4	3	4	9	5	3	2	1	2
02/05/2022	15	8	7	5	3	5	17	9	7	5	4	5	9	5	3	2	1	2
01/06/2022	15	9	8	6	4	6	17	9	8	6	5	6	10	6	4	4	2	4
01/07/2022	15	9	8	6	4	6	17	9	8	6	5	6	13	9	7	6	2	5
01/08/2022	15	9	8	6	4	6	17	9	8	6	5	6	13	9	7	6	2	5
01/09/2022	16	10	9	7	4	7	18	10	9	7	5	7	14	10	8	7	2	6
03/10/2022	16	10	9	7	5	7	17	10	9	7	5	7	13	9	8	7	2	6
01/11/2022	16	10	9	7	5	7	17	10	9	7	5	7	13	9	8	7	2	6
01/12/2022	15	9	8	6	5	6	16	9	8	6	5	6	13	9	8	7	2	6
03/01/2023	12	8	7	6	5	6	13	8	7	6	5	6	14	9	8	7	2	6

Table 41: QQ.O, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	12	11	10	10	8	10	12	10	9	8	8	8	11	10	9	8	8	8
01/03/2021	8	7	7	7	5	7	10	7	6	6	6	6	8	7	6	6	6	6
01/04/2021	5	4	4	4	3	4	7	4	4	4	4	4	5	4	4	4	4	4
03/05/2021	5	4	4	4	3	4	7	4	4	4	4	4	5	4	4	4	4	4
01/06/2021	7	4	4	4	3	4	9	4	4	4	4	4	7	4	4	4	4	4
01/07/2021	5	3	3	3	2	3	7	3	3	3	3	3	5	3	3	3	3	3
02/08/2021	5	3	3	3	2	3	7	3	3	3	3	3	5	3	3	3	3	3
01/09/2021	5	3	3	3	2	3	7	3	3	3	3	3	5	3	3	3	3	3
01/10/2021	5	2	2	1	0	1	7	3	2	2	2	2	5	2	2	2	1	2
01/11/2021	4	1	1	0	0	0	7	2	1	1	1	1	5	1	1	1	0	1
01/12/2021	5	2	1	0	0	0	7	3	2	1	1	1	6	2	2	1	0	1
03/01/2022	8	2	1	0	0	0	10	3	2	1	1	1	9	2	2	1	0	1
01/02/2022	11	3	2	1	0	1	14	4	3	2	1	1	11	3	3	1	0	1
01/03/2022	12	4	3	2	1	2	14	5	4	3	2	2	12	4	4	2	1	2
01/04/2022	13	4	3	2	1	2	15	5	4	3	2	2	12	4	4	2	1	2
02/05/2022	15	6	4	3	2	3	17	7	6	4	3	3	14	6	6	3	2	3
01/06/2022	14	7	5	4	3	4	16	8	7	5	4	4	13	7	7	4	3	4
01/07/2022	18	10	7	6	4	6	19	11	9	7	4	5	16	10	9	6	3	5
01/08/2022	18	10	7	6	4	6	19	11	9	7	4	5	16	10	9	6	3	5
01/09/2022	19	11	8	7	4	7	20	12	10	8	4	5	17	11	10	7	3	5
03/10/2022	19	11	8	8	5	8	20	11	10	8	4	5	17	11	10	7	4	5
01/11/2022	20	12	8	8	5	8	20	12	10	8	4	5	17	12	10	7	4	5
01/12/2022	20	11	8	8	5	8	20	11	9	8	4	5	17	11	9	7	4	5
03/01/2023	18	11	8	8	5	8	18	11	9	8	4	5	15	11	9	7	4	5

Table 42: VO, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	22	17	16	14	14	14	18	14	13	12	12	12	20	16	12	11	10	10
01/03/2021	19	14	13	11	11	11	15	11	10	9	9	9	17	14	11	10	9	9
01/04/2021	11	8	7	5	5	5	9	6	6	5	5	5	11	8	6	5	5	5
03/05/2021	11	8	7	5	5	5	9	6	6	5	5	5	11	8	6	5	5	5
01/06/2021	11	9	8	6	6	6	9	7	7	6	6	6	11	9	7	6	6	6
01/07/2021	9	8	7	5	5	5	8	6	6	5	5	5	9	8	6	5	5	5
02/08/2021	9	8	7	5	5	5	8	6	6	5	5	5	9	8	6	5	5	5
01/09/2021	9	8	7	5	5	5	8	6	6	5	5	5	9	8	6	5	5	5
01/10/2021	6	6	4	3	3	3	7	4	3	3	3	3	6	6	3	3	3	3
01/11/2021	3	3	2	1	1	1	4	2	1	1	1	1	3	3	1	1	1	1
01/12/2021	5	5	4	3	3	3	6	4	3	3	3	3	5	5	3	3	3	3
03/01/2022	8	6	5	4	4	4	9	5	4	4	4	4	8	6	4	4	4	4
01/02/2022	11	7	6	5	5	5	12	7	6	5	5	5	10	6	5	5	5	5
01/03/2022	13	8	7	5	5	5	15	8	6	5	5	5	11	6	5	5	5	5
01/04/2022	14	9	8	6	6	6	16	9	7	6	6	6	12	7	6	6	5	6
02/05/2022	18	12	11	9	8	9	20	12	10	9	9	9	16	10	9	9	7	9
01/06/2022	18	12	11	9	8	9	22	12	10	9	9	9	17	10	9	9	7	9
01/07/2022	20	12	11	9	8	9	24	13	10	9	9	9	19	10	9	9	7	9
01/08/2022	20	12	11	9	8	9	24	13	10	9	9	9	19	10	9	9	7	9
01/09/2022	21	13	12	10	8	9	25	14	11	10	9	10	20	11	10	10	7	10
03/10/2022	21	13	13	11	9	10	24	14	12	11	10	11	20	11	11	11	8	11
01/11/2022	21	13	13	11	9	10	24	14	12	11	10	11	20	11	11	11	8	11
01/12/2022	20	11	11	9	7	8	23	12	10	9	8	9	19	9	9	9	6	9
03/01/2023	17	10	10	8	6	7	20	11	9	8	7	8	16	8	8	8	5	8

Table 43: VooiV.P, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	15	13	12	10	8	10	21	16	15	12	11	12	19	13	12	11	10	11
01/03/2021	13	10	9	7	5	7	19	14	13	9	8	9	17	11	9	8	7	8
01/04/2021	10	7	6	4	3	4	12	8	8	5	5	5	11	8	6	5	4	5
03/05/2021	10	7	6	4	3	4	12	8	8	5	5	5	11	8	6	5	4	5
01/06/2021	10	7	6	4	3	4	11	8	8	5	5	5	11	8	7	5	4	5
01/07/2021	8	6	5	3	2	3	9	7	7	4	4	4	9	7	6	4	3	4
02/08/2021	9	6	5	3	2	3	10	7	7	4	4	4	10	7	6	4	3	4
01/09/2021	9	6	5	3	2	3	10	7	7	4	4	4	10	7	6	4	3	4
01/10/2021	8	5	4	2	1	2	8	5	5	3	2	2	8	4	3	2	1	2
01/11/2021	7	4	3	1	1	1	7	4	4	2	1	1	7	3	2	1	0	1
01/12/2021	10	6	5	3	3	3	9	6	6	3	2	2	9	4	3	1	0	1
03/01/2022	11	6	5	3	3	3	9	6	6	3	2	2	9	4	3	1	0	1
01/02/2022	14	8	7	6	3	6	10	7	7	3	1	2	10	4	3	0	0	0
01/03/2022	17	11	10	9	4	8	12	9	7	4	1	3	12	5	4	1	0	1
01/04/2022	18	12	11	10	5	9	13	10	8	5	1	4	13	6	5	2	0	2
02/05/2022	22	15	12	11	6	10	15	11	9	5	1	4	16	7	6	2	0	2
01/06/2022	24	17	14	13	8	12	17	12	9	5	1	4	18	8	6	2	0	2
01/07/2022	26	18	15	13	8	12	18	13	9	5	1	4	19	8	6	2	0	2
01/08/2022	25	18	15	13	8	12	17	13	9	5	1	4	18	8	6	2	0	2
01/09/2022	27	19	16	14	9	13	18	14	10	5	1	4	20	9	7	3	1	3
03/10/2022	25	18	15	14	8	13	17	14	10	5	1	5	19	10	8	4	1	4
01/11/2022	26	18	15	14	8	13	18	14	10	5	1	5	20	10	8	4	1	4
01/12/2022	23	16	13	12	6	11	16	12	8	4	0	4	18	9	7	4	1	4
03/01/2023	22	16	13	12	6	11	16	12	8	4	0	4	18	9	7	4	1	4

Table 44: VTWO.O, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	17	15	11	10	9	10	17	14	12	10	10	10	15	13	10	10	9	10
01/03/2021	15	14	12	10	9	10	15	13	11	10	10	10	13	13	11	10	9	10
01/04/2021	10	9	7	5	4	5	9	7	5	5	5	5	10	9	6	5	4	5
03/05/2021	10	9	7	5	4	5	9	7	5	5	5	5	10	9	6	5	4	5
01/06/2021	11	10	7	5	4	5	10	8	5	5	5	5	11	10	6	5	4	5
01/07/2021	10	9	6	4	3	4	9	7	4	4	4	4	10	9	5	4	3	4
02/08/2021	10	9	6	4	3	4	9	7	4	4	4	4	10	9	5	4	3	4
01/09/2021	10	9	6	4	3	4	9	7	4	4	4	4	10	9	5	4	3	4
01/10/2021	7	5	2	1	1	1	7	4	1	1	1	1	8	5	2	1	1	1
01/11/2021	5	3	1	0	0	0	5	2	0	0	0	0	6	3	1	0	0	0
01/12/2021	6	4	2	1	0	1	6	3	1	1	1	1	7	4	2	1	1	1
03/01/2022	9	4	2	1	0	1	9	4	1	1	1	1	10	5	2	1	1	1
01/02/2022	11	6	4	3	1	2	11	6	3	3	2	2	12	7	4	3	2	2
01/03/2022	11	5	3	3	1	2	10	5	3	3	2	2	11	6	3	3	2	2
01/04/2022	10	4	3	3	1	2	10	5	3	3	2	2	10	5	3	3	2	2
02/05/2022	14	6	4	4	1	2	13	6	4	3	2	2	13	6	4	3	2	2
01/06/2022	17	8	7	7	3	4	16	8	7	6	4	4	16	8	7	6	4	4
01/07/2022	20	11	8	7	3	4	19	11	8	6	4	4	19	10	8	6	4	4
01/08/2022	20	11	8	7	3	4	19	11	8	6	4	4	19	10	8	6	4	4
01/09/2022	21	12	9	8	3	4	20	12	9	7	4	4	20	11	9	7	4	4
03/10/2022	21	13	10	9	3	4	19	13	10	8	4	4	19	12	10	8	4	4
01/11/2022	22	13	10	9	3	4	20	13	10	8	4	4	20	12	10	8	4	4
01/12/2022	22	12	9	8	3	3	20	12	9	7	3	3	20	11	9	7	3	3
03/01/2023	20	12	9	8	3	3	18	11	9	7	3	3	18	10	9	7	3	3

Table 45: VUG, GARCH (1,1) traffic light test results

	Normal						Student-t						Skew student-t					
Date	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%	95,000%	97,500%	98,125%	98,750%	99,375%	99,000%
01/02/2021	16	12	11	10	10	10	13	10	10	10	9	10	16	10	10	9	8	8
01/03/2021	13	9	9	8	7	7	10	8	8	7	6	7	14	8	8	7	6	7
01/04/2021	8	6	6	5	4	4	8	6	5	4	4	4	14	6	6	6	4	5
03/05/2021	8	6	6	5	4	4	8	6	5	4	4	4	15	6	6	6	4	5
01/06/2021	10	7	7	6	4	4	10	7	6	5	4	5	16	8	7	7	5	6
01/07/2021	9	5	5	5	3	3	9	6	5	4	3	4	15	7	6	6	4	5
02/08/2021	9	5	5	5	3	3	10	6	5	4	3	4	15	7	6	6	4	5
01/09/2021	9	5	5	5	3	3	10	6	5	4	3	4	15	7	6	6	4	5
01/10/2021	8	4	4	3	1	1	9	5	4	2	1	2	13	6	5	5	2	3
01/11/2021	8	4	4	2	0	0	9	5	4	1	0	1	12	6	4	4	1	2
01/12/2021	10	5	5	3	1	1	10	6	5	2	1	2	13	7	5	5	2	3
03/01/2022	14	8	8	4	1	2	14	9	7	3	1	3	17	10	8	7	2	4
01/02/2022	16	10	10	6	3	4	16	11	9	5	3	5	19	12	10	9	4	6
01/03/2022	15	9	9	5	3	4	15	10	8	5	3	5	18	11	9	8	4	5
01/04/2022	14	9	9	5	3	4	14	9	8	5	3	5	15	10	8	7	4	5
02/05/2022	15	9	9	5	3	4	15	9	8	5	3	5	15	10	8	7	4	5
01/06/2022	15	10	10	6	4	6	15	10	9	6	4	6	15	10	9	8	4	6
01/07/2022	14	10	10	6	4	6	14	10	9	6	4	6	14	10	9	8	4	6
01/08/2022	14	10	10	6	4	6	13	10	9	6	4	6	14	10	9	8	4	6
01/09/2022	15	11	11	6	4	6	14	11	10	6	4	6	15	11	10	8	4	6
03/10/2022	14	10	10	6	4	6	13	10	9	6	4	6	14	10	9	7	4	6
01/11/2022	13	9	9	6	4	6	12	9	8	6	4	6	13	9	9	7	4	6
01/12/2022	11	8	8	5	3	5	11	8	7	5	3	5	11	8	8	6	3	5
03/01/2023	7	5	5	4	3	4	7	5	5	4	3	4	7	5	5	4	3	4

VaR-95% and VaR-99% Traffic Light Test

Table 46: Dia, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95 %**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	16	8	12	6	14	12
01/03/2021	13	6	10	5	11	9
01/04/2021	3	0	7	5	2	1
03/05/2021	2	0	7	5	1	1
01/06/2021	3	0	9	5	2	1
01/07/2021	3	0	9	4	2	0
02/08/2021	4	0	10	5	3	0
01/09/2021	4	0	10	5	3	0
01/10/2021	7	0	10	4	6	0
01/11/2021	6	0	8	3	6	0
01/12/2021	8	1	9	4	8	2
03/01/2022	9	1	10	4	11	2
01/02/2022	10	1	9	3	13	2
01/03/2022	12	1	8	2	19	2
01/04/2022	15	2	9	2	22	4
02/05/2022	18	3	11	3	25	7
01/06/2022	20	6	11	4	27	9
01/07/2022	23	6	12	4	30	11
01/08/2022	22	6	11	3	29	11
01/09/2022	24	7	13	4	31	12
03/10/2022	22	8	13	5	29	13
01/11/2022	23	8	13	5	30	13
01/12/2022	21	7	13	4	28	11
03/01/2023	21	7	13	4	26	11

Table 47: IWF, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	19	8	25	14	19	13
01/03/2021	15	5	21	11	15	10
01/04/2021	6	0	18	8	6	2
03/05/2021	4	0	17	8	4	2
01/06/2021	5	0	15	8	5	2

01/07/2021	4	0	12	6	4	1
02/08/2021	4	0	10	6	4	1
01/09/2021	4	0	9	6	4	1
01/10/2021	3	1	6	4	4	1
01/11/2021	3	1	3	3	4	1
01/12/2021	4	1	3	3	6	2
03/01/2022	5	1	2	2	9	2
01/02/2022	10	2	1	0	14	5
01/03/2022	14	3	4	0	19	8
01/04/2022	15	4	5	1	21	9
02/05/2022	19	6	8	1	26	11
01/06/2022	22	9	10	1	29	14
01/07/2022	25	10	13	2	34	16
01/08/2022	25	10	13	2	34	16
01/09/2022	26	10	15	3	35	16
03/10/2022	26	10	17	4	34	16
01/11/2022	26	10	19	4	34	16
01/12/2022	25	10	20	4	33	15
03/01/2023	24	10	21	5	30	15

Table 48: QQQ.O, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	19	6	25	10	19	12
01/03/2021	16	4	21	9	15	9
01/04/2021	8	0	18	6	7	3
03/05/2021	6	0	17	6	5	3
01/06/2021	6	0	16	6	7	3
01/07/2021	5	0	13	5	6	2
02/08/2021	5	0	11	5	6	2
01/09/2021	5	0	10	5	6	2
01/10/2021	4	0	7	4	6	1
01/11/2021	3	0	4	3	6	1
01/12/2021	3	0	4	3	7	1
03/01/2022	4	0	3	2	10	2
01/02/2022	8	1	0	0	15	4
01/03/2022	11	3	1	0	20	7
01/04/2022	11	4	2	0	21	8
02/05/2022	14	6	4	0	28	10
01/06/2022	18	8	6	0	30	13
01/07/2022	22	9	8	0	36	15
01/08/2022	22	9	8	0	36	15
01/09/2022	23	9	11	1	37	16
03/10/2022	23	10	13	2	36	16
01/11/2022	24	10	14	2	36	16
01/12/2022	24	10	15	2	36	16
03/01/2023	23	10	16	3	33	15

Table 49: VO, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**

01/02/2021	18	8	16	9	14	14
01/03/2021	15	5	14	9	11	11
01/04/2021	5	0	10	6	2	2
03/05/2021	4	0	10	6	1	1
01/06/2021	4	0	11	6	2	2
01/07/2021	2	0	9	5	1	1
02/08/2021	3	0	9	5	2	1
01/09/2021	3	0	9	5	2	1
01/10/2021	4	0	7	4	4	1
01/11/2021	4	0	5	2	4	1
01/12/2021	6	1	7	2	6	3
03/01/2022	8	1	7	2	8	4
01/02/2022	12	1	6	2	12	5
01/03/2022	16	1	7	1	17	6
01/04/2022	18	2	8	2	19	7
02/05/2022	22	4	12	3	23	10
01/06/2022	24	7	14	5	25	12
01/07/2022	28	9	18	7	29	14
01/08/2022	27	9	18	7	28	14
01/09/2022	28	9	21	8	29	15
03/10/2022	27	9	21	9	28	16
01/11/2022	29	9	23	9	30	16
01/12/2022	28	8	24	10	29	14
03/01/2023	26	8	27	11	27	13

Table 50: VooiV.P, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	19	8	22	13	18	12
01/03/2021	16	5	20	11	15	9
01/04/2021	5	0	17	8	4	1
03/05/2021	3	0	16	8	3	1
01/06/2021	4	0	15	8	4	1
01/07/2021	3	0	12	6	3	0
02/08/2021	4	0	12	6	4	0
01/09/2021	4	0	12	6	4	0
01/10/2021	5	0	10	4	5	1
01/11/2021	4	0	7	2	4	1
01/12/2021	6	1	8	3	6	3
03/01/2022	6	1	7	2	6	3
01/02/2022	10	1	6	1	10	5
01/03/2022	15	2	8	1	15	7
01/04/2022	16	3	10	2	18	8
02/05/2022	19	6	15	3	22	11
01/06/2022	21	9	17	4	24	14
01/07/2022	25	10	21	4	28	17
01/08/2022	24	10	20	4	27	17

01/09/2022	25	10	23	5	28	18
03/10/2022	24	11	24	6	27	18
01/11/2022	25	11	25	6	28	18
01/12/2022	23	10	27	6	27	16
03/01/2023	23	10	28	7	27	16

Table 51: VTWO.O, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	19	6	9	4	15	11
01/03/2021	17	4	7	2	12	8
01/04/2021	8	0	5	1	4	2
03/05/2021	5	0	5	1	2	1
01/06/2021	4	0	5	1	3	1
01/07/2021	3	0	4	0	3	0
02/08/2021	3	0	4	0	3	0
01/09/2021	3	0	4	0	3	0
01/10/2021	5	0	3	0	5	0
01/11/2021	5	0	2	0	5	0
01/12/2021	6	1	3	0	6	1
03/01/2022	9	1	3	0	9	1
01/02/2022	11	1	4	0	11	2
01/03/2022	11	1	3	0	12	2
01/04/2022	11	1	3	0	12	2
02/05/2022	15	2	4	0	16	3
01/06/2022	18	4	6	0	19	5
01/07/2022	21	6	9	2	21	7
01/08/2022	21	6	9	2	21	7
01/09/2022	22	6	10	3	22	7
03/10/2022	21	6	11	4	21	8
01/11/2022	22	6	12	4	22	8
01/12/2022	22	5	14	5	22	7
03/01/2023	20	5	16	5	20	7

Table 52: VUG, VaR-95% and VaR-99% Traffic Light Test

	Historical		Parametric		Monte Carlo	
Date	**95%**	**99%**	**95%**	**99%**	**95%**	**99%**
01/02/2021	20	6	27	15	20	12
01/03/2021	16	4	23	12	16	9
01/04/2021	7	0	20	9	6	2
03/05/2021	5	0	19	9	4	2
01/06/2021	6	0	17	9	6	2
01/07/2021	5	0	14	7	5	1
02/08/2021	5	0	12	7	5	1
01/09/2021	5	0	11	7	5	1
01/10/2021	4	1	7	5	5	1
01/11/2021	4	1	3	3	5	1
01/12/2021	5	1	3	3	7	1
03/01/2022	7	1	2	2	10	1
01/02/2022	12	2	1	0	15	4
01/03/2022	16	3	4	0	20	7

01/04/2022	16	4	5	0	22	8
02/05/2022	20	6	8	0	27	10
01/06/2022	23	9	10	0	29	13
01/07/2022	27	10	13	0	34	15
01/08/2022	27	10	13	0	34	15
01/09/2022	28	10	16	1	35	15
03/10/2022	28	10	18	2	34	15
01/11/2022	28	10	20	2	34	15
01/12/2022	28	10	21	2	33	15
03/01/2023	26	10	22	3	30	15

Cross sectional analysis

1. ESG statistics

Dia

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	77,646	76,575	80,645	74,763
Median	77,510	76,983	80,667	74,591
Maximum	78,331	77,705	81,212	76,454
std.Dev	0,301	0,967	0,326	1,222

IWF

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	72,340	66,235	76,055	71,490
Median	72,505	66,450	76,219	71,313
Maximum	73,781	67,336	77,065	74,164
std.Dev	1,163	0,782	0,770	2,088

QQQ.O

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	74,266	68,380	76,680	74,234
Median	75,010	68,909	77,019	75,271
Maximum	75,808	69,155	77,850	76,587
std.Dev	1,331	0,878	0,802	2,074

VO

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	60,960	51,771	64,576	62,304
Median	61,623	52,391	65,098	63,209
Maximum	62,878	53,572	66,034	64,924
std.Dev	1,752	1,724	1,368	2,365

VooiV.P

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	73,443	69,720	76,711	70,790
Median	73,855	70,089	76,838	71,532
Maximum	74,186	70,220	77,219	72,220
std.Dev	0,822	0,561	0,383	1,546

VTWO.O

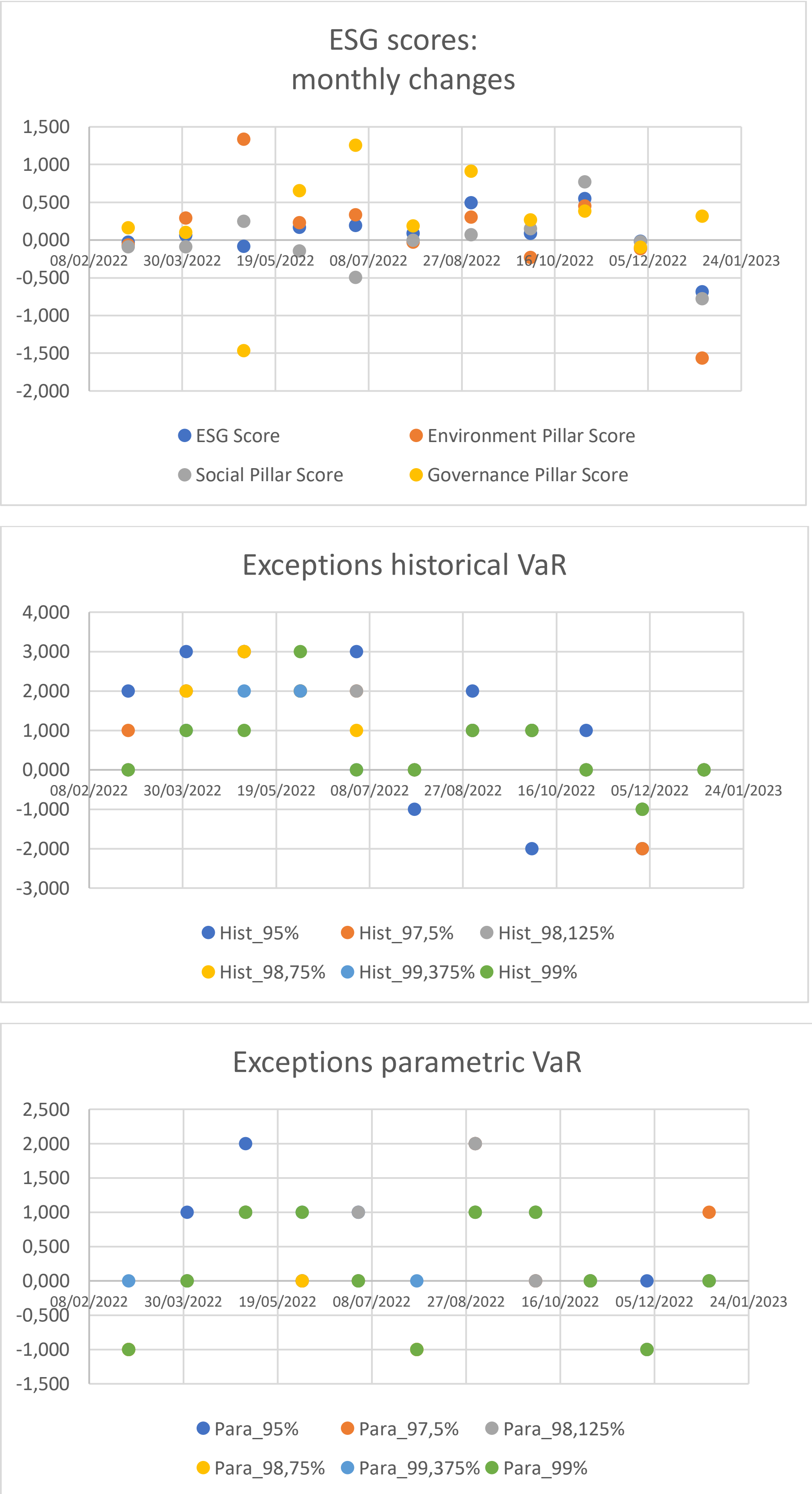
	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
Mean	43,213	25,144	45,642	53,387
Median	43,174	25,057	45,472	53,616
Maximum	44,599	25,943	46,631	55,611

VUG

	ESG Score	Environment Pillar Score	Social Pillar Score	Governance Pillar Score
	72,738	66,130	76,084	72,269
Mean	73,333	66,450	76,264	73,372
Median	73,711	66,819	76,941	73,967
Maximum	1,134	0,733	0,614	1,988

2. Graph: evolution of monthly changes

Figure 3: Dia, Cross sectional analysis results



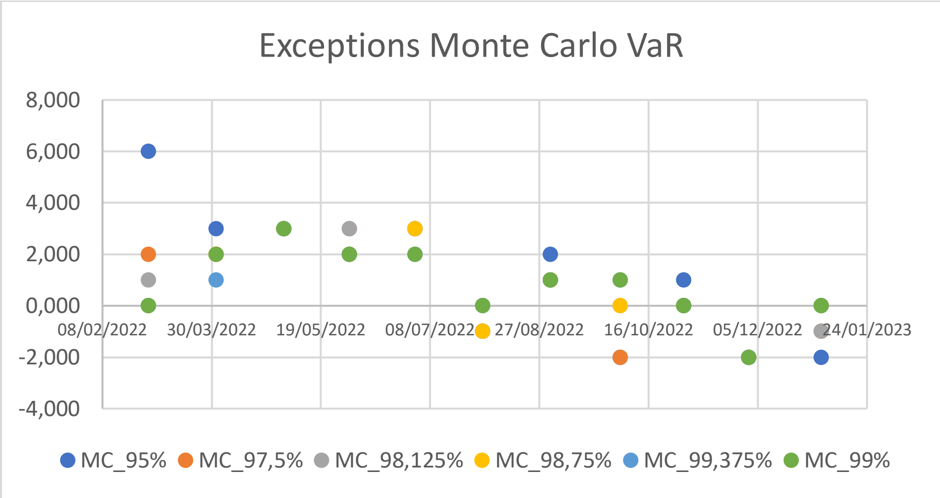
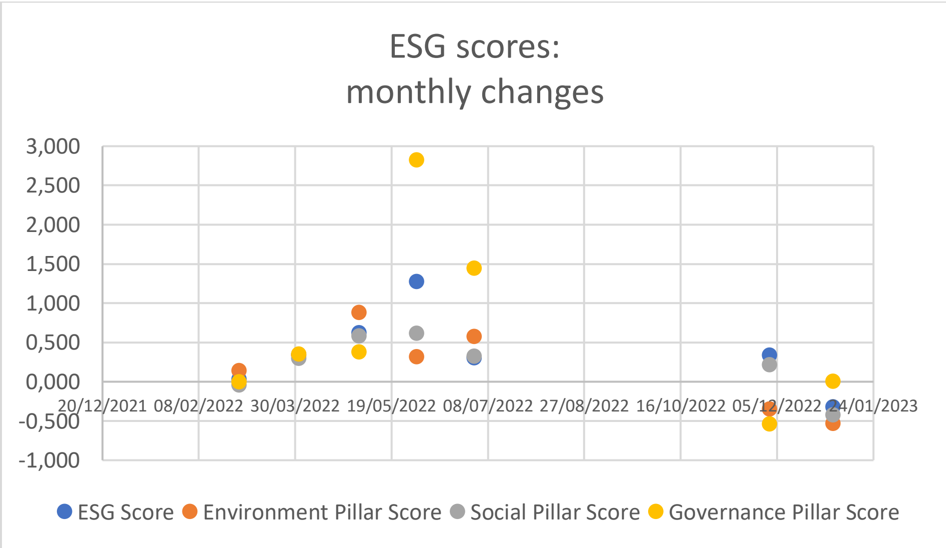
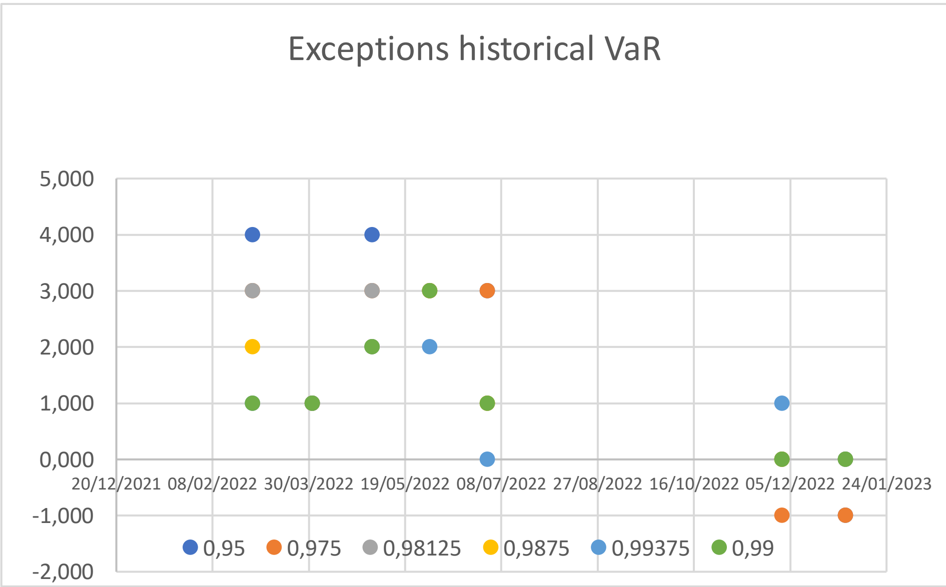
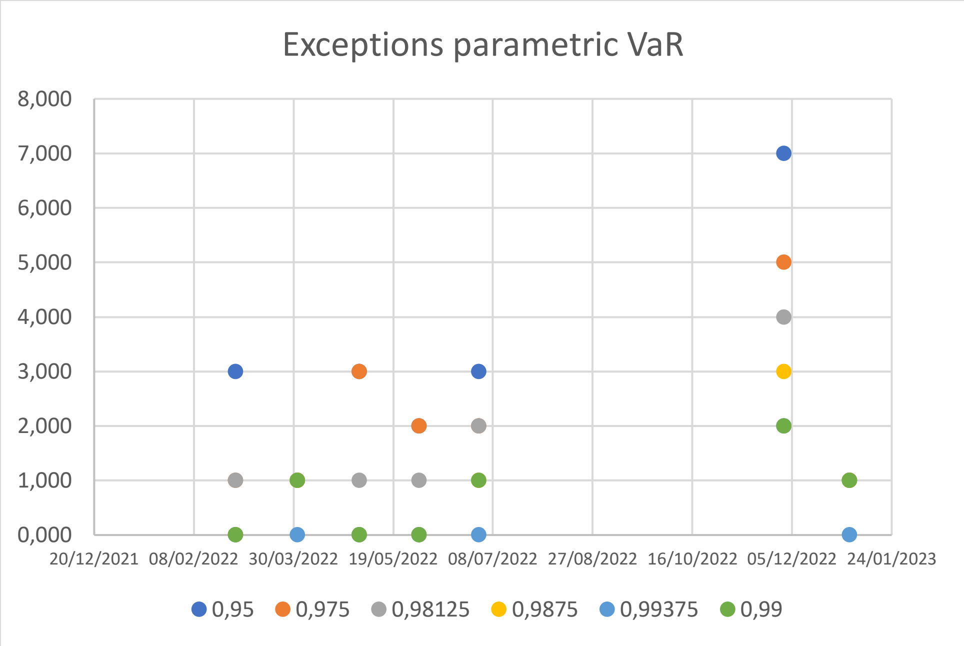


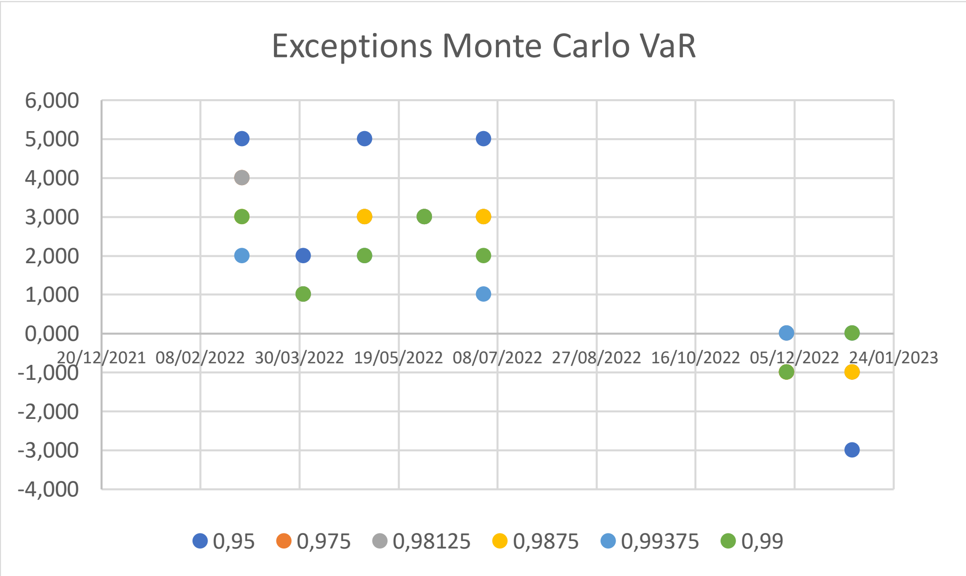
Figure 4, IWF, Cross sectional analysis results





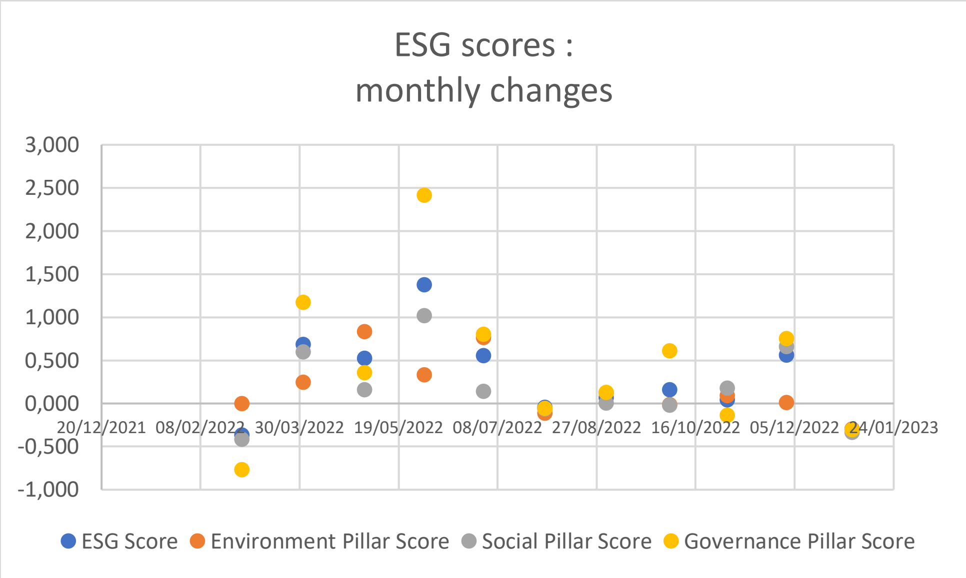


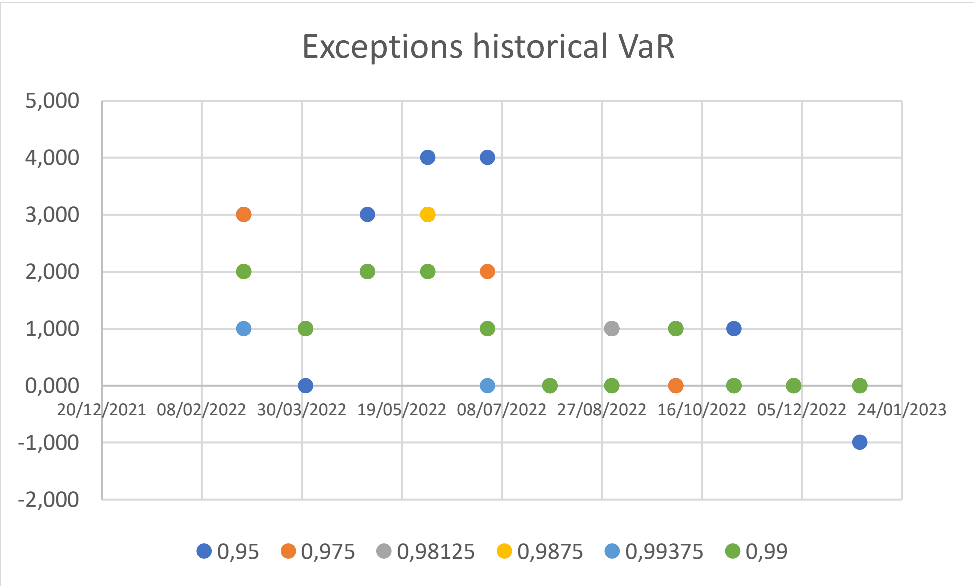
Exceptions parametric VaR
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

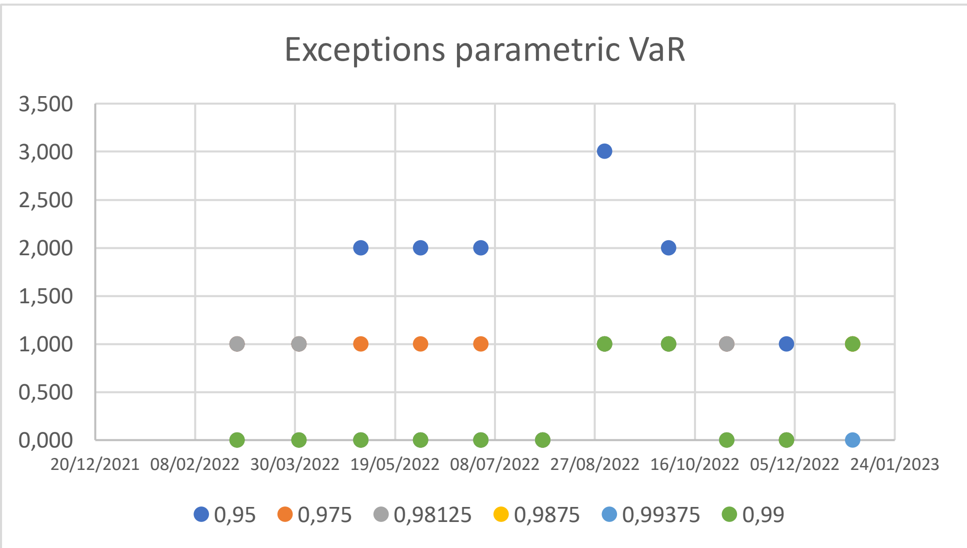


Exceptions Monte Carlo VaR
6,000
5,000
4,000
3,000
2,000
1,000
0,000
-1,000
-2,000
-3,000
-4,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

Figure 5: QQQ.O, Cross sectional analysis results







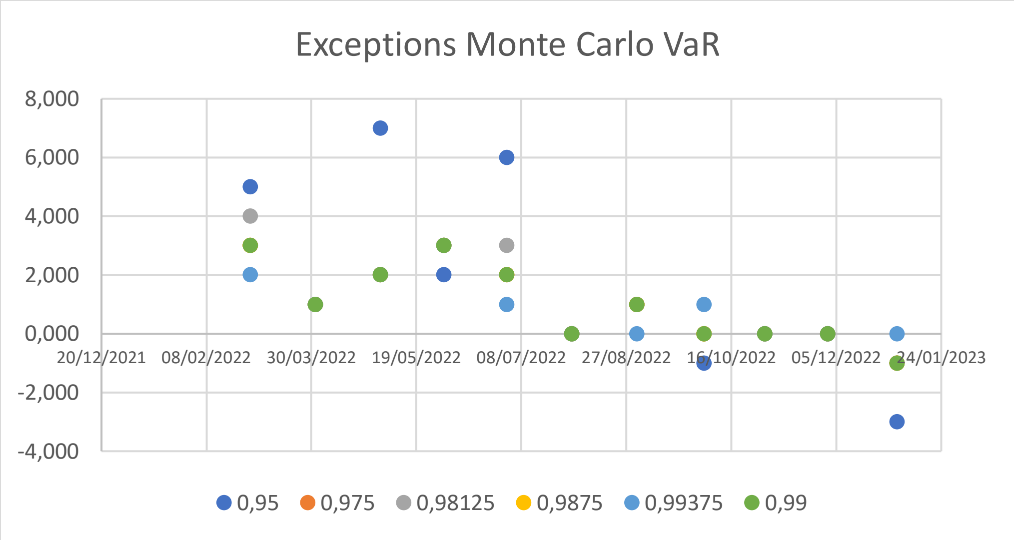
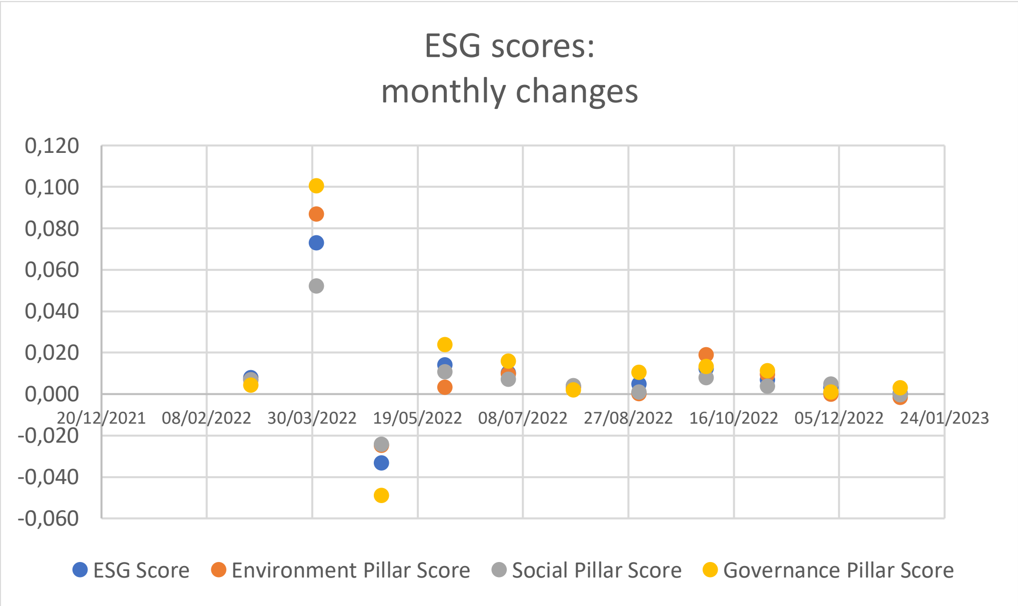
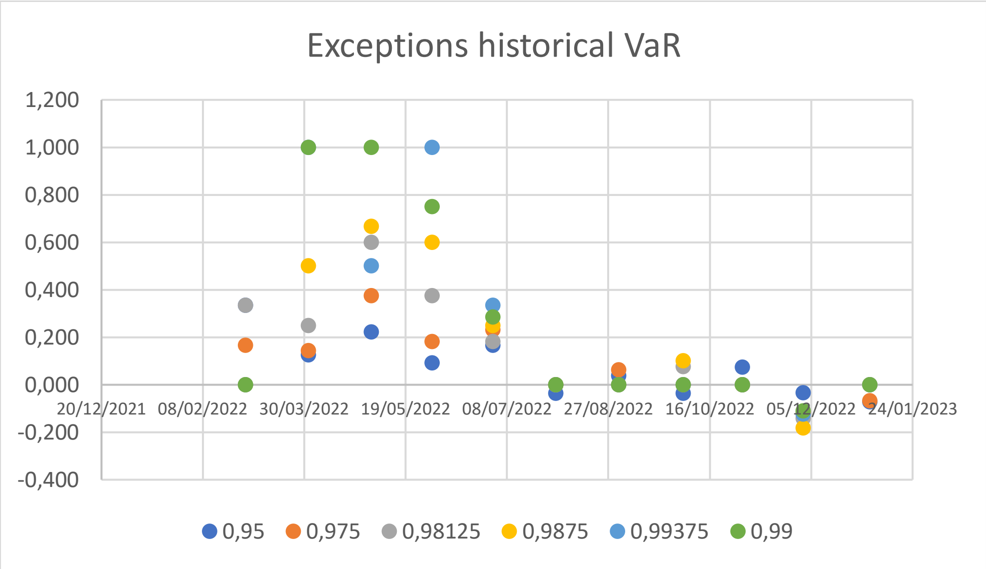
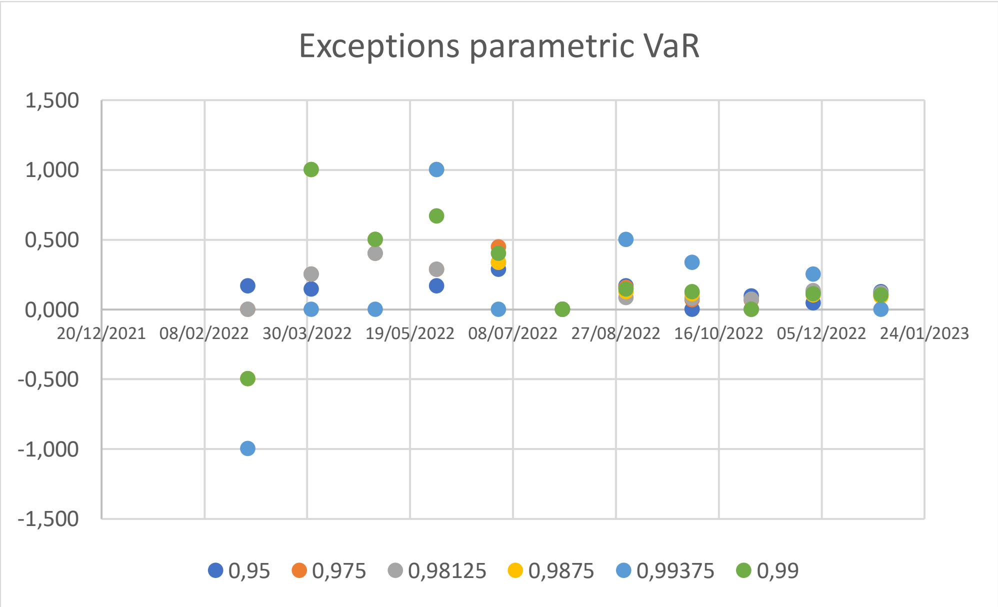


Figure 6: VO, Cross sectional analysis results

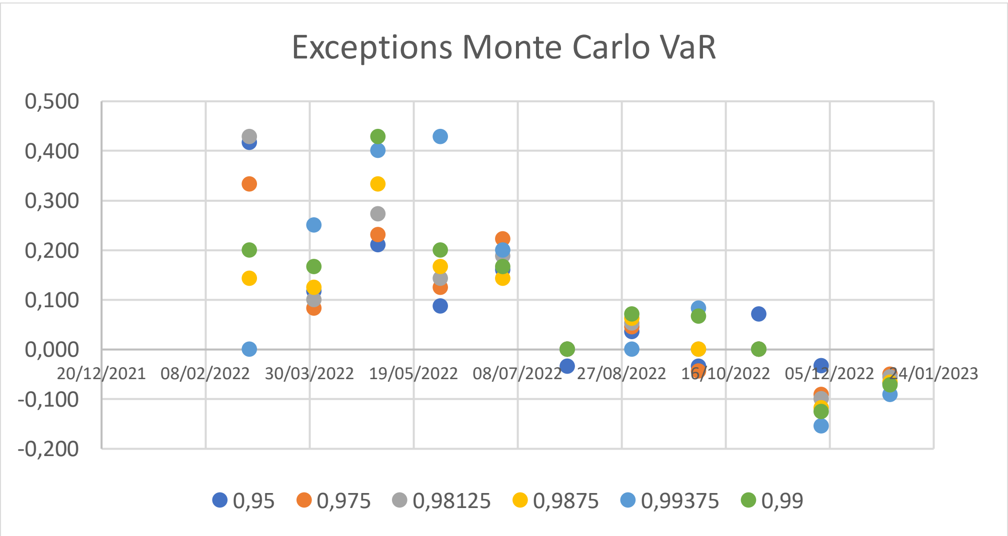




Exceptions historical VaR
1,200
1,000
0,800
0,600
0,400
0,200
0,000
-0,200
-0,400
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

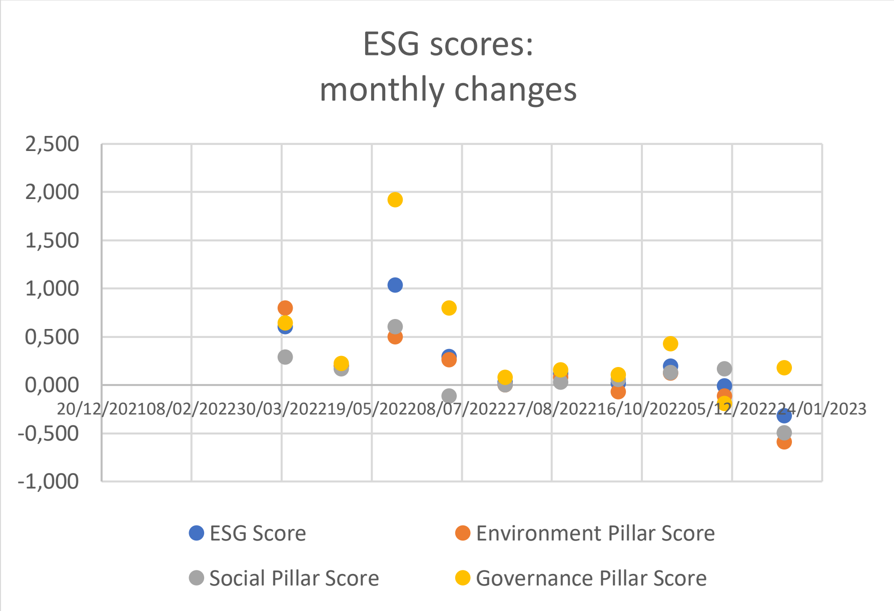


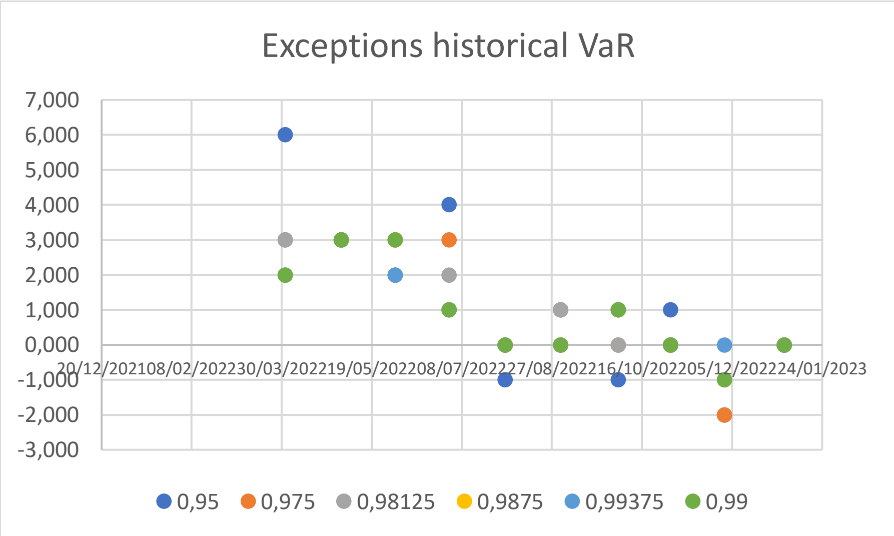
Exceptions parametric VaR
1,500
1,000
0,500
0,000
-0,500
-1,000
-1,500
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

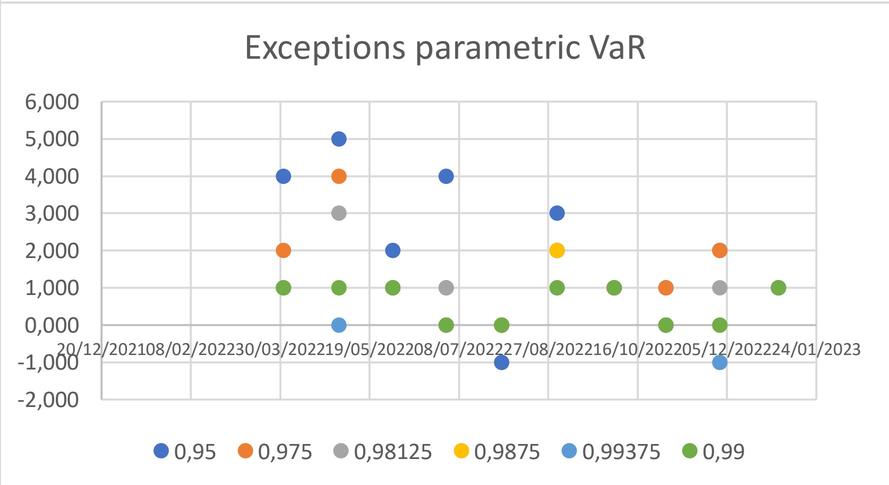


Exceptions Monte Carlo VaR
0,500
0,400
0,300
0,200
0,100
0,000
-0,100
-0,200
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

Figure 7; VooiV.P, Cross sectional analysis results







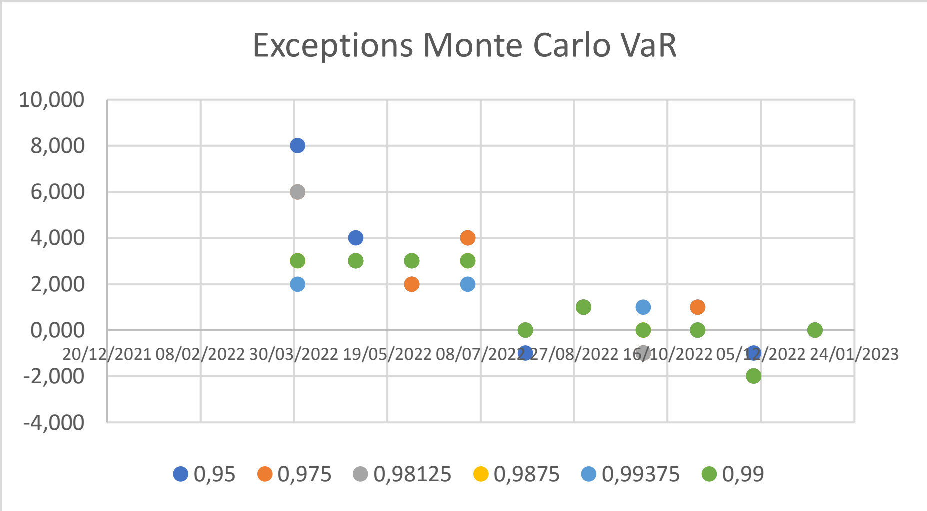
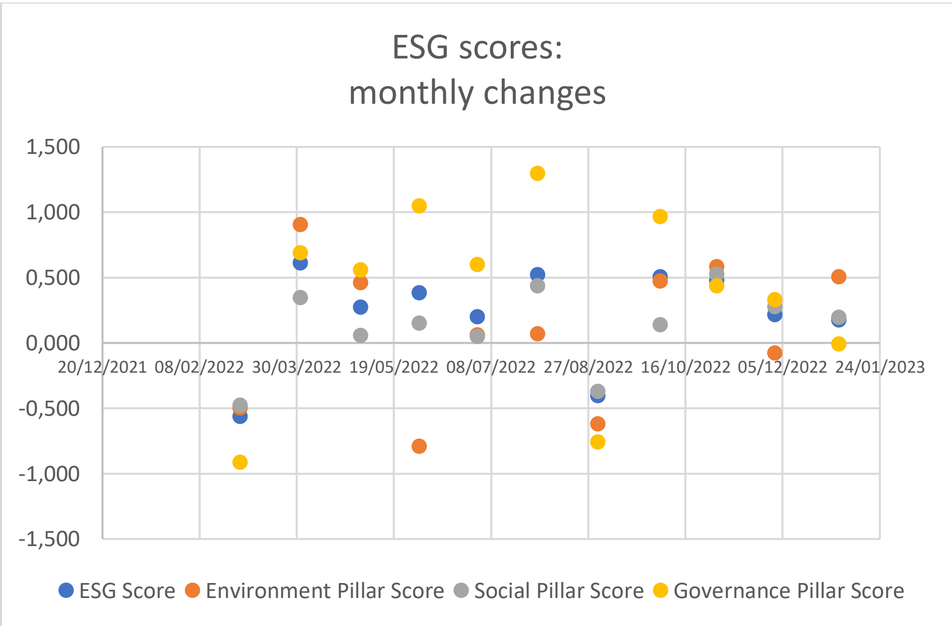
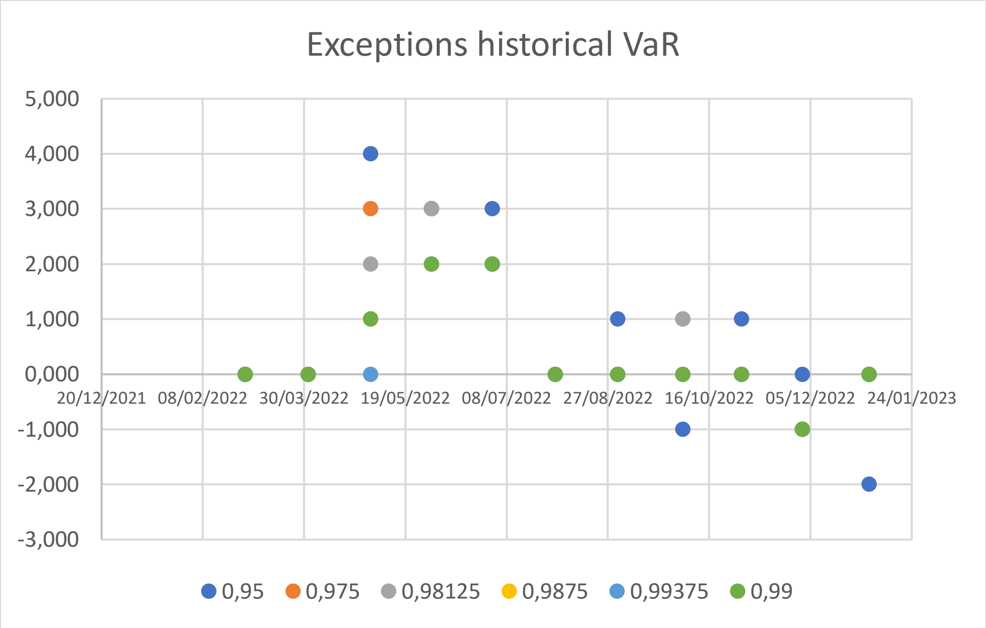
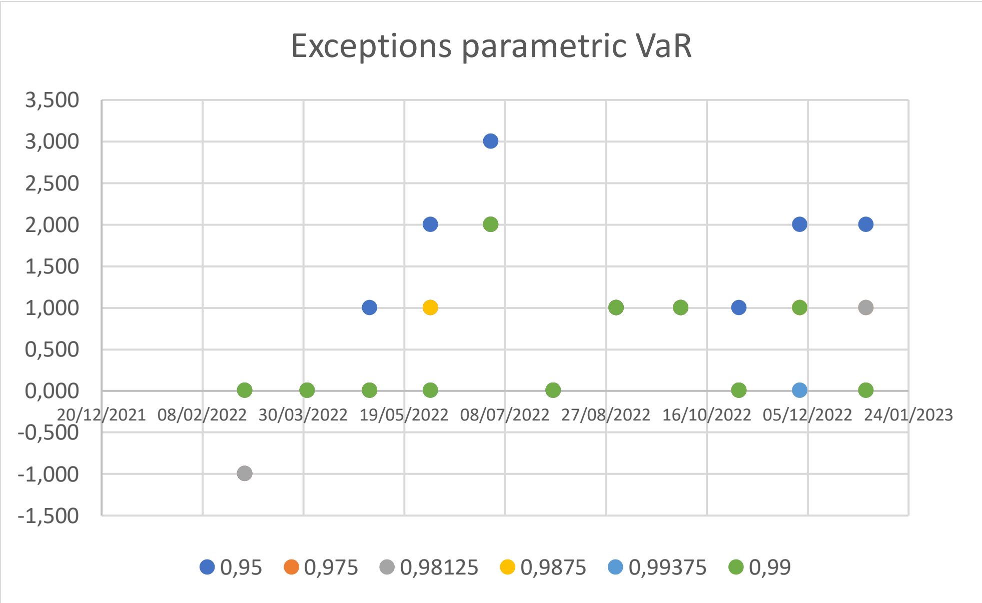


Figure 8: VTWO, Cross sectional analysis results




Exceptions historical VaR
5,000
4,000
3,000
2,000
1,000
0,000
-1,000
-2,000
-3,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99


Exceptions parametric VaR
3,500
3,000
2,500
2,000
1,500
1,000
0,500
0,000
-0,500
-1,000
-1,500
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

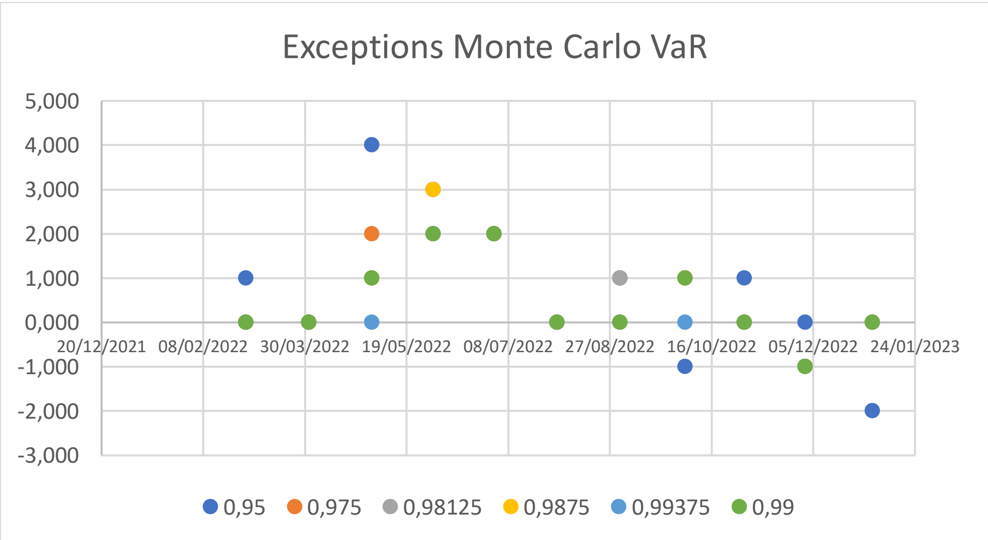
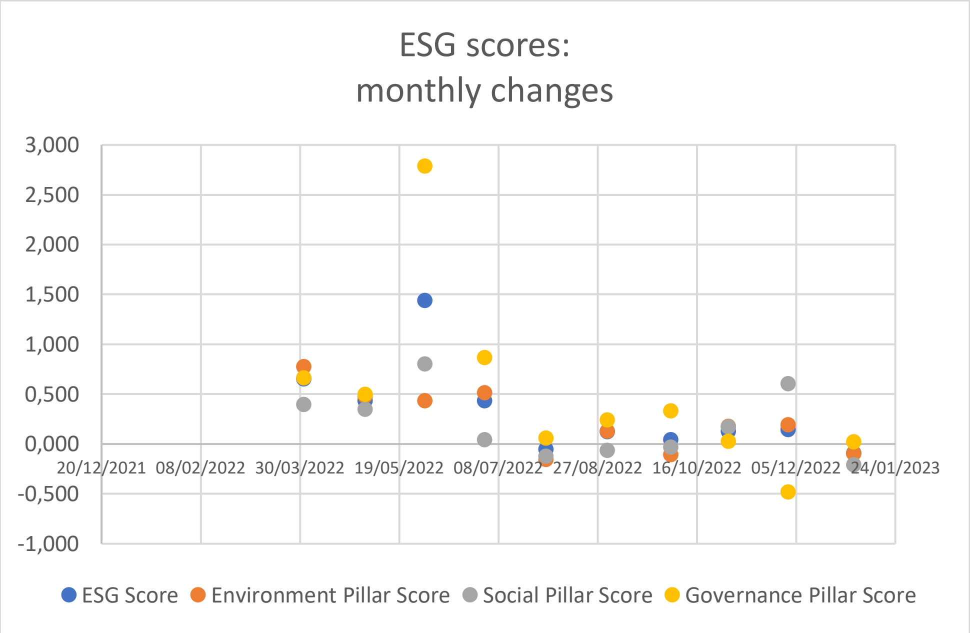
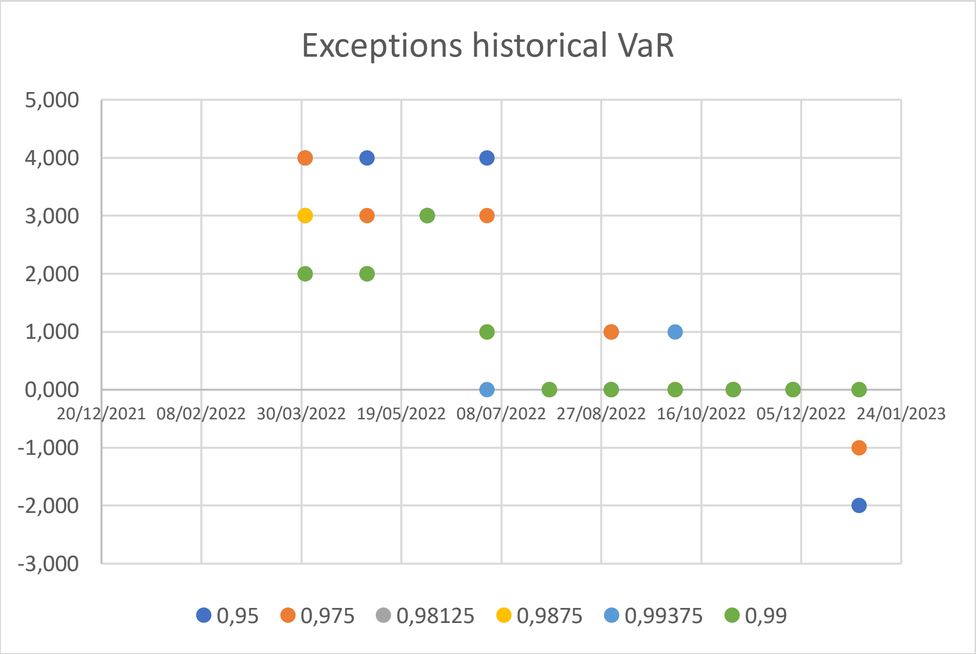
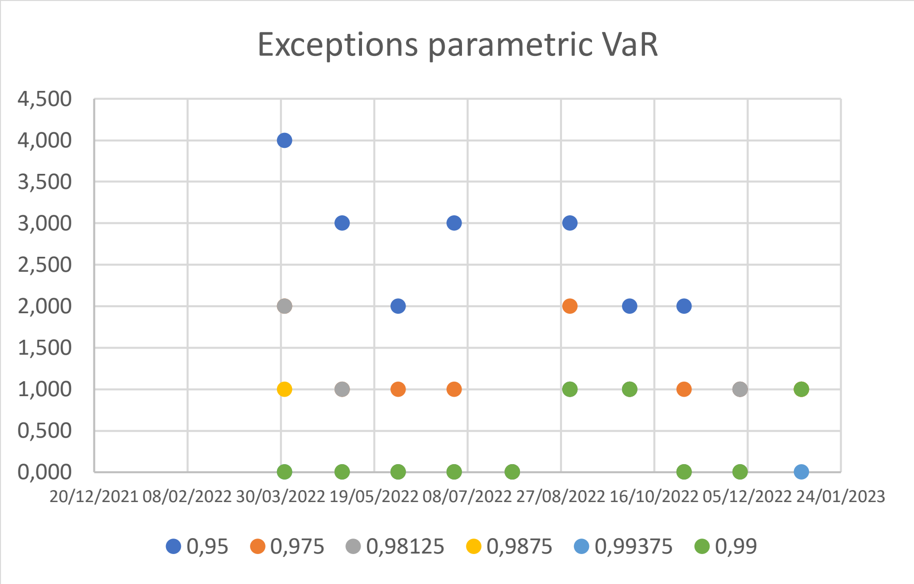
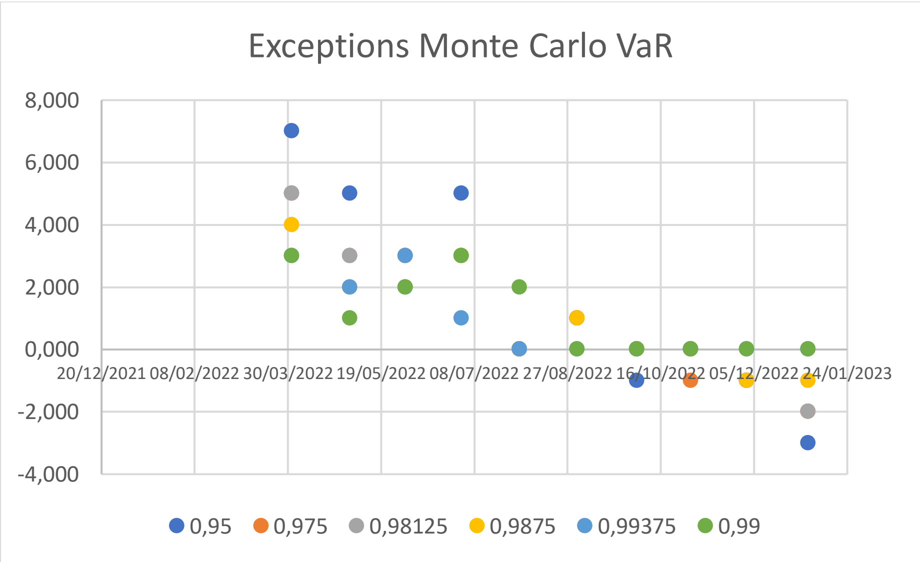

Exceptions Monte Carlo VaR
5,000
4,000
3,000
2,000
1,000
0,000
-1,000
-2,000
-3,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99

Figure 9: VUG, Cross sectional analysis results






Exceptions parametric VaR
4,500
4,000
3,500
3,000
2,500
2,000
1,500
1,000
0,500
0,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99


Exceptions Monte Carlo VaR
8,000
6,000
4,000
2,000
0,000
-2,000
-4,000
20/12/2021 08/02/2022 30/03/2022 19/05/2022 08/07/2022 27/08/2022 16/10/2022 05/12/2022 24/01/2023
0,95 0,975 0,98125 0,9875 0,99375 0,99