Financial Uncertainty and Asset Volatility Dynamics: Insights from an Extended Stochastic Volatility Model

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1 Introduction

Volatility modelling and forecasting play a vital role in various economic and financial applications, making them of utmost importance to a wide range of economic actors. Investors, portfolio managers, financial institutions, policymakers and even industrial firms, rely on understanding and predicting volatility to make informed decisions regarding asset allocation, hedging strategies, risk management or the formulation of well-sounded economic policies. The dispersion of asset returns notably holds significant importance in the Modern Portfolio Theory (MPT) pioneered by Markowitz (1952) and is also integral to the pricing and hedging of derivatives products. In addition, the monitoring of volatility stands as a crucial responsibility for financial regulators in maintaining stability within the financial system and safeguarding against excessive systemic risk. In the broader context, understanding the dynamic of asset volatility extends beyond financial markets by providing invaluable insights into forthcoming economic conditions, thereby helping institutions in the formulation of sound economic policies.

Research by Corradi et al. (2013) and Engle et al. (2013) has shown that volatility and large price variations are determined not only by the equilibrium between supply and demand but also by exogenous factors such as macroeconomic determinants. This highlights the importance of considering broader economic factors when modelling and forecasting volatility. Among potential candidates, one can find economic uncertainty, which has become increasingly popular in the aftermath of the Global Financial Crisis (GFC). Although economic uncertainty remains an amorphous concept, it is generally accepted to be a latent state faced by economic agents and characterized by a lack of predictability or clarity regarding future economic conditions (Bloom, 2014).

Many studies have demonstrated that uncertainty shocks have negative effects on various economic variables, including investment levels, consumption, unemployment, and credit conditions (see Bloom, 2009; Bloom et al., 2007; Gilchrist et al., 2014, among others). Given the widespread repercussions of uncertainty shocks, it is, therefore, reasonable to postulate that such a state marked by a prevailing information asymmetry could also influence investor behaviour and, in turn, have a cascading effect on asset volatility. Among the different components of economic uncertainty, financial uncertainty is systematically regarded as a key driver of asset returns and volatility, in contrast to other forms of uncertainty. As a result, our first and main research contribution is focused on the central question: *"Is there a proven* *relationship between financial uncertainty and financial asset volatility, and to what extent does financial uncertainty help in the modelling and forecasting of asset volatility?"*.

To explore this research question, we initially examine the potential transmission channels that may exist between financial uncertainty and asset volatility, and then leverage the Extended Stochastic Volatility model (SVX) of Ulm and Hambuckers (2022) to explore the dynamics of the relationship between the two variables. In this model, a measure of lagged financial uncertainty is incorporated as an additional covariate to enhance the explanation and modelling of asset returns' dispersion. By doing so, we aim to better understand how past financial uncertainty might influence the current level of asset conditional volatility. Our study's originality lies in its methodological approach, as it departs from the more common use of Heterogeneous Autoregressive (HAR) and Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models for investigating the predictive ability of uncertainty on asset volatility (see for example Asgharian et al., 2023; Liu & Zhang, 2015; Yu & Huang, 2021). Instead, we have relied on a discrete stochastic volatility model which offers numerous advantages and generally outperforms other volatility models (Bauwens et al., 2012; Chan & Grant, 2016), but has remained unexplored for this application. With this study, we, therefore, aim to fill this gap and inspect if our results are coherent with other models in use.

Despite the inherent unquantifiable nature of uncertainty, researchers have still attempted to gauge it using various proxies, but no theoretical consensus has been reached so far. As raised by Jurado et al. (2015), this situation is rather concerning and usually implies that the results of research works are dependent on the proxy used by the author. In order to tackle this issue, the second objective of this study is to present a comprehensive review of commonly employed techniques to quantify uncertainty, and in the end to construct a daily synthetic financial uncertainty index for the United States (US) region. Using a Principal Component Analysis (PCA), we measure financial uncertainty as the first principal component of a bunch of widely accepted proxies. Hence, this factor approach assumes that financial uncertainty materializes if all the proxies move together and allows us to overcome specific weaknesses associated with individual proxies. To the best of our knowledge, we are the first to propose a synthetic and general measure of financial uncertainty for the US region at a daily frequency. This synthetic index will eventually be fed into our SVX model in an attempt to answer our main research question.

Our two other contributions, though more minor in scope, still offer fresh perspectives on the relationship between financial uncertainty and asset volatility. On the one hand, we investigate how financial uncertainty might heterogeneously affect the volatility of different asset classes. By applying our SVX model to various asset types such as stocks, oil, exchange rates, bonds, and real estate, we try to shed light on potential variations in the relationship between financial uncertainty and volatility across these assets. This analysis is particularly valuable as different asset classes may respond differently to uncertainty shocks due to their inherent characteristics and market dynamics. Additionally, within the stocks category, the study further delves into potential differences by classifying them into eight industries. This approach recognizes that heightened uncertainty can have varying impacts on specific industries during times of uncertainty (Baek et al., 2020). On the other hand, we present an application of our SVX model through the creation of a parametric Value-at-Risk. This is intended to demonstrate that our model coupled with a measure of financial uncertainty can have significant practical value for practitioners in risk management.

Our empirical study yields compelling evidence that one-day past financial uncertainty level positively influences current conditional volatility for all asset classes. It means that a positive financial uncertainty shock will boost the overall market volatility in the next period. In terms of magnitude, we observed that the volatility of high-yield bonds, oil, and stocks in the energy sector is particularly sensitive to changes in financial uncertainty. At the other end of the scale, volatility of stocks in the industrial, consumer staples, and healthcare sectors shows less responsiveness to financial uncertainty while being still impacted. In addition, we found that financial uncertainty is proved to be a statistically significant predictor of volatility and clearly improves the in-sample and out-of-sample model performance. This improvement is particularly pronounced during periods of market stress and economic turmoil, subject to financial uncertainty peaks. Such periods are particularly challenging for most volatility models, but our SVX model demonstrates its effectiveness in capturing the dynamics of heightened volatility during these challenging phases. These findings not only bear statistical significance but also align seamlessly with economic intuition. Importantly, they confirm the conclusions of Asgharian et al. (2023) regarding the sign of the relationship between these two variables and is well aligned with the findings of Corradi et al. (2013) who asserted that volatility demonstrates countercyclical behaviour and is substantively explicable through exogenous factors. Additionally, we also confirmed the robustness of our results by showing that financial uncertainty was consistently found to be a powerful predictor against additional uncertainty components emanating from other sources (i.e. macroeconomic, economic policy, geopolitical). Contrary to the studies of S. R. Baker et al. (2016) and Liu and Zhang (2015), we found that economic policy uncertainty is only a relevant predictor for a couple of particular assets when considered in combination with financial uncertainty.

We identified corporate financial performance and investor sentiment as the primary transmission channels conveying financial uncertainty effects to asset return and volatility. The close link between financial uncertainty and investor sentiment has been little investigated until now, but through a regression analysis, we support that a higher financial uncertainty level can lead to more pessimistic investor sentiment. As described by Berardi (2022) and Birru and Young (2022), investor sentiment may in turn impact asset valuation and market volatility.

Lastly, our Value-at-Risk (VaR) application further supports the utility of financial uncertainty in volatility modelling. The improved backtesting results of VaR constructed using our SVX model, as compared to a simple stochastic volatility model, underscore its superior capacity to capture market risk and tail events.

The remaining sections of this article are structured as follows: Section 2 provides a comprehensive literature review on financial uncertainty and asset volatility, establishing the theoretical foundations underpinning this work. In Section 3, we outline the empirical methods employed, with a particular focus on the extended stochastic volatility model utilized to capture the volatility of our diverse financial assets, along with an in-depth explanation of the estimation procedure. Furthermore, we describe the construction of our synthetic financial uncertainty index in a separate subsection, employing a PCA method. Section 4 offers a detailed overview of the data used in our analysis. Next, in Section 5, we engage in a thorough discussion and interpretation of the results obtained. We also check the robustness of our findings and make some diagnostics to verify the accuracy of SVX model's outputs. Finally, Section 6 concludes this study by summarizing the findings and highlighting potential avenues for future research and exploration.

2 Literature Review

This research work builds upon two strands in the economic and financial literature. First, it participates in fuelling the debate on economic uncertainty, its proxies, and its impact on the financial world. Second, it contributes to the tasks of volatility modelling and forecasting by relying on the use of exogenous factors. To this end, let us review the existing literature on these two subject matters that we eventually connect so that it will lay the groundwork for this research.

2.1 Nature of uncertainty

The modern notion of uncertainty is not recent and dates back over a century ago as it was first introduced in 1921 by Frank Knight in his book *Risk, Uncertainty, and Profit*, who coined the term "Knightian uncertainty" (Knight, 1921). At that time, he was the first to move away from the classical perspective and make a sharp distinction between the concepts of risk and uncertainty. From his point of view, risk pertains to situations where the probabilities of different outcomes are known or can be estimated, whereas uncertainty arises in situations where economic agents lack sufficient information to estimate and assign probabilities to the different outcomes. In fact, uncertainty even goes beyond a mere lack of information as it also describes a total inability to list all future events, as famously described by the "Black Swans" of Taleb (2010). This idea of unpredictability was also put forward by Davidson (1991), who rejected the use of probability theory as the basis for understanding uncertainty and defined it as an *"ignorance about future consequences"*. More recently, various authors have come up with slight variations in the definition of uncertainty, but they all maintain this fundamental idea of unforeseeable changes. For instance, Jurado et al. (2015) adopt a more econometric perspective and define uncertainty as *"the conditional volatility of a disturbance that is unforecastable from the perspective of economic agents."*. In this work, we will adopt this vision of uncertainty reflected in these definitions, as opposed to the mainstream perspective (see for example Machina, 1987) which blends the notions of risk and uncertainty and for which past is a reliable predictor of future.

Although the literature on uncertainty and its impact on the economy is quite old (see for example Dixit & Pindyck, 1994), it has only gained particular interest in the last decade driven by several influential factors. Firstly, the aftermath of the global financial crisis of 2008-2009, has spurred researchers to inspect the role played by uncertainty in shaping and exacerbating this financial cataclysm and economic downturn. Secondly, the emergence of modern computing techniques and the rise of Big Data have greatly facilitated the estimation of novel uncertainty proxies. While uncertainty remains an unmeasurable and latent variable, these technological advancements have enabled more refined and comprehensive estimations of uncertainty proxies, fueling a growing interest in their use in economic models. Before detailing a list of uncertainty measurement methods (see Section 2.2), it is possible to draw some stylized facts and patterns that economic uncertainty regularly exhibits. Here are the main ones:

Fact 1: Uncertainty has various origins

One notable observation is that economic uncertainty can arise from various sources. In this respect, Himounet (2022) surveyed the literature and identified that scholars commonly distinguish between different uncertainty components, including macroeconomic, policy, financial, geopolitical, and microeconomic uncertainty. More precisely, macroeconomic uncertainty relates to fluctuations and unpredictability in macroeconomic variables such as GDP growth, inflation, interest rates, and employment as well as monetary policies. Financial uncertainty is specific to financial markets and is often associated with market instability and a high degree of information asymmetry in asset prices and between investors themselves. Geopolitical uncertainty specifically emerges from geopolitical events, such as conflicts, wars, and international tensions, but can also stem from natural disasters. Policy uncertainty is characterized by unpredictability in government policies and regulations, including fiscal policies, trade policies, and health policies. Finally, microeconomic uncertainty manifests at the firm or industry level due to erratic consumer preferences, technological disruptions, or market competition. Although there may be some underlying commonalities among certain uncertainty components, they typically represent distinct phenomena with unique characteristics, driven by different events and factors.

Fact 2: Uncertainty rises in recessions

A well-established finding is that uncertainty tends to rise sharply during recessions and periods of economic turmoil. Empirical evidence consistently demonstrates that uncertainty, whatever its origin and the way it is proxied, exhibits a countercyclical pattern (Bloom, 2014). However, whether uncertainty is an exogenous driver of business cycles or an endogenous response to a shock in economic fundamentals remains a subject of debate. Ludvigson et al. (2021) addressed this question and identified that uncertainty can have both endogenous and exogenous characteristics, usually depending on its origin. Specifically, heightened financial uncertainty was found to be an exogenous (i.e. causal) factor driving recessions, while increased macroeconomic and policy uncertainty were identified as negative consequences of business fluctuations and shocks in economic fundamentals. Carriero, Clark, and Marcellino (2018) reached similar conclusions regarding macroeconomic uncertainty but in addition found that a part of financial uncertainty can be, to some extent, an endogenous response to certain macroeconomic developments, which has been confirmed in the context of monetary policy by Bekaert et al. (2013) and Crucil et al. (2023).

Fact 3: Uncertainty is contagious

Economic uncertainty is not confined to national boundaries. Major foreign events and uncertainty shocks can transmit to countries' domestic uncertainty levels. These spillover effects have been studied in particular by Beckmann et al. (2023) and Karanasos et al. (2021), who highlight the interconnectedness and interdependence of today's economies facilitating the propagation of uncertainty overseas. Their research also underscores two key points. Firstly, these contagious effects of uncertainty are more likely to occur from developed and major economies to emerging markets. In particular, the US, the Eurozone, the UK and China contribute significantly to the global common uncertainty level. Secondly, interest rates and exchange rates are pointed out as the main transmission channels.

2.2 Measurement of uncertainty

Uncertainty, by its nature, remains theoretically unquantifiable and unmeasurable. Despite this fact, numerous researchers have made multiple attempts to gauge it using various proxies without reaching any theoretical consensus. Broadly speaking, following Cascaldi-Garcia et al. (2021), methods for measuring uncertainty can be categorized into four groups: survey-based methods, textual analysis measures, econometric measures and market data measures. We further extend this categorization with a fifth group encompassing factor-based methods.

(1) Survey-based methods

Survey-based methods involve collecting data through surveys and examining the degree of disagreement in forecasts among economic forecasters. One of the earliest methods for proxying uncertainty is the dispersion of forecasts as introduced by Zarnowitz and Lambros (1987), where they modelled inflation uncertainty as the standard deviation of predictive probability distributions attached to point forecasts. Generally, a strong disagreement or a high degree of dispersion in economic forecasts reveals that economists have divergent views and there is greater uncertainty surrounding the future path of the economy (Bomberger, 1996). More recently, Bachmann et al. (2013) extracted a measure of economic uncertainty for Germany using the disagreement among business leaders in a forward-looking survey that reflects their expectations for future business conditions. Survey methods are also popular for evaluating macroeconomic uncertainty. For example, Scotti (2016) worked out a real-time macroeconomic uncertainty index based on the surprise level between Bloomberg forecasters' expectations and actual releases of macroeconomic variables. In this case, it represents an ex-post survey-based measure of uncertainty.

(2) Textual analysis based methods

Textual analysis methods have recently emerged as effective approaches for gauging economic uncertainty, leveraging machine learning algorithms and natural language processing techniques. The pioneering work of S. R. Baker et al. (2016) laid the foundation for employing these methods in this context. By analyzing the frequency of words related to "economic", "policy" and "uncertainty" in major newspapers, they developed the Economic Policy Uncertainty Index. Initially devised for the United States, this approach has been replicated for other countries as well[1]. Consequently, the Economic Policy Uncertainty index is now available for 22 countries, which enables global comparison of domestic uncertainty levels. Building upon this methodology, S. R. Baker et al. (2019) derived the US Equity Market Volatility tracker (EMV), which intends to capture financial uncertainty. The newspaper coverage, in this case, is based on words linked to the equity market, economy and uncertainty. By recognizing the importance of news as a valuable source of market uncertainty information, the EMV index provides a sweeping measure of the uncertainty surrounding the equity market, and distinguishes itself from traditional implied volatility indices that solely rely on historical data or options pricing models (e.g. CBOE VIX or VXO). Finally, still in a similar fashion, Caldara and Iacoviello (2022) constructed the Geopolitical Risk Index (GPR), which measures uncertainty emanating from geopolitical tensions and events. Without any surprise, this index peaks during notable periods such as the Gulf War, the 9/11 attacks, the Iraqi War but as well as during the Covid-19 pandemic and the Russia-Ukrainian War.

(3) Econometric-based methods

An alternative approach to computing uncertainty involves the use of econometric and statistical models. Two prominent works in this field are the studies of Jurado et al. (2015) (JLN) and Ludvigson et al. (2021), who estimate the h-period ahead forecasts of macroeconomic and financial uncertainty for the United States as the common variations in the unforecastable component of a large panel of macroeconomic and financial observations. Recently, their approach was extended to European countries by Fortin et al. (2023). Rossi and Sekhposyan (2015) quantified quarterly macroeconomic uncertainty through the lens of nowcasting methods and the distribution of forecast errors of macroeconomic variables. If the observed forecast error corresponds to a high quantile in their historical empirical distribution, they conclude that there is a high level of uncertainty. With this methodology, Ismailov and Rossi (2018) constructed a monthly exchange uncertainty index for a range of developed countries. On a different note, some researchers have employed dynamic volatility models to directly estimate uncertainty. In this case, they assume that conditional volatility serves as a relevant proxy for uncertainty. Baum and Wan (2010), for instance, estimated macroeconomic uncertainty from the conditional variance of the GDP's growth rate estimated with a GARCH model. On the same idea, stochastic volatility models can also be used to represent uncertainty. For example, Chan and Song (2018) extracted inflation uncertainty from the stochastic volatility component within an augmented univariate unobserved components model (UCSV-X) or

[1] Please refer to the "Economic Policy Uncertainty" website available under the hereafter URL to find the complete reference of other Economic Policy Uncertainty indexes. https://www.policyuncertainty.com/all_country_data.html

Carriero, Clark, and Marcellino (2018) carried out macroeconomic and financial uncertainty estimates from a large vector autoregression (VAR) with stochastic volatility.

(4) Market data based methods

Lastly, a large part of uncertainty proxies emerges directly from observable market data that are known to co-move with uncertainty. Currently, a vast amount of data are freely available from financial markets and are a real goldmine for constructing uncertainty proxies. Among them, options-implied volatility indices such as VIX or VXO are among the most widely used proxies for financial uncertainty (Bekaert et al., 2013; Bloom, 2009; Caggiano et al., 2014; Crucil et al., 2023; Hambuckers et al., 2018). VIX Index is often referred to as the "fear gauge" because when market stress and uncertainty increase, investors tend to demand higher premiums for options, resulting in higher implied volatility (Whaley, 2000). Incidentally, it provides insights into investors' expectations regarding future market conditions and can thus be interpreted as a relevant indicator of anticipated market uncertainty. Moreover, options data have also been harnessed by Dew-Becker and Giglio (2023) to construct a firm-level uncertainty index based on a cross-sectional implied volatility measure of individual firm options.

Apart from options data, in both the equity and debt markets, metrics like bond spreads, credit default swap spreads, and market liquidity have surfaced as robust uncertainty indicators. Indeed, in times of uncertainty and market stress, the first two generally widen (Baum & Wan, 2010; Gilchrist et al., 2014), whereas the latter is shown to dry up (Easley & O'Hara, 2010; Rehse et al., 2019).

(5) Factor-based methods

As raised by Jurado et al. (2015), the absence of consensus poses a significant challenge. Indeed, depending on the measurement method employed, research papers may reach differing conclusions regarding uncertainty and its impact on the economy. Indeed, each method of calculating uncertainty has its own strengths and flaws and performance may vary depending on the specific situation. Interestingly, different methods of measuring uncertainty often capture distinct dimensions of uncertainty even when they aim proxying the same underlying uncertainty source. Consequently, some authors have attempted to overcome these limitations by leveraging the advantages of multiple approaches and constructing composite uncertainty indexes. Two commonly used dimensionality reduction algorithms for this purpose are the Dynamic Factor Model (DFM) and Principal Component Analysis (PCA). These techniques aim to identify the common factors among different proxies of uncertainty, providing a more comprehensive and robust measure of uncertainty. While these dimensionality reduction algorithms have gained popularity in constructing investor sentiment indexes (M. Baker & Wurgler, 2007), their application in the context of uncertainty measurement is relatively limited compared to other methods. However, the literature on this topic is gradually expanding, with some recent works. For example, Caggiano and Castelnuovo (2023) extracted a monthly Global Financial Uncertainty Index using a dynamic hierarchical factor model (DHFM), whereas Himounet (2022) constructed a monthly general measure of uncertainty for the United States using PCA. Although these methods seem to be primarily used to construct global uncertainty indexes by aggregating domestic uncertainty indexes, it would also be valid to employ them in designing a synthetic index based on different proxies for a specific type of uncertainty within a specific economic region. This reflection motivates one of our research objectives, which aims to create a synthetic financial uncertainty index for the United States available at a daily frequency. This will bridge a gap in the literature since, as far as we know, there is up to now no comprehensive financial uncertainty index available at this high frequency and for this economic region.

To go further, comprehensive surveys on distinct uncertainty proxy methodologies are proposed by Cascaldi-Garcia et al. (2021) and Himounet (2022).

2.3 Economic and financial impacts of uncertainty shocks

We know from a great number of research works that shocks in economic uncertainty negatively impact both a large series of economic parameters and the general level of economic outcome. The most notable negative effects are the following. In times of heightened uncertainty, Bloom et al. (2007) demonstrated that firms dampen investments and adopt a more cautious approach in their capital expenditures policy. In a subsequent study, Bloom et al. (2022) quantified that a two-standard deviation increase in uncertainty, as proxied by a survey, leads to a drop in firms' investment level of 6%. This fact originates essentially from the interplay of three phenomena. First, the "" behaviour of firms, as evidenced by Dixit and Pindyck (1994), where the option value of waiting is increasing due to information asymmetry resulting from uncertainty shocks. This is commonly referred to as the "real-option" effect. Second, the "financial frictions" effect, identified by Gilchrist et al. (2014), arises from widening credit spreads and exacerbates the decline in investments. Credit spreads, in this context, act as transmission factors for uncertainty to firms. Finally, Jurado et al. (2015) identified another effect resulting from the higher risk aversion of economic agents, known as the "precautionary savings" effect. During times of uncertainty, individuals and firms tend to increase their savings and reduce spending as a precautionary measure against future adverse events. Similar to investment decisions, hiring decisions are also impacted by rising uncertainty. In a more general study about the economic impacts of uncertainty shocks, Bloom (2009) found that hiring follows a similar pattern as investment, with firms adopting a wait-and-see approach resulting from the adjustment costs associated with hiring and firing. In this case, the lack of cost flexibility represents the friction mechanism. Overall, this raises the unemployment level but reinforced the demand for temporary workers (Bloom et al., 2022). On top of that, they showed that productivity and sales growth also experienced a drastic decline as a result of the cuts in investments and hiring. We know from a great number of research works that shocks in economic uncertainty negatively impact both a large series of economic parameters and the general level of economic outcome. The most notable negative effects are the following. In times of heightened uncertainty, Bloom et al. (2007) demonstrated that firms dampen investments and adopt a more cautious approach in their capital expenditures policy. In a subsequent study, Bloom et al. (2022) quantified that a two-standard deviation increase in uncertainty, as proxied by a survey, leads to a drop in firms' investment level of 6%. This fact originates essentially from the interplay of three phenomena. First, the "wait-and-see" behaviour of firms, as evidenced by Dixit and Pindyck (1994), where the option value of waiting is increasing due to information asymmetry resulting from uncertainty shocks. This is commonly referred to as the "real-option" effect. Second, the "financial friction" effect, identified by Gilchrist et al. (2014), arises from widening credit spreads and exacerbates the decline in investments. Credit spreads, in this context, act as transmission factors for uncertainty to firms. Finally, Jurado et al. (2015) identified another effect resulting from the higher risk aversion of economic agents, known as the "precautionary savings" effect. During times of uncertainty, individuals and firms tend to increase their savings and reduce spending as a precautionary measure against future adverse events. Similar to investment decisions, hiring decisions are also impacted by rising uncertainty. In a more general study about the economic impacts of uncertainty shocks, Bloom (2009) found that hiring follows a similar pattern as investment, with firms adopting a wait-and-see approach resulting from the adjustment costs associated with hiring and firing. In this case, the lack of cost flexibility represents the friction mechanism. Overall, this raises the unemployment level but reinforced the demand for temporary workers (Bloom et al., 2022). On top of that, they showed that productivity and sales growth also experienced a drastic decline as a result of the cuts in investments and hiring.

Similarly, financial parameters and variables are also sensitive to uncertainty shocks but with mixed evidence depending on the uncertainty source. When it comes to financial uncertainty, there is almost unanimous support for the view that it reduces asset returns and boosts asset volatility (Asgharian et al., 2023; Caldara et al., 2016; Carriero, Clark, & Marcellino, 2018; Fortin et al., 2023). Financial uncertainty also leads to a significant decline in the valuation ratios of financial assets (Bansal et al., 2005). Regarding macroeconomic uncertainty, Arnold and Vrugt (2008) identified a positive relationship between survey-based macro-uncertainty and asset volatility from 1969 to 1996, but this relationship diminished from 1997 onwards, coinciding with a regime change in volatility. These findings are relatively coherent with the ones found by Carriero, Clark, and Marcellino (2018), who stated that macroeconomic uncertainty is barely significant to explain changes in stock prices. Nevertheless, the debate regarding the effect of economic policy uncertainty on financial variables is less clear-cut. On one side, some authors argued that policy uncertainty results in negative and more dispersed stock returns (S. R. Baker et al., 2016; Liu & Zhang, 2015; Pástor & Veronesi, 2012). Conversely, Asgharian et al. (2023) estimates that economic policy uncertainty does not have any predictive power compared to financial uncertainty when forecasting stock market volatility. Overall, the general conclusion that can be drawn is that financial variables are more responsive to financial uncertainty compared to other sources of uncertainty. Therefore, as we perceive it, it is more economically meaningful to focus our analysis on how financial uncertainty can be leveraged to model and forecast asset volatility.

In a large majority of scientific papers, the study of the impacts on economic and financial variables of uncertainty shocks is conducted by means of vector autoregressive (VAR) models and their numerous variants. These models are commonly employed in research works such as S. R. Baker et al. (2016), Bloom (2009), Carriero, Clark, and Marcellino (2018), Crucil et al. (2023), Ludvigson et al. (2021), and Scotti (2016), among others. In contrast, the use of this type of model is more complicated for studying the effect of uncertainty on volatility and is not favoured for this application in the literature. Unlike macroeconomic variables, financial variables, especially volatility, exhibit highly non-linear behaviours (e.g., volatility clustering), which may not be adequately captured by the linearity assumption of VAR models. Instead, scholars have turned to augmented time-varying volatility models, incorporating the predictive power of exogenous variables, such as measures of economic uncertainty, to better understand and model volatility dynamics. Several types of volatility model exist and compete with each other.

Among them, the Heterogeneous Auto-Regressive (HAR) model developed by Corsi (2009) along with its modified versions, are the most popular ones for this application. For instance, Asgharian et al. (2023) studied the impact of both time-varying uncertainty and risk aversion on the volatility of the S&P500 and seven stock indices using an extended HARQ-RV model. Specifically, they found that financial uncertainty and risk aversion have significant predictive power for stock market volatility, but economic policy uncertainty does not. This contrasts with the conclusion of Liu and Zhang (2015), who empirically demonstrated that past economic policy uncertainty improves the in-sample and out-of-sample performance of S&P 500 volatility modelling through eight variants of the HAR model. Another notable study by Wen et al. (2019) revealed that equity market uncertainty serves as a significant predictor of short-term crude oil futures realized volatility, based on their examination of six HAR-type models. Due to their ability to parsimoniously capture the long-memory effect observed in realized volatility and their simple estimation via OLS, HAR models are currently considered the workhorse in volatility forecasting.

Generalized AutoRegressive Conditional Heteroskedasticity (GARCH)-type models are the second type of model extensively popular in the literature. More particularly, the GARCH-MIDAS alternative proposed by Engle et al. (2013) has gained prominence, as it allows the incorporation of low-frequency data to model the conditional variance of high-frequency data. Relying on this model, Yu and Huang (2021) showed that the low-frequency information contained in the monthly Chinese EPU index improves the in-sample and out-of-sample performance of Chinese stock volatility modelisation.

The last class of volatility model that could have been utilized for this task, though seemingly unexplored, are discrete stochastic volatility (SV) models. In comparison to GARCH or HAR models, SV models offer, nevertheless, distinct benefits (Bauwens et al., 2012). Economically speaking, SV models better align with the true nature of volatility by considering it as an unobserved and latent state, as it behaves in the real world. In other words, SV models treat volatility as a random variable, while GARCH models treat it as a deterministic function of the model's parameters and past observations. Consequently, SV models hold greater economic significance in their representation of conditional volatility. From a statistical standpoint, SV models possess better capacities and greater flexibility in fitting both fat-tailed distributions of innovations and data exhibiting large variations. As a result, it is often unnecessary to relax the normality assumption of innovations in these models. On the flip side, the estimation of SV models is far more complex compared to GARCH models, as its likelihood function is intractable and needs to be numerically approximated. Despite this increased complexity in their estimation, a model comparison study conducted by Chan and Grant (2016) concluded that stochastic volatility models generally outperform their GARCH counterparts, further justifying their use. Similarly to HAR and GARCH models, an extended version of the log-volatility model of Taylor (1986) with additional explanatory covariates can be found in the literature and has been proposed by Ulm and Hambuckers (2022). They primarily used it to model exchange rate volatility using an interest rate differentials factor, but there are all reasons to believe that this model could be suitable for explaining the relationship between uncertainty and asset volatility.

2.4 From uncertainty to asset volatility: transmission channels

A causal relationship between financial uncertainty and asset volatility is predicated on the existence of transmission channels, whereby any changes in the level of financial uncertainty are passed on to the level of asset volatility. From the existing literature, we identified investor sentiment, and corporate financial performance as possible transmission channels from financial uncertainty to market volatility. Each of these channels contributes differently to the impact of uncertainty on market volatility.

A relevant starting point is a study conducted by Berardi (2022), who suggested that sentiment is closely tied to the level of uncertainty present in the economy. According to his view, in the absence of uncertainty, sentiments would not arise at all, and it is the degree of uncertainty that provides room for individuals' psychological attitudes to significantly influence their expectations. To illustrate this concept, he explains that if investors possess complete information and were fully aware of the fundamental value of an asset there would be no room for psychological attitudes to impact beliefs about its value. However, in reality, it is the presence of imperfect information and uncertainty that allow subjective beliefs to come into play and affect how investors value assets. On the other hand, Dicks and Fulghieri (2021) showed that investors' beliefs and sentiment are endogenous to uncertainty and are the driver of innovation waves and equity valuation. In times of heightened uncertainty, investor sentiment can be seriously impacted by the prevailing information asymmetry in the market. Typically, investors might have divergent opinions and face more difficulties in accurately assessing the future value of assets impacting their risk aversion level and their confidence level in their own actions. These results, therefore, highlight an inability of investors to rely on the "wisdom of the crowd" that may push them to adopt herding behaviours based on their current sentiment. In addition, Birru and Young (2022) provide empirical evidence showing that sentiment has a significant predictive ability for stock returns during high-uncertainty periods, while this predictive effect fades away in low-uncertainty times. Similarly, they also noticed that sentiment has deeper effects on high-volatility stocks when uncertainty is high. All these studies corroborate the conclusion of M. Baker and Wurgler (2007) that lower investor sentiment increases risk aversion and affects stock returns.

Based on these facts, it is reasonable to consider that investor sentiment and beliefs act as important transmission channels, allowing the effects of financial uncertainty to propagate to asset valuation and volatility.

The second pathway through which financial uncertainty affects asset volatility is related to corporate performance, which is the result of a large number of idiosyncratic factors likely to be affected in times of high uncertainty. From the previous section, we know that uncertainty shocks can impact firms at different levels. Firstly, uncertainty shocks tend to push firms to become more cautious, leading them to halt investment, pause expanding and reduce hiring (Bloom et al., 2007). These actions, in turn, result in a decline in productivity and sales growth (Bloom, 2009; Bloom et al., 2022). Then, firms may encounter difficulties in obtaining financing from the debt market. This is often attributed to widening credit spreads and deteriorating financial conditions rising the risk aversion of credit lenders (Gilchrist et al., 2014). It has also been demonstrated that rising financial uncertainty can increase the likelihood of incurring more extreme operational losses for firms (Hambuckers et al., 2018). Eventually, as shown by Gambetti et al. (2019), uncertainty can exacerbate credit losses by reducing the mean of recovery rate distributions. All of these impacts collectively contribute to poorer corporate performance and hinder investors' capacity to accurately forecast future cash flows for these firms. This can trigger a domino effect on asset prices, ultimately leading to heightened volatility in a firm's stock. Empirical confirmation of this relationship was provided by Brunnermeier and Sannikov (2014) who showed that increased stock volatility can be attributed to deteriorated balance sheet conditions.

Although we have identified investor sentiment and corporate performance as the main transmission channels linking financial uncertainty to asset volatility, it is crucial to acknowledge that the relationship between uncertainty and asset dynamics is multifaceted. There may exist additional transmission channels that warrant further investigation. Unfortunately, the present literature exploring the structured understanding of how financial uncertainty impacts the returns and volatility of financial assets is too thinly explored, leaving room for future research to delve deeper into this area. Since our research does not primarily focus on exploring these potential transmission channels but rather focuses on explaining how financial uncertainty drives asset volatility, we wisely leave the investigation of these aspects to future studies.

3 Models and methodology

In this section, a detailed description of all methods and methodologies employed to construct our synthetic financial uncertainty index and model the volatility of the different financial assets will be introduced. We start by presenting the Extended Stochastic Volatility (SVX) Model and by detailing all the estimation procedure to get the model parameters and latent volatility measures. We then review the construction of our uncertainty index which is worked out through the application of a PCA approach.

3.1 Extended stochastic volatility model (SVX)

To investigate whether financial uncertainty explains and helps to model the volatility of diverse financial assets, the stochastic volatility model proposed by Ulm and Hambuckers (2022) will be used. This model expands upon the classical standard Gaussian autoregressive model initially introduced by Taylor (1986) by incorporating additional covariates that have the potential to influence and explain the volatility level. Unlike the GARCH model, Taylor's approach considers volatility as an unobserved variable governed by its own stochastic process, taking the form of a stationary autoregressive model. It can be viewed as a discrete counterpart to the Hull and White (1987) continuous stochastic volatility model. The Taylor model can be represented as follows:

$$y_t - \mu_t = \sigma_t \epsilon_t, \qquad \epsilon_t \overset{\text{iid}}{\sim} N(0,1), \tag{3.1}$$

$$\sigma_t^2 = \exp(h_t), \tag{3.2}$$

$$h_t = \mu_h + \phi(h_{t-1} - \mu_h) + \eta_t, \quad \eta_t \overset{\text{iid}}{\sim} N(0, \omega^2), \tag{3.3}$$

In the above equations, $y_t - \mu_t$ represents the demeaned log returns, σ_t denotes the conditional volatility, and h_t represents the unobserved logarithmized volatility, following a stationary autoregressive model with an unconditional mean of μ_h. Naturally, this model relies on two main assumptions: (1) The innovation terms ϵ_t and η_t are assumed to be independent ($\epsilon_t \perp\!\!\!\perp \eta_t$) and normally distributed. (2) The volatility persistence parameter ϕ, which measures the degree of autocorrelation in squared returns, must fall within the range of -1 to 1 to ensure the stationarity and the ergodicity of the autoregressive process.

In their research, Ulm and Hambuckers (2022) proposed a modification to Equation 3.2 by incorporating additional covariates to account for their regression effects on the volatility level. As a result, the model now takes the following form:

$$y_t - \mu_t = \sigma_t \epsilon_t, \qquad \epsilon_t \overset{\text{iid}}{\sim} N(0,1), \tag{3.4}$$

$$\sigma_t^2 = \exp(h_t + \mathbf{x_{t-1}}\boldsymbol{\beta}), \tag{3.5}$$

$$h_t = \mu_h + \phi(h_{t-1} - \mu_h) + \eta_t, \quad \eta_t \overset{\text{iid}}{\sim} N(0,\omega^2) \tag{3.6}$$

where the term $\mathbf{x_{t-1}}\boldsymbol{\beta}$ incorporates the influence of additional covariates to explain the volatility of financial assets. In our study, we consider three such covariates. The first one (x_1) captures the impact of the financial uncertainty level one period before (u_{t-1}). The second covariate, x_2, accounts for ARCH effects by including the lagged log-return (y_{t-1}). Lastly, x_3 captures the leverage effect through a dummy variable that takes the value 1 if the lagged return (y_{t-1}) is negative, and 0 otherwise.

By introducing these additional explanatory variables into the model, we aim to capture and quantify their respective influences on the volatility of financial assets. This approach allows us to account for various factors that may contribute to the observed volatility patterns in the market.

3.2 Estimation procedure

The estimation problem poses a significant challenge in stochastic volatility models, which is one of the reasons why GARCH models are often favoured. This is primarily due to the intractable likelihood function associated with stochastic volatility models, making classical estimation techniques like maximum likelihood inapplicable. In the context of our SVX model, the likelihood function corresponds to the density defined by:

$$L(\theta; Y_T) \propto \int p(Y_T \mid \alpha_T; \theta) \times p(\alpha_T \mid \theta) d\alpha_T, \tag{3.7}$$

Here, $\theta = (\mu_h, \phi, \omega^2)'$ represents the parameter vector, α_T denotes the vector of unobserved components, and Y_T encompasses the vector of log-returns. However, this high-dimensional integral of dimension T, i.e. the sample size, lacks a closed-form solution, rendering it analytically unsolvable. Consequently, alternative estimation techniques must be employed.

Over the years, researchers have explored various methods to tackle this issue, broadly categorized into two main approaches: estimators based on moment expressions and estimators based on simulation procedures. While moment methods offer the advantage of relative simplicity in implementation, they tend to exhibit inefficiency and lack consistency in accuracy. On the other hand, simulation-based techniques for inference are more computationally expensive and time-consuming but they provide an effective means to approximate various likelihood functions, including those of non-linear state-space models such as our SVX model.

Moment-based inference

With moment-based inference techniques, we try to learn about the vector of parameter $\boldsymbol{\theta} = (\theta_1, ..., \theta_K)'$ given a set of data $\mathbf{Y} = (y_1, ..., y_T)'$, which represents in our case a sequence of daily returns. One approach, proposed by Andersen et al. (1996) relies on Generalized Methods of Moments (GMM), which involves constructing moments conditions based on observed data and then finding the parameter values that minimize the distance between sample moments and analytical moments implied by the model. However, GMM estimators are rather inefficient and may not consistently provide accurate results depending on the moments used. In addition, this approach requires a separate estimation of the latent volatilities. On a different note, Harvey et al. (1994) produced a Quasi-Maximum Likelihood estimator (QML), that jointly estimates model parameters and the unobserved states via a Kalman Filter. However, it suffers from high inefficiency attributable to the skewed log chi-squared distribution of the error term in Equation 3.6, which is poorly approximated by a Gaussian distribution, thereby introducing estimation inaccuracies.

Simulation-based inference

The second class of estimation methods encompasses simulation techniques that are specifically designed to approximate the likelihood function. Within this category, Bayesian Markov Chain Monte Carlo (MCMC) algorithms have gained considerable popularity. These algorithms offer a solution by circumventing the need for calculating the likelihood function and instead allow direct sampling from the posterior distribution of the model parameters. By adopting this Bayesian framework, Kim et al. (1998) successfully addressed the challenge posed by the skewed log chi-squared distribution of the error term as faced by the QML estimator of (Harvey et al., 1994). To overcome this issue, they approximated the problematic distribution with an offset mixture of seven Gaussian distributions. Building on this foundation, Omori et al. (2007) further extended this estimation procedure to account for stochastic volatility models with leverage effects. At the same time, they also introduced a ten-component Gaussian mixture that provides an even better approximation of the log chi-squared distribution of the error term. Furthermore, for more sophisticated and efficient estimation techniques, options like importance sampling or particle filtering can be explored. For deeper insights into these methods, Andersen (2009) serves as a valuable resource

In the context of our SVX model, we will adopt the estimation procedure of Kim et al. (1998) and Omori et al. (2007), as did Ulm and Hambuckers (2022). In the upcoming sections, we will provide a detailed description of this approach, offering insights into its practical implementation and considerations.

3.2.1 Linearization into a Gaussian linear state-space model

Under its initial form, our SVX model is a non-linear state-space model and has to be linearized such that the Kalman filter and smoother can be applied to get an estimation of the unobserved states h_t and regression coefficients $\boldsymbol{\beta}$. This filtering algorithm only deals with Gaussian linear state-space models, and therefore, the SVX model must be transformed into a linear form to make use of this estimation method. While the Kalman filter might not be the most efficient way to sample h_t, it is the most suitable method for accommodating additional covariates in the volatility equation and estimating their regression coefficients.

In order to obtain a linear state-space model, we followed the linearization procedure of Kim et al. (1998) and Omori et al. (2007). Let's first rewrite Equation 3.1 in the following form by injecting Equation 3.2:

$$y_t - \mu = \sqrt{\exp(h_t + \mathbf{x_{t-1}}\boldsymbol{\beta})}\epsilon_t \tag{3.8}$$

The presence of the square and exponential functions in this equation makes it non-linear. To overcome this, we can eliminate non-linearities by squaring and taking the logarithm of Equation 3.9:

$$\log\left[(y_t - \mu)^2\right] = \log\left[\left(\sqrt{\exp(h_t + \mathbf{x_{t-1}}\boldsymbol{\beta})}\epsilon_t\right)^2\right] \tag{3.9}$$

$$= h_t + \mathbf{x_{t-1}}\boldsymbol{\beta} + \log\epsilon^2 \tag{3.10}$$

If we pose $\kappa_t = \log\epsilon^2$ and $g_t = \log((y_t - \mu)^2 + c)$, where $c = 0.001$, we can express the equation as:

$$g_t = h_t + \mathbf{x_{t-1}}\boldsymbol{\beta} + \kappa_t \tag{3.11}$$

where $\log\epsilon^2$ is distributed according to a log chi-squared distribution with the following density function:

$$f(\kappa_t) = \frac{1}{\sqrt{2\pi}}\exp\left\{\frac{\kappa_t - \exp(\kappa_t)}{2}\right\}, \qquad \kappa_t \in \mathbb{R} \tag{3.12}$$

We introduced the small positive constant c to guarantee and robustify the QML estimator of the model when y_t^2 takes extremely small values, following the suggestion of Fuller (1996). Although the model has been linearized, it still exhibits non-Gaussian characteristics due to the log chi-squared distribution of the error term. To address this issue, Kim et al. (1998) and Omori et al. (2007) stated that the model can be converted into a conditional Gaussian state-space model by approximating this distribution by a mixture of normal distributions. Kim et al. (1998) employ a seven-component Gaussian mixture, while Omori et al. (2007) utilize a ten-component Gaussian mixture for this purpose. In our work, we adopt the ten-component mixture as Omori et al. (2007) demonstrate its comparable performance to the seven-component mixture while being applicable under wider conditions. Now, based on the ten-component mixture of normal densities, the distribution of κ_t may be reexpressed as:

$$g(\kappa_t) = \sum_{k=1}^{K} q_k N(\kappa_t \mid m_k, v_k^2), \qquad \kappa \in \mathbb{R} \tag{3.13}$$

where $N(\kappa_t \mid m_k, v_k^2)$ stands for the density function of a Gaussian distribution with mean m_k and variance v_k^2. The mixture weights q_k correspond to the probabilities, defined as:

$$\Pr(s_t = k) = q_k, \qquad s_t \in \{1, 2, ..., 10\} \tag{3.14}$$

In this equation, s_t represents a stochastic indicator that is sampled for each time period t from a uniform distribution such that $\kappa_t \mid (s_t = k) \sim N(m_k, v_k^2)$. The specific values of the mixture parameters $\{q_k, m_k, v_k^2\}$ are obtained from Omori et al. (2007) and reported in Table 3.1. Thanks to this mixture approach, Equation 3.11 can be rewritten as:

$$g_t - m_t = h_t + \mathbf{x_{t-1}}\boldsymbol{\beta} + \tilde{\kappa}_t \tag{3.15}$$

Table 3.1

Parameters of the 10-component Gaussian mixture of Omori et al. (2007) to approximate the log chi-squared distribution of κ_t

K	q_k	m_k	v_k^2
1	0.00609	1.92677	0.11265
2	0.04775	1.34744	0.17788
3	0.13057	0.73504	0.26768
4	0.20674	0.02266	0.40611
5	0.22715	-0.85173	0.62699
6	0.18842	-1.97278	0.98583
7	0.12047	-3.46788	1.57469
8	0.05591	-5.55246	2.54498
9	0.01575	-8.68384	4.16591
10	0.00115	-14.6500	7.33342

Under this revised formulation, the state-space model is now linear and conditionally Gaussian, which allows us to leverage classical sampling techniques such as the Kalman filter. Combining Equation 3.15 with Equation 3.6, we end up with the following linear Gaussian state-space model:

$$g_t - m_t = h_t + \mathbf{x_{t-1}}\boldsymbol{\beta} + \tilde{\kappa}_t, \qquad \tilde{\kappa}_t \overset{\text{iid}}{\sim} N(0, v_{tk}^2) \tag{3.16}$$

$$h_t = \mu_h + \phi(h_{t-1} - \mu_h) + \eta_t, \qquad \eta_t \overset{\text{iid}}{\sim} N(0, \omega^2) \tag{3.17}$$

The next sections will delve into the core of the Bayesian estimation procedure used to estimate the vector of model states $\alpha = (h_t, \boldsymbol{\beta})'$ and the vector of model parameters $\theta = (\mu_h, \phi, \omega^2)'$.

As it might be beneficial for the non-initiated reader, we provide a bit of background information about the different methods and concepts employed, and then we expose their practical implementation in the context of our reformulated SVX model. The first theoretical reminder, which can be found in the next subsection is dedicated to state-space models.

3.2.2 State-space models

State-space models are dynamic time series models characterized by two fundamental equations, namely the state equation and the measurement equation, which are both interconnected. There exist several variations of state-space models, however, we will concentrate our attention on the study of linear Gaussian state-space models which may be stated in their generic form as:

$$y_t = Z_t \alpha_t + \kappa_t, \qquad \kappa_t \overset{\text{iid}}{\sim} N(0, H_t), \tag{3.18}$$

$$\alpha_t = d + T_t \alpha_{t-1} + R_t \nu_t, \qquad \nu_t \overset{\text{iid}}{\sim} N(0, Q_t), \qquad \alpha_1 \sim N(a_1, P_1) \tag{3.19}$$

These equations can represent our reformulated SVX model using the same matrix notations as in Ulm and Hambuckers (2022):

$$y_t = g_t - m_{tk}, \quad Z_t = \begin{bmatrix} 1 & \mathbf{x_{t-1}} \end{bmatrix}, \quad \alpha_t = \begin{bmatrix} h_t \\ \boldsymbol{\beta_t} \end{bmatrix}, \quad H_t = v_{tk}^2 \tag{3.20}$$

$$d = \begin{bmatrix} \mu_h & \mathbf{0} \end{bmatrix}, \quad T_t = \begin{bmatrix} \phi & \mathbf{0}' \\ \mathbf{0} & I_N \end{bmatrix}, \quad R_t = \begin{bmatrix} 1 & \mathbf{0}' \\ \mathbf{0} & I_N \end{bmatrix}, \quad Q_t = \begin{bmatrix} \omega^2 & \mathbf{0}' \\ \mathbf{0} & Q^* \end{bmatrix} \tag{3.21}$$

The first equation (Equation 3.18), known as the *measurement equation*, links the observation vector y_t to the unobserved states α_t plus a noise κ_t. On the other hand, the second equation (Equation 3.19), referred to as the *state equation*, describes the evolution of the unobserved states α_t (e.g. the log-volatility h_t) over time. These state variables are referred to as "unobserved" because they cannot be directly measured or known from the available data. In the context of stochastic volatility modelling, they represent the true values of the logarithmized volatility and regression coefficients $\beta_{i,t}$, which cannot be directly measured in the data. On the other side, the returns correspond to the observed data as they can be precisely quantified. Consequently, the purpose of the measurement equation is to infer information about the dynamics of the unobserved states α_t from our knowledge of the set of observations y_t. In other words, our analysis relies on the observed data y_t as the unobserved states α_t remain beyond direct observation (Durbin & Koopman, 2012). Graphically, the fundamental principle of a state space model may be represented by Figure 3.1.

Finally, we adopt the same matrix specifications as outlined by Ulm and Hambuckers (2022). In particular, the variances of the regression coefficients represented by the $N \times N$ diagonal matrix Q^* are set to $Q^* = 10^{-10} I_N$. This choice is motivated by the fact that the

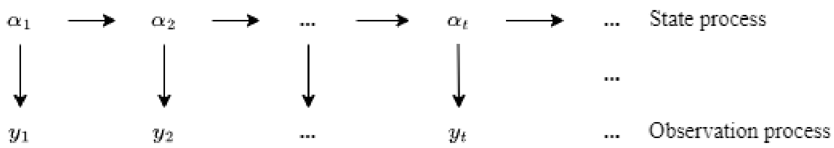


Figure 3.1

Graphical representation of a state space model. Information about α_t is inferred each time we observed a new y_t. (Source: Author's contribution)

Kalman filter does not allow us to consider constant regression coefficients β_t over time. By fixing their variance to a very small value, the coefficients remain nearly time-invariant while adhering to the assumptions of the Kalman filter. In the subsequent section, we will delve into the details of the Kalman filter algorithm which allows us to derive an estimation of the unobserved states α_t.

3.2.3 Kalman filter and smoother

The estimation of latent state components and their covariance matrix over time typically involves the use of an iterative filter algorithm, with the Kalman filter as being one of the leading method. A filter is a recursive algorithm that is designed to estimate the unobserved components or the time-varying parameters of a system. The Kalman filter is specifically tailored for linear Gaussian state space models, making it well-suited for our reformulated SVX model, which now falls into this class. In addition to the forward pass algorithm of the Kalman filter that sequentially estimates state components, there is also a backward pass version called the Kalman Smoother. The Kalman Smoother refines the state estimates using a different set of information. In the following sections, we will provide detailed explanations of both the Kalman filter and the Kalman Smoother recursions.

Kalman filter

During the filtering recursion, first of all, the algorithm sequentially provides optimal estimates of the unobserved variables $\alpha_{t|t}$ based on the information brought by past and current observations Y_t. The filter utilizes Bayesian inference, incorporating prior knowledge and current measurements to generate updated estimates of the system's state (Figure 3.2). It also derives the uncertainty and variability associated with the errors present in both the measurement and state equations. Over a second phase, the filter generates a one-step ahead prediction of the future state of the system α_{t+1} using the current set of observation Y_t. Initially, it assumes that $\alpha_1 \sim N(a_1, P_1)$ but all subsequent α_t (for all $t = 1, ..., T$) are also normally distributed, as a linear combination of Gaussian distributions remains Gaussian.

Mathematically, the filtering recursion estimates the following quantities:

$$a_{t|t} = \mathbb{E}[\alpha_t \mid Y_t], \qquad P_{t|t} = Var[\alpha_t \mid Y_t] \tag{3.22}$$

$$a_{t+1} = \mathbb{E}[\alpha_{t+1} \mid Y_t], \qquad P_{t+1} = Var[\alpha_{t+1} \mid Y_t] \tag{3.23}$$

by means of the hereafter recursion updating the system from t to $t+1$:

$$v_t = y_t - Z_t\alpha_t, \qquad F_t = Z_t P_t Z_t' + H_t, \tag{3.24}$$

$$a_{t|t} = \alpha_t + P_t Z_t' F_t^{-1}, \qquad P_{t|t} = P_t - P_t Z_t' F_t^{-1} Z_t P_t, \tag{3.25}$$

$$K_t = T_t P_t Z_t' F_t^{-1}, \qquad L_t = T_t - K_t Z_t, \tag{3.26}$$

$$a_{t+1} = T_t a_t + K_t v_t + d_t = T_t a_{t|t} + d_t, \qquad P_{t+1} = T_t P_t L_t' + R_t Q_t R_t' = T_t P_{t|t} T_t' + R_t Q_t R_t' \tag{3.27}$$

for $t = 1, ..., T$, where v_t represents the one-step-ahead forecast error of y_t given Y_{t-1}, with $\mathbb{E}[v_t \mid Y_{t-1}] = 0$, and F_t is a nonsingular matrix representing $Var[v_t \mid Y_{t-1}]$. Equation 3.26 is referred to as the *updating step* of the Kalman filter, wherein our knowledge of the state is updated each time a new observation is made. Equation 3.27, on the other hand, is called the *prediction step*, in which we forecast the next system's state as a linear function of the previous state and prediction errors. The matrix K_t is defined as the *Kalman gain*, and acts as a key component of the Kalman filter. This factor represents the trade-off between the reliability of the prediction and the reliability of the measurement. It determines how much weight should be given to the predicted state of a system and the measurement of that system in the process of updating the state estimate. A higher Kalman gain means that more weight is given to the measurement because our predictions of states are uncertain. On the opposite, a lower Kalman gain emphasizes the prediction because the error in the measurement is large relative to the error in the predicted states. The Kalman gain is set such that it minimizes the mean square error $\mathbb{E}[(\alpha_{t+1} - a_{t+1})]$ in the posterior estimate of α_{t+1} at each point in time conditionally on all observed data. Consequently, the Kalman filter is qualified as an optimal unbiased minimum variance estimator for linear systems.

Kalman smoother

Moving on to the Kalman smoother, the smoothing recursion can be envisioned as a backward pass that updates the previously predicted state values α_t using the entire information set provided by $Y_T = y_1, ... y_T$. As a result, this smoothing process refines the estimates of the state variables by incorporating additional information since we consider all the observations to re-estimate our prediction.

The state smoother shall calculate the following quantities:

$$\alpha_t^s = \mathbb{E}[\alpha_t \mid Y_T], \qquad V_t = Var[\alpha_t \mid Y_T] \tag{3.28}$$

with the help of the below backward recursion (or state smoothing recursion):

$$r_{t-1} = Z_t' F_t^{-1} v_t + L_t' r_t, \qquad N_{t-1} = Z_t' F_t^{-1} Z_t + L_t' N_t L_t, \tag{3.29}$$

$$\alpha_t^s = a_t + P_t r_{t-1}, \qquad V_t = P_t - P_t N_{t-1} P_t \tag{3.30}$$

for $t = T, ..., 1$ and initialised with $r_T = 0$ and $N_T = 0$.

For the interested reader, a complete and detailed derivation of both the filtering and smoothing recursion can be found in Durbin and Koopman (2012), while a summarized version is proposed by Durbin et al. (2004).

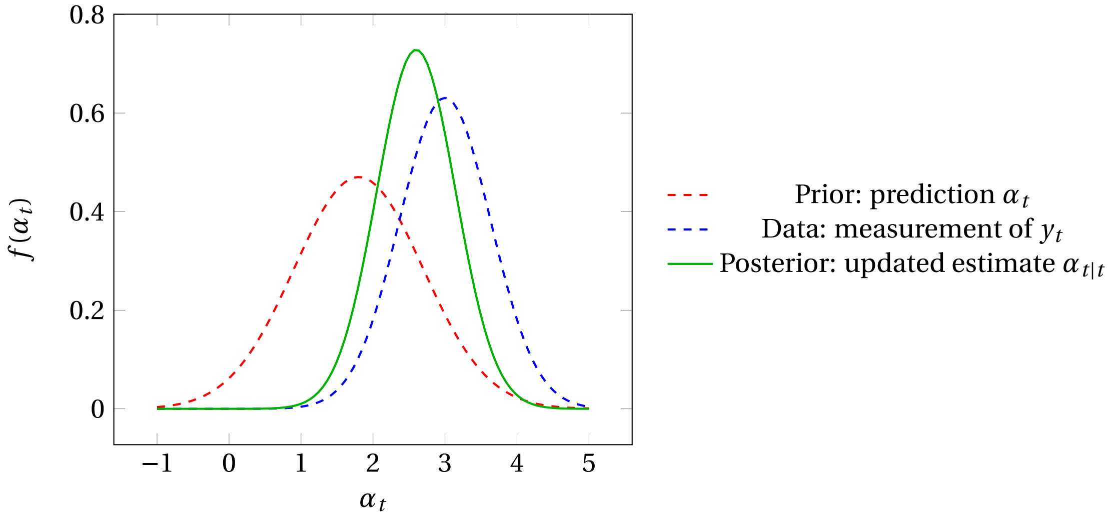


Figure 3.2

Bayesian intuition behind the Kalman filter, summarized by the following process: (1) Prediction of α_t and P_t, which represents our prior beliefs about the system's states before observing y_t. (2) Measuring y_t and learning from the forecast error v_t. (3) The measurement of y_t and the forecast error v_t are fused with the prior distribution to work out the posterior distribution of the system's state estimate. This refined estimate enjoys a smaller variance because a part of the uncertainty in the conditional distribution of α_t given Y_t is removed with the measurement of y_t. (Source: Author's contribution)

3.2.4 Markov Chain Monte Carlo methods

While the unobserved states $\alpha = (h_t, \beta_t)$ can be estimated by means of a filter algorithm (i.e. Kalman filter), the estimation of the remaining stochastic volatility model parameters $\theta = (\mu_h, \phi, \omega^2)'$ still needs to be addressed. In line with the approach outlined by Kim et al. (1998), Omori et al. (2007), and Ulm and Hambuckers (2022), the estimation can take place within a Bayesian framework through the sequential application of a Gibbs sampler. This particular algorithm belongs to the class of Markov chain Monte Carlo (MCMC) methods, which constitute a set of techniques used in Bayesian statistical inference for estimating the posterior distribution of model parameters. Let's elaborate on why this method was chosen.

When performing Bayesian inference to estimate parameters, our goal is to determine their posterior distribution given the available data. In accordance with Bayes' theorem, the joint posterior distribution of model parameters conditional on the data can be expressed as follows:

$$P(\theta|Y) = \frac{P(Y|\theta)P(\theta)}{P(Y)}, \tag{3.31}$$

$$P(\theta|Y) \propto P(Y|\theta)P(\theta) \tag{3.32}$$

The normalizing constant $P(Y)$ can be disregarded as it does not depend on θ. Therefore, the joint posterior distribution of model parameters $P(\theta|Y)$ given the data is proportional to the product of the likelihood function $P(Y|\theta)$ and the prior density $P(\theta)$ of the model parameters. Applying this principle to our linearized SVX model (Equation 3.17), the joint prior distribution of model parameters is defined by:

$$p(\theta;\alpha_1,\alpha_2,...,\alpha_T) = p(\theta) \times \prod_{t=1}^{T} p(\alpha_t|\alpha_{t-1};\theta) \tag{3.33}$$

On the other hand, by application of the law of total probability where the partitioning is done over the possible values of α_t, the likelihood function is obtained as:

$$p(y_1,y_2,...,y_T|\theta) = \prod_{t=1}^{T} p(y_t|\alpha_t;\theta) \times p(\alpha_t|\theta) \tag{3.34}$$

$$= \int_T p(\mathbf{Y_T}|\boldsymbol{\alpha_T};\theta) \times p(\boldsymbol{\alpha_T}|\theta)\, d\boldsymbol{\alpha_T} \tag{3.35}$$

where $\mathbf{Y_T} = (y_1, y_2, ..., y_T)'$ and $\boldsymbol{\alpha_T} = (\alpha_1,\alpha_2,...,\alpha_T)'$. Then the joint posterior distribution of our SVX model parameters is:

$$p(\theta;\alpha_1,\alpha_2,...,\alpha_T|y_1,y_2,...,y_T) \propto p(\theta) \times \prod_{t=1}^{T} p(\alpha_t|\alpha_{t-1};\theta) \times \prod_{t=1}^{T} p(y_t \mid \alpha_t;\theta) \times p(\alpha_t \mid \theta) \tag{3.36}$$

Unfortunately, this posterior distribution cannot be analytically estimated due to the high-dimensional integral involved. In this situation, the Gibbs Sampler algorithm offers the very practical advantage of circumventing the challenges posed by the intractable likelihood function by directly sampling from the posterior distribution. If we pose $\boldsymbol{\alpha_{-t}} = (\alpha_1,\alpha_2,...,\alpha_{t-1},\alpha_{t+1},...,\alpha_T)'$, then the Gibbs sampler sequentially sample parameters in blocks from the following conditional posterior distributions by executing the following steps:

1. Initialize $\theta = (\mu_h,\phi,\omega^2)'$ and α_t for $t = 1,...,T$.
2. Sample α_t from $p(\boldsymbol{\alpha_t}|\boldsymbol{\alpha_{-t}},\theta,\mathbf{Y_T})$ for $t = 1,...,T$, which results from the application of the Kalman filter and smoother detailed in Section 3.2.3.
3. Sample ω^2 from $p(\omega^2|\mathbf{H_T},\phi,\mu_h)$.

4. Sample ϕ from $p(\phi|\mathbf{H_T}, \mu_h, \omega^2)$, with the use of an independent Metropolis-Hastings algorithm.
5. Sample μ_h from $p(\mu_h|\mathbf{H_T}, \phi, \omega^2)$.
6. Go to step 2 and repeat $N-1$ times.

As in Kim et al. (1998) and Ulm and Hambuckers (2022), we assumed the subsequent prior distributions for our different parameters:

- $h_1 \mid \omega^2, \phi, \mu_h \sim N(\mu_h, \frac{\omega^2}{(1-\phi^2)})$
- $h_t \mid h_{t-1}, \omega^2, \phi, \mu_h \sim N(\mu_h + \phi(h_{t-1} - \mu_h), \omega^2)$
- $\beta_1 \sim N(0, Q)$
- $\beta_t \mid \beta_{t-1} \sim N(\beta_{t-1}, Q^*)$
- $\omega^2 \mid \mathbf{H_T}, \phi, \mu_h \sim IG(\nu_0 + T/2, S_0 + \lambda)$, where $\lambda = ((h_1)^2(1-\phi^2) + \sum_{t=2}^{T}(h_t - \phi h_{t-1})^2)/2$
- $\phi \mid \mathbf{H_T}, \mu_h, \omega^2 \sim N(\hat{\phi}, D_\phi)$ if and only if $|\phi| < 1$ (i.e. proposal density of an independent Metropolis-Hastings chain), where

$$D_\phi = \left(\frac{1}{V_\phi} + \frac{1}{\omega^2}\sum_{t=1}^{T-1} h_t^2\right)^{-1} \tag{3.37}$$

$$\hat{\phi} = D_\phi\left(\frac{1}{V_\phi}\phi_0 + \frac{1}{\omega^2}\sum_{t=1}^{T} h_{t-1}h_t\right) \tag{3.38}$$

- $\mu_h \mid \mathbf{H_T}, \phi, \omega^2 \sim N(\hat{\mu}_h, D_{\mu_h})$, where

$$D_{\mu_h} = \left(\frac{1}{V_{\mu_h}} + \frac{1}{\omega^2}\left[(T-1)(1-\phi)^2 + 1 - \phi^2\right]\right)^{-1} \tag{3.39}$$

$$\hat{\mu}_h = D_{\mu_h}\left(\frac{1}{V_{\mu_h}}\left[\frac{(1-\phi^2)}{\omega^2}h_1 + \frac{(1-\phi)}{\omega^2}\sum_{t=2}^{T} h_t - \phi h_{t-1}\right]\right) \tag{3.40}$$

The different distribution hyperparameters are summarized in Table 3.2. Our Gibbs sampler procedure is therefore similar to Ulm and Hambuckers (2022). The entire sampling process constitutes a Gibbs sampler, integrating both an independent Kalman filter step and an independent Metropolis-Hastings step. Among these steps, the Kalman filter is the most computationally intensive as it updates the T unobserved components $\boldsymbol{\alpha_T}$ and all the matrices in Equation 3.20 across N iterations.

In our study, the sampler is set to run for 30,000 iterations, discarding the initial 5,000 sweeps as a burn-in sample. This approach ensures that the sampler converges to the target distribution before collecting samples for analysis. It's essential to recognize that the drawn samples constitute only a numerical approximation of the posterior distribution and a sufficient sequence of draws has to be generated before they can be viewed as actual draws from the true posterior distribution.

Table 3.2

Hyperparameters prior distributions - SVX model

Parameter	Distribution	Hyperparameters
μ_h	Normal	$\mu_0 = 1 \quad V_{\mu_h} = 10$
ϕ	Normal	$\phi_0 = 0.90 \quad V_\phi = 0.01$
ω^2	Inverse-Gamma	$v_0 = 5 \quad S_h = 0.16$
β_1	Normal	$Q = 0.04$
β_t	Normal	$Q^* = 10^{-10}$
h_1	Normal	$\mu_h = 0$

3.2.5 Model specifications

Throughout this study, we have examined three distinct specifications for our SVX model. These specifications are outlined and summarized in Table 3.3. The first specification serves as the foundation for addressing our primary research question. It contains three additional covariates, including one-day lagged financial uncertainty level (x_1), one-day lagged asset log-return (x_2) and a dummy variable (x_3) accounting for leverage effect if $x_2 < 0$. The second specification entails a simplified AR(1)-SV model without any regression effects, as illustrated in Equation 3.1. This specification is employed as a benchmark for performance comparison against our more SVX specification 1. The last specification aims to validate the robustness of our findings by incorporating additional economic uncertainty variables. It is an extension of our model specification 1, augmented with three additional uncertainty covariates. Further details about this specification can be found in Section 5.6.

Table 3.3

List of model specifications

Specification	Name	Covariates	Sample
1	SVX	$x_1 = \text{fu}_{t-1}$ $x_2 = r_{t-1}$ $x_3 = \begin{cases} 1, & \text{if } x_2 < 0 \\ 0, & \text{otherwise} \end{cases}$	2017-11-15 - 2023-05-01
2	AR(1)-SV	—	2017-11-15 - 2023-05-01
3	SVX-U	x_1, x_2, x_3 see spec. 1 $x_4 = \text{epu}_{t-1}$ $x_5 = \text{gri}_{t-1}$ $x_6 = \text{mu}_{t-1}$	2017-11-15 - 2023-05-01

Notes: "fu" denotes our synthetic financial uncertainty index, "epu" represents the economic policy uncertainty index of S. R. Baker et al. (2016), "gri" is the geopolitical risk index of Caldara and Iacoviello (2022), and "mu" stands for the macroeconomic uncertainty index of Bekaert et al. (2022).

3.3 Uncertainty index construction

In this section, the set-up of our daily synthetic US financial uncertainty index, which will be used later as a predictor for modelling and forecasting asset volatility, is presented. The particularity of this index is that it captures financial uncertainty from different perspectives, providing a comprehensive representation of the uncertainty hovering over the US financial sector. To construct this index, a traditional PCA method is employed to aggregate a bunch of correlated uncertainty proxies that have been widely recognized in the literature to reliably gauge financial uncertainty. Thanks to this factor approach, we aim to avoid favouring one method over another and instead overcome the specific limitations and flaws associated with each individual proxy in order to construct a robust and comprehensive uncertainty index. This is the reason why, we decided to name it by the term "synthetic". We assume that financial uncertainty is expected to arise if all its proxies move together. Based on this assumption, the measurement of our synthetic financial uncertainty index is derived as the first principal component from a principal components analysis of the selected proxies. We follow then the same assumption that M. Baker and Wurgler (2007) and Jurado et al. (2015).

3.3.1 Principal Component Analysis

Principal Component Analysis (PCA) is a multivariate statistical technique used to transform a set of correlated variables in $\mathbf{R}^p$ into a smaller set of uncorrelated variables in $\mathbf{R}^q$, called principal components, with $q \leq p$. Each successive principal component is constructed such as it captures the maximum amount of the remaining variability within the data. Although principal component analysis is basically a data-rotation technique for reducing the dimension of a data set, it can also be applied to construct factors. It is this latter application that we are interested in for the construction of our uncertainty index. If $U_t = (u_{1,t}, u_{2,t}, ..., u_{N,t})'$ denotes our time series vector of N scaled uncertainty predictors for $t = 1, ..., T$, then PCA can be expressed as the hereafter linear application transforming our original data U_t into a smaller new set of orthogonal (i.e. uncorrelated) variables Z_t:

$$U_t = \mu + \Gamma Z_t + \epsilon_t \tag{3.41}$$

where Γ is a matrix of eigenvectors of the covariance matrix of U_t, and μ is the sample mean vector of U. For the construction of our financial uncertainty index, we were mainly interested in the first component of Z_t.

4 Data

This section presents all the data collected in order to carry out our empirical study and to investigate our research questions. The first subsection details the financial assets that will be used to test our SVX model, whereas the second subsection focuses on the constituents of our synthetic financial uncertainty index. Moving forward, the third subsection sheds light on the hourly data employed to compute the daily realized volatility series for each financial asset. Finally, in each subsection data summary statistics, pre-diagnostics and treatment procedures are also discussed.

4.1 Financial assets data

As mentioned earlier, one objective of this work is to investigate whether financial uncertainty causes heterogeneous effects on different classes of financial assets. To answer this question, a diversified range of 12 financial assets was selected. These assets can be classified into 5 main asset classes, namely oil, bonds, real estate, equities and currencies. In the case of equities, we further segmented the stock market into eight different industries. We believe this decision is justified because stock market volatility varies across economic sectors during periods of high uncertainty, such as the Covid-19 pandemic crisis (Baek et al., 2020). The majority of the chosen financial assets, excluding oil and exchange rate assets, are represented through Exchange-traded Funds (ETFs). This choice allows us to reflect a more global and holistic view of economic sectors as opposed to relying on isolated individual assets. In a similar vein, the U.S. Dollar Index (USDX), which is a collection of foreign currencies against the U.S. dollar, is employed to represent exchange rate assets. Most of the time series prices of the selected assets were retrieved from the Refinitiv Eikon Database at a daily frequency for a period spanning from November 15, 2017, to May 1, 2023, corresponding to 1370 days of data. Only the US dollar index has been collected for a shorter period. More details about the different series of financial assets are given in Table 4.1. From the daily closing asset prices, we computed the daily log-returns of each asset i using $r_{i,t} = \log(p_{i,t}) - \log(p_{i,t-1})$. A summary description and descriptive statistics of the log-return series are available in Table 4.2.

Table 4.1

Financial asset categories, description and sources

Categories	Assets	Abreviation	Source
a. Stocks	Energy Select Sector SPDR Fund	XLE	Eikon - Refinitiv
	Financial Select Sector SPDR Fund	XLF	Eikon - Refinitiv
	Industrial Select Sector SPDR Fund	XLI	Eikon - Refinitiv
	Technology Select Sector SPDR Fund	XLK	Eikon - Refinitiv
	Consumer Staples Select Sector SPDR Fund	XLP	Eikon - Refinitiv
	Utilities Select Sector SPDR Fund	XLU	Eikon - Refinitiv
	Health Select Sector SPDR Fund	XLV	Eikon - Refinitiv
	Consumer Discretionary Select Sector SPDR Fund	XLY	Eikon - Refinitiv
b. Oil	West Texas Intermediate - Light Crude Oil	WTI	Eikon - Refinitiv
c. Real Estate	iShares US Real Estate ETF	IYR	Eikon - Refinitiv
d. Bonds	SPDR Barclays Capital High Yield Bond ETF	JNK	Eikon - Refinitiv
e. Currencies	US Dollar Index	USDX	Eikon - Refinitiv

Table 4.2

Descriptive statistics of financial asset variables

Categories	Assets	Obs.	Mean	Max	Min	Std.Dev.	Skewness	Kurtosis
a. Stocks	XLE	1,370	0.000186	0.1487	-0.2249	0.0227	-0.9232	16.4297
	XLF	1,370	0.000170	0.1236	-0.1474	0.0167	-0.05855	15.9672
	XLI	1,370	0.000255	0.1191	-0.1204	0.0149	-0.6213	14.5027
	XLK	1,370	0.000628	0.1109	-0.1487	0.0173	-0.4351	10.9637
	XLP	1,370	0.000250	0.0817	-0.0987	0.0108	-0.5299	17.7031
	XLU	1,370	0.000150	0.1204	-0.1206	0.0139	-0.2079	18.9496
	XLV	1,370	0.000368	0.0742	-0.1038	0.0120	-0.4589	12.6811
	XLY	1,370	0.00032	0.0897	-0.1355	0.0159	-0.7440	10.6882
b. Oil	WTI	1,370	0.000230	0.3196	-0.6017	0.0367	-3.1503	70.3475
c. Real Estate	IYR	1,370	0.000021	0.0819	-0.1848	0.0151	-1.6606	25.6165
d. Bond	JNK	1,370	-0.000136	0.0649	-0.0593	0.0064	-0.2959	25.2266
d. Currencies	USDX	1,149	-0.000049	0.0164	-0.0214	0.0043	-0.0968	5.0578

Table 4.3

Augmented Dickey-Fuller unit root test results uncertainty index constituents

Variable	Test statistics	P-value	Lag order	Alternative
VIX	-5.8137	0.01***	18	Stationary
FSI	-4.2814	0.01***	18	Stationary
IV_FIRM	-4.2593	0.01***	17	Stationary
EMV	-8.3511	0.01***	18	Stationary
VOLUME	-5.2201	0.01***	18	Stationary
BOND_MOVE	-3.8774	0.015**	17	Stationary
OVX	-5.0071	0.01***	15	Stationary
GVZ	-4.1171	0.01***	15	Stationary

Notes: Lag order is determined automatically using the rule $k = \lfloor (T-1)^{1/3} \rfloor$ where T is the sample length of the tested series and $\lfloor . \rfloor$ is the floor function. Statistical significance level is given by *p<0.1; **p<0.05; ***p<0.01.

4.2 Uncertainty index constituents

To construct our synthetic financial uncertainty index, we performed a PCA on a comprehensive list of eight financial uncertainty proxies that have been widely acknowledged by academia. These proxies are the following, (1) CBOE VIX index; (2) OFR Financial Stress index (FSI); (3) Cross-sectional firm uncertainty index (IV_firm); (4) Equity Market Volatility (EMV) index; (5) S&P500 trading volume (VOLUME); (6) ICE BofAML MOVE Index (BOND_MOVE) (7) CBOE Crude Oil Volatility Index (OVX); (8) Cboe Gold Volatility Index (GVZ). We retrieved them at a daily frequency for the period from January 3, 2000, to May 31, 2023. Figure 4.1 shows their time series plots. All uncertainty proxies are strongly correlated with one another, as shown by the correlation matrix in Table 4.5, which is a good sign that they are indeed all proxying financial uncertainty. Moreover, we observed that the different uncertainty index constituents have varying scales (see Table 4.4). To prevent larger scales from disproportionately influencing the PCA results, we standardized the variables by centering each one around its mean and dividing by its standard deviation. It is worth noting that some constituents might not be available for all periods (i.e., BOND_MOVE, OVX, GVZ, IV_Firm). In such cases, we replaced the missing values with their mean to enable PCA analysis, as it cannot handle missing values.

As an initial diagnostic step, we tested for the presence of a unit root and the stationarity of each proxy's time series. Indeed, PCA does not perform well on non-stationary data. For this purpose, we conducted the Augmented Dickey-Fuller (ADF) unit root test. This statistical test relies on the following augmented lag regression model:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \epsilon_t \tag{4.1}$$

The null hypothesis is that $\gamma = 0$, indicating the presence of a unit root (i.e., non-stationary process with a drift). As a result under the alternative, the time series is considered stationary. The test statistic can be computed with:

$$DF_T = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \tag{4.2}$$

and then compared to Dickey-Fuller's test critical values to assess the stationarity. We determined the proper number of lags to be included in the regression model using the rule $k = \lfloor (T-1)^{1/3} \rfloor$ where T is the sample length of the tested series and $\lfloor . \rfloor$ is the floor function. Based on the results of the Augmented Dickey-Fuller tests, we reject the null hypothesis for all uncertainty proxies, indicating that their time series are stationary. Results are reported in Table 4.3.

Subsequently, we continue this section by providing a detailed description of each uncertainty proxy along with justifications for their inclusion in our synthetic financial uncertainty index.

(1) CBOE VIX

The first employed financial uncertainty proxy is the CBOE S&P500 Options Implied Volatility Index (VIX), which is often referred to as the "fear gauge" because when market stress

Table 4.4

Descriptive statistics uncertainty index constituents

	Min	Q1	Median	Mean	Q3	Max
VIX	9.14	13.99	18.27	20.18	23.87	82.69
FSI	-2.01	-1.15	-0.40	0.25	1.17	13.28
IV_FIRM	0.17	0.22	0.25	0.30	0.35	0.93
EMV	4.8	14.87	36.01	69.74	81.79	1811.33
VOLUME	356,070,000	2,002,395,000	3,420,210,000	3,310,797,572	4,163,585,000	11,456,230,000
BOND_MOVE	36.62	60.99	76.30	84.28	99.90	264.60
OVX	14.50	29.31	35.77	39.36	45.38	325.14
GVZ	8.88	14.84	17.5	18.90	21.19	64.52

Notes: Descriptive statistics of the different financial uncertainty proxies show that they exhibit very different scales. Data standardization is applied before PCA analysis.

and uncertainty increase, investors tend to demand higher premiums for options, leading to higher implied volatility (Whaley, 2000). It is worth mentioning that VIX is primarily a forward-looking measure of "risk-neutral" expected return volatility of the S&P500 over the next 30 days. As such, it provides insights into investors' expectations regarding future market conditions and can thus be interpreted as a relevant indicator of anticipated market uncertainty. Incidentally, in the literature, it's one of the most commonly used proxies for financial uncertainty (See Section 2).

(2) OFR Financial Stress Indicator

The OFR Financial Stress Index (FSI) is a market-based daily indicator which offers an overall assessment of the health and stability of financial systems on a global and regional level. In the words of Monin (2019), financial stress is defined as *"disruptions in the typical functioning of financial markets"*. We decided to incorporate the FSI into our index because of its profound interdependencies with financial uncertainty. Indeed, as highlighted by Hakkio and Keeton (2009) certain key phenomena characterizing periods of financial stress are inherently linked to financial uncertainty. Firstly, financial stress manifests as *"an increased uncertainty about the fundamental value of financial assets"*, resulting in reduced investors' confidence in the present value of future cash flows. Secondly, *"an increased uncertainty about the behaviours of other investors"* also contributes to a surge in financial stress, as they struggle to anticipate the actions and strategies of their counterparts. Thirdly, *"an information asymmetry between lenders and borrowers or buyers and sellers"* often depicts periods of financial stress and uncertainty leading to higher average risk premiums asked by investors and wider credit spread imposed by lenders. Lastly, *"Decreased willingness to hold risky assets"* known as *"the flight to safety phenomenon"* is also typical during growing periods of financial stress and financial uncertainty. The FSI is based on 5 categories of indicators: (i) Credit; (ii) Equity valuation; (iii) Funding; (iv) Safe assets; (v) Volatility. Among these categories, some financial stress indicators are known to be excellent proxies for financial uncertainty. A detailed composition of the FSI can be found in Appendix (...). In particular, bond spreads, swap spreads, FX options' implied volatility and valuation ratios will be indirect components of our synthetic index through the inclusion in their FSI. Finally, it should be specified that for the purpose of this research, we will solely utilize the component of the index that pertains to the United States since our focus is exclusively on capturing a snapshot of financial stress within this region.

(3) Cross-sectional firm uncertainty

The Cross-sectional firm uncertainty proxy[1], as developed by Dew-Becker and Giglio (2023), is a novel uncertainty index that aims to capture firm-level uncertainty using the implied volatility of stock options on individual firms. They postulated that the total uncertainty faced by firms is a combination of two components: aggregate market uncertainty, commonly represented by the VIX, and firm-specific idiosyncratic uncertainty, which is orthogonal and uncorrelated with aggregate uncertainty. Therefore, by isolating this orthogonal component, they find a cross-sectional estimate of firm uncertainty. The distinguishing feature of this firm-level uncertainty measure is its forward-looking nature, unlike realized volatility, which offers a retrospective view of firm-specific uncertainty based on historical data.

(4) US Equity Market News Uncertainty Index

The US Equity Market Volatility Index (EMV), proposed by S. R. Baker et al. (2019), is an equity market implied volatility index that gauges the level of uncertainty by analyzing the frequency of co-occurring instances of terms related to uncertainty and equities in newspapers. It provides a sweeping measure of the uncertainty surrounding the equity market based on news and is, therefore, a nice candidate for our synthetic index.

(5) S&P500 trading volume

Market liquidity as measured by the S&P500 daily trading volume can be regarded as an information flow indicator which can reflect rising information asymmetry and adverse selection problems on the market. As we know that these phenomena appear concurrently with uncertainty, market liquidity seems to be a relevant uncertainty indicator.

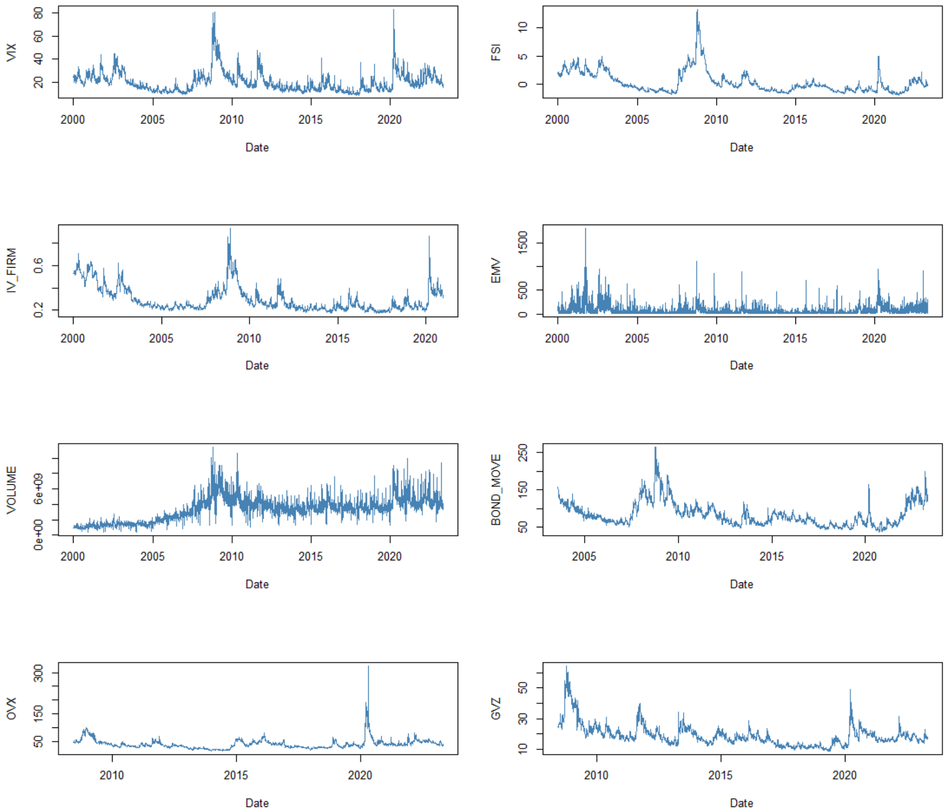
(6,7,8) Bond MOVE index, Gold implied volatility and Oil implied volatility

As we attempt to construct a synthetic and comprehensive financial uncertainty measure, we believe it is necessary not to only consider equity market uncertainty but also uncertainty associated with other classes of financial assets. To achieve this, we retrieved the ICE BofAML MOVE Index, the CBOE Gold implied volatility index, and the CBOE Oil implied volatility index. These indices mirror the concept of the VIX but for different asset categories: bonds, gold, and oil. They offer valuable insights into the market's expectations of future volatility and uncertainty surrounding their respective financial assets. Specifically, the Bond MOVE

[1]This uncertainty index is not publicly available. This is why, I would like to express my warmest gratitude to Ian Dew-Becker (Kellogg School of Management), who kindly agreed to share it with me.

Figure 4.1

Time series plots of financial uncertainty proxies



index will reflect investors' uncertainty about interest rates and debt market conditions. On the other hand, the Gold implied volatility and Oil implied volatility will gauge investors' perceptions of risk and uncertainty in these commodities markets.

Table 4.5

Correlation matrix of the uncertainty index constituents

	VIX	FSI	IV_FIRM	EMV	VOLUME	BOND_MOVE	OVX	GVZ
VIX	1							
FSI	0.806	1						
IV_FIRM	0.842	0.864	1					
EMV	0.473	0.357	0.433	1				
VOLUME	0.358	0.073	0.048	0.037	1			
BOND_MOVE	0.631	0.817	0.690	0.219	0.289	1		
OVX	0.719	0.599	0.710	0.463	0.513	0.440	1	
GVZ	0.802	0.852	0.832	0.324	0.549	0.703	0.547	1

4.3 Realized volatility

To assess the forecasting and modelling performance of our SVX model, we compare its one-day ahead volatility predictions against the daily realized volatility measure, acting as a proxy for the true volatility. Following the recommendations of Bauwens et al. (2012) we decided not to use simple daily squared returns as true volatility proxy because they tend to be excessively noisy. Realized volatility can be computed as the square root of the realized variance, which is defined as the sum of n intraday squared returns. Mathematically, the daily realized volatility is therefore calculated as follows:

$$RV_t^d = \sqrt{\sum_{j-1}^{n} r_{j,n}^2} \tag{4.3}$$

where n represents the number of intraday periods, and $r_{j,n}$ denotes the intraday return for period $j = 1, ..., n$. Under correct circumstances, realized variance has been shown to be an asymptotically unbiased estimator of the integrated variance when $n \to \infty$ (Hansen & Lunde, 2006).

To construct our daily realized volatility measure for each of the 12 financial assets, we utilize one-hour intraday asset price data available from the Dukascopy historical database, resulting in 24 intraday periods ($n = 24$) for each trading day. This hourly frequency is deliberately chosen to avoid adverse market microstructure noise that can contaminate and bias very high-frequency data (such as minute data) (see Hansen & Lunde, 2006). The only lower data frequency available was minute-based data, which exhibits too much autocorrelation and is, therefore, more prone to return a poor measure of realized volatility. This is the reason why we keep an hourly frequency.

The main advantage of the Dukascopy's data is that they are available free of charge and for sufficiently long historical periods (over 5 years), which is required in our analysis. However, it is important to acknowledge certain critical limitations related to the use of this data source. For some assets, especially stocks, only Contract for Difference (CFD) prices are available, rather than direct asset prices. CFDs are derivatives products whose prices track the underlying asset's price. It allows investors to establish investing strategies on a financial asset without owning this asset. While CFDs are designed to reflect movements in the asset's value, there may be slight discrepancies between the CFD price and the actual underlying asset price. This divergence can arise due to factors such as market liquidity and demand, making CFDs only proxies for the true asset price. Therefore, the use of CFDs for computing realized volatility introduces some degree of inaccuracy in the volatility measurement. However, we argue that this limitation is acceptable compared to the limitations associated with other volatility proxies such as the daily squared returns, that are too noisy and do not accurately capture high-volatility periods. In contrast, even though CFDs are not exact asset prices, they still provide a reasonable approximation of price movements and outweigh the important flaws associated with other volatility proxies.

Before obtaining daily realized volatility for each asset, we cleaned our intraday data based on the following rules: (1) Data entries that fall outside market opening hours are deleted. Trading hours considered are specific to each market and financial asset; (2) Data entries whose market volume is strictly null, which corresponds to additional periods when exchanges are closed, are removed; (3) Data entries with null or missing realized volatility are also deleted (extremely rare).

5 Results and discussion

This section is dedicated to the presentation of our empirical results regarding the volatility modelisation for our 12 financial assets. We start first by presenting our synthetic financial uncertainty index, its features and how it compares to other uncertainty indexes available in the literature. Next, we provide empirical evidence of the investor sentiment transmission channel. We then address our main research question and how the results we have obtained from our SVX model provide us with some answers. In the following subsections, we evaluate our SVX model in-sample and out-of-sample performance, and we diagnose our Monte Carlo Markov Chain to check the quality of the sample generated. Lastly, we test the robustness of our findings by considering another model specification including additional uncertainty predictors.

5.1 Synthetic Financial Uncertainty Index

Figure 5.1 displays our resulting daily synthetic financial uncertainty index for the US region from January 2000 to May 2023. As a recall, it is constructed as the first principal component of a group of financial uncertainty indicators. When observing its shape and behaviour, it is evident that it exhibits peaks during periods of economic crises and market turmoil. Notable events such as the Global Financial Crisis (GFC) of 2008-2009, the European sovereign debt crisis of 2011-2012, the Chinese stock market turbulence of 2015-2016, the US Regional banking crisis in March 2023, as well as the outbreak of the Covid-19 pandemic at the beginning of the year 2020, are correctly captured by the index. All these spikes correspond to times of heightened uncertainty and escalated market strain driven by an increased information asymmetry and an inability of market participants to anticipate future market directions. It, therefore, confirms the stylized facts of Bloom (2014), that all forms of uncertainty rise sharply in economic stress periods. As a second observation, it is also worth noting that even though events like the European sovereign debt crisis and the Chinese stock market turbulence in 2015 did not have a direct impact on the United States, the interdependence and international interconnectedness of financial markets led to contagion effects. These effects, in turn, contributed to heightening the uncertainty level in the US region. Our index notably captures this aspect, echoing the conclusions of studies by Beckmann et al. (2023) and Karanasos et al. (2021) which point out significant uncertainty spillover effects among economic regions, especially from major economies like China or the Eurozone.

Additionally, the presence of spikes in the index following the 9/11 attacks and the declaration of war on Iraq in 2003 supports the notion of interaction and contagion effects between different sources of uncertainty, as discussed by Himounet (2022). In this case, geopolitical uncertainty sows doubt in the financial world and leads to a sudden rise in financial uncertainty level. In conclusion, the behaviour of our synthetic financial uncertainty index well underscores its ability to capture significant financial events and periods characterized by heightened uncertainty and information asymmetry in the financial world.

To assess the performance of our synthetic financial uncertainty index, we compared it to the monthly US financial uncertainty index of Jurado et al. (2015) (hereinafter referred to as the FU JLN index). The FU JLN index is widely considered as a leading financial uncertainty indicator in the literature. To facilitate this comparison, we converted the time scale of index to a monthly frequency by taking the average uncertainty value within each month. We observed strikingly similar characteristics and a high positive correlation between the two indexes of 0.785, 0.772, and 0.738 for forecast horizons of 1, 3, and 12 months respectively. This indicates that our synthetic index captures similar patterns of financial uncertainty as the FU JLN index, further validating its effectiveness in measuring US financial uncertainty. More importantly, our synthetic financial uncertainty index holds the particular advantage of being available at a daily frequency, allowing for real-time tracking of the dynamics of US financial uncertainty, unlike the FU JLN index.

If we now turn the comparison against the Global Financial Uncertainty index of Caggiano and Castelnuovo (2023), which provides a measure of global financial uncertainty, the correlation coefficient between the global and our US uncertainty proxy was found to be 0.929, indicating a strong positive relationship. Unsurprisingly, it translates that the United States plays a significant role in driving global financial uncertainty levels as they are one of the major financial player in the world.

Finally, to gain further insights into the behaviour of our synthetic financial uncertainty index, we also benchmarked it against other indexes that reflect other components of economic uncertainty. This analysis aims to identify whether our index exhibits distinct or common behaviours with other types of uncertainties. The US geopolitical uncertainty as proxied by the Geopolitical Risk Index (GPR) of Caldara and Iacoviello (2022) is our first reference point. As expected, the correlation between our synthetic index and the GPR index is rather low, amounting to only 0.041 for the period 2000-2023. This suggests that financial uncertainty and geopolitical uncertainty are very distinct phenomena with limited overall correlation. However, if we focus on the specific period of 2001-2004, which encompasses the 9/11 attacks and the beginning of the Iraqi war, the correlation between the two indexes increases to 0.358. This observation supports there the existence of some contagion effects between geopolitical uncertainty and financial uncertainty during these periods of geopolitical tensions.

In terms of macroeconomic uncertainty, which is the second uncertainty type of interest, the linear correlation coefficients found between our financial uncertainty index and both the daily macro-uncertainty index of Scotti (2016) and Bekaert et al. (2022) amounted to 0.158 and 0.362, respectively. This indicates a moderate positive relationship between financial and macroeconomic uncertainty, which is not surprising given the well-documented influences between the macroeconomic and financial sectors, particularly in the context of monetary policies (Bekaert et al., 2013; Crucil et al., 2023). Overall, this benchmarking analysis reveals that our synthetic financial uncertainty index is able to catch some commonalities with other types of uncertainties but also provides unique information on uncertainty beyond what is captured, for instance, by macroeconomic or geopolitical uncertainty indexes.

For our empirical analysis on whether financial uncertainty contributes to the volatility of financial assets, we will therefore be using the dynamic uncertainty level present in the US financial sector as returned by our synthetic financial uncertainty index.

Figure 5.1

Synthetic Financial Uncertainty Index for the US region along with major events leading to uncertainty peaks.

5.2 Transmission channels: empirical evidence

Before delving into the interpretation of the SVX model results and investigating the intricate relationship between asset volatility and financial uncertainty, we have chosen to present empirical evidence of an existing investor sentiment transmission channel, which conveys the effects of financial uncertainty shocks to asset volatility. While a number of studies have documented that investor sentiment negatively impacts returns and boosts market volatility (M. Baker & Wurgler, 2007; Birru & Young, 2022), few, if any, have effectively established an empirical causal relationship between financial uncertainty and investor sentiment. Our objective is then to fill this gap in the literature and set up a basis on which we can construct our main econometric study. This section, however, does not dwell upon demonstrating an empirical link between the corporate transmission channel and financial uncertainty shocks. This aspect lies beyond the scope of our current study. As outlined in Section 2, many authors have already shown that financial uncertainty shocks can have adverse effects on various economic parameters that are drivers of firm performance (Bloom, 2009; Bloom et al., 2022; Gilchrist et al., 2014). Consequently, based on the findings of those studies, we assume that a link between financial uncertainty and firm performance is a reality.

In order to determine whether a causality relationship between financial uncertainty and investor sentiment exists, we conducted the following regression analysis between these two variables:

$$\log(Y)_t = \alpha + \beta \log(X)_{t-1} + \epsilon_t \tag{5.1}$$

where Y represents the investor sentiment level and X stands for the financial uncertainty level. Our aim is, therefore, to prove that lagged value of financial uncertainty does indeed negatively affect the investor sentiment level in the next period. We captured investor sentiment through three distinct proxies: the investor sentiment (IS) index introduced by M. Baker and Wurgler (2007), the news-based investor sentiment (NIS) index proposed by Shapiro et al. (2022), and the consumer sentiment (CS) index from the University of Michigan. As a result, $Y \in \{IS, NIS, CS\}$ and three individual regression models for each pairing of our financial uncertainty measure and investor sentiment proxy were constructed. This analysis was performed at a monthly frequency utilizing data from January 2000 to June 2022, which accounts for a total of 268 observations.

Detailed regression analysis results and plots of fitted curves are gathered in Appendix 7.2. The results show a statistically significant negative relationship between two of the three investor sentiment proxies (NIS and CS) and lagged financial uncertainty. However, no significant relationship is observed for the remaining investor sentiment proxy (IS). To assess the statistical significance of the regression coefficients, we employed Newey-West heteroscedastic robust standard error estimators for computing t-stats.

Collectively, these empirical outcomes tend to indicate that financial uncertainty could indeed deteriorate investor sentiment in the subsequent period, as theoretically assumed in Section 2.4. Although this observed effect may vary depending on the chosen investor sentiment proxy, it still provides empirical support for the existence of the investor sentiment transmission channel which would spread financial uncertainty effects until asset volatility. Furthermore, despite the fact that our variables are $I(1)$ and subject to a unit root (see Table 7.3), the Johansen (1991) cointegration test reveals that they are all cointegrated of order 1 at least (see Table 7.4), confirming that the hypothesis of spurious results can be ruled out.

Finally, it is important to note that our findings remain robust even at a daily frequency. Indeed, repeating the procedure with the news-based investor sentiment index, which is the sole proxy available at this frequency, upholds the integrity of our results and conclusions.

5.3 MCMC diagnostics

Throughout this subsection, we try to evaluate the quality and convergence of the samples generated by our MCMC algorithm. We evaluated the performance of our MCMC along two dimensions: (1) Convergence: Has the Markov chain successfully converged to a stationary distribution with an invariant mean and variance? (2) Mixing: Has the chain efficiently explored the posterior distribution and the parameter space?

To estimate the model parameters and log-volatilities, we executed the chain for 30,000 iterations in total and discarded the initial 5,000 iterations as a burn-in sample to avoid the effects of a "bad starting point". We then limit autocorrelation between draws by thinning the chain by a factor of 5, resulting in 5,000 remaining samples used to compute the posterior mean estimates for each parameter. The posterior mean of each parameter was computed as the sample mean on the 5,000 generated samples.

We assessed the MCMC convergence to the target distribution with the help of the convergence diagnostic statistics proposed by Geweke (1991), which take the form of a test for equality of the means between the early and late draws generated by the chain. Thus, it relies on the underlying assumption that if the means of these two segments are not significantly different, then the chain has converged to a stationary distribution. The test is constructed such that we aim to fail to reject the null hypothesis:

$$H_0 : \theta_A = \theta_B \qquad\qquad H_1 : \theta_A \neq \theta_B \tag{5.2}$$

Usually, by default, we compare the first 10% (A) and the last 50% (B) generated draws. These are the values recommended by Geweke (1991), and we follow this rule in this work. If the chain has converged and the two subsample means θ_A and θ_B are equal, then the Geweke test statistic is asymptotically distributed according to a standard normal distribution such that:

$$Z_n = \frac{(\theta_A - \theta_B)}{\sqrt{\frac{1}{n_A}\hat{S}_A + \frac{1}{n_B}\hat{S}_B}} \xrightarrow{d} N(0,1) \tag{5.3}$$

Hence, the null hypothesis is rejected when Z is larger than the critical test level (i.e. $|Z| > \alpha$). Because Z is normally distributed, p-values can be easily computed. In Table 5.1, we reported the values of convergence diagnostic statistics along with their corresponding p-values. Based on these elements, we can confidently assert that our Markov Chain has converged well to a stationary distribution for all model configurations and asset series. Indeed, p-values do not allow us to spot significant differences between the mean of early and the mean of late draws of the chain.

Additionally, we cross-checked the conclusions of these Geweke diagnostic tests with traceplots[1], which represent the sampled values of each parameter as iterations progress.

[1]These traceplots are not available in this work due to their very large number

With this graphical tool, we were able to confirm that the chain, for all parameters, converged relatively quickly and exhibited a stationary behavior. Thus, it further supports the conclusion of convergence.

Table 5.1

Geweke convergence diagnostics of the MCMC algorithm

SVX Sample (2017-11-15 - 2023-05-01)													
Parameter	Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
h_t	CD	-0.058	0.094	-0.198	0.093	0.358	-0.099	0.565	0.122	-0.137	-0.231	0.239	-0.211
	p-value	0.954	0.925	0.843	0.926	0.721	0.922	0.572	0.903	0.891	0.817	0.811	0.833
β_1	CD	-1.101	0.628	0.861	1.204	0.344	-0.332	0.310	1.680	0.428	0.682	0.611	-0.423
	p-value	0.271	0.530	0.390	0.229	0.731	0.740	0.756	0.093	0.669	0.495	0.541	0.672
β_2	CD	0.327	-0.038	0.110	-1.054	-0.493	-0.765	-0.478	-0.257	0.212	-1.249	-0.367	-0.750
	p-value	0.744	0.970	0.913	0.292	0.622	0.444	0.633	0.798	0.832	0.212	0.713	0.453
β_3	CD	0.063	0.052	-0.114	1.078	0.646	0.631	0.671	0.360	-0.302	1.866	0.489	0.399
	p-value	0.950	0.959	0.909	0.281	0.518	0.528	0.502	0.719	0.763	0.062	0.625	0.690
ω_h^2	CD	-0.418	0.688	-0.538	1.174	-1.052	-1.318	0.656	1.739	0.577	0.114	0.258	0.807
	p-value	0.676	0.492	0.590	0.241	0.293	0.187	0.512	0.082	0.564	0.909	0.797	0.420
ϕ	CD	0.492	-0.901	-0.208	-1.267	1.048	1.461	-0.995	-1.763	-0.920	0.801	0.494	-0.421
	p-value	0.623	0.368	0.835	0.205	0.295	0.144	0.320	0.078	0.358	0.423	0.621	0.674
μ_h	CD	-0.759	0.976	0.236	0.578	1.447	0.113	0.874	0.043	-0.545	-1.650	-1.013	-0.413
	p-value	0.448	0.329	0.814	0.563	0.148	0.910	0.382	0.966	0.586	0.099	0.311	0.680

AR(1) SV Sample (2017-11-15 - 2023-05-01)													
Parameter	Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
h	CD	0.212	-0.207	-0.143	-0.445	-0.547	0.389	-0.934	-0.083	-0.426	-0.089	1.276	0.477
	p-value	0.832	0.836	0.886	0.657	0.584	0.697	0.350	0.934	0.670	0.929	0.202	0.633
ω_h^2	CD	0.4135	-1.3728	-1.4188	0.9522	0.2511	-1.0608	-2.2697	-0.6405	0.9306	0.4143	-0.1066	-2.0517
	p-value	0.679	0.170	0.156	0.341	0.802	0.289	0.023	0.522	0.352	0.679	0.915	0.040
ϕ	CD	-0.471	1.704	1.363	-1.476	-0.002	0.959	1.726	0.487	-1.032	-0.258	0.041	0.580
	p-value	0.638	0.088	0.173	0.140	0.998	0.337	0.084	0.627	0.302	0.796	0.967	0.562
μ_h	CD	1.084	0.425	-0.703	-0.768	-0.773	-0.652	-1.726	0.031	-1.355	-0.073	0.973	-0.951
	p-value	0.278	0.671	0.482	0.442	0.440	0.515	0.084	0.976	0.175	0.942	0.331	0.342

Notes: CD denotes Geweke Convergence Diagnostics and represents the z-score value.

A second main issue with MCMC algorithms is that they cannot produce independent samples. High autocorrelation in the chain induces slow mixing and inefficiency, as the samples are not effectively exploring all the parameter space of the posterior distribution. To address this issue and ensure efficient mixing, we previously thinned the chain by retaining only 1 iteration over 5. To confirm whether it had a good impact on chain mixing, we relied on autocorrelation plots and on the Effective Sample Size (ESS) metric. On the one hand, for each model specification and financial asset series, autocorrelation is decaying rapidly implying efficient mixing. On the other hand, the ESS metrics reported in Table 5.2, indicate that we generated a sufficient number of independent samples containing the same amount of information as all the dependent draws of our MCMC algorithm. The larger the ESS, the better it is, but an indicative reference value of 1,000 independent draws is given by Bürkner (2017). Based on this reference value, we can state that for all assets, we have generated enough "independent" samples.

Collectively, these indicators underscore the reliability of our sampling procedure and the proper execution of our MCMC algorithm.

Table 5.2

Effective sample size for the SVX model

	SVX model (2017-11-15 - 2023-05-01)											
	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
ESS	2613	2563	2571	2803	2386	2460	2519	2541	2441	2646	1930	1943

Notes: ESS denotes Effective Sample Size, which refers to the number of i.i.d. samples that would provide the same amount of information as the correlated samples generated by the MCMC algorithm. An indicative reference of 1,000 independent draws is given by Bürkner (2017).

5.4 Impact of financial uncertainty on assets volatility

In this section, we present and interpret the empirical findings that address the central research question of this study: "Does financial uncertainty affects and help to model asset volatility?" As a recall, to investigate this, we relied on the first specification of our stochastic volatility model with three additional covariates: the one-day lagged level of synthetic financial uncertainty (u_{t-1}), the one-day lagged return (r_{t-1}), and a dummy variable taking a value of 1 if the previous period's return was negative, and 0 otherwise (x_{t-1}). We have reported all the in-sample estimation results for each model configuration and for the 12 asset series in Table 5.3. By analyzing the estimated regression coefficients, we can gain insights into the sign and strength of the relationship between financial uncertainty and asset volatility, and how these effects vary across different asset classes.

A first observation is that the level of financial uncertainty in the period before (x_1) has a positive impact (i.e. $\beta_1 > 0$) on the current conditional volatility level (σ_t) for each type of financial asset. This indicates that a positive variation in financial uncertainty level generally leads to an increase in the whole market volatility during the following period. This finding supports the notion that financial uncertainty plays a crucial role in driving fluctuations in asset prices and volatility, and is coherent with the results found by Asgharian et al. (2023). However, unlike this study, we extended this finding to other asset classes than stocks. Based on our hypothetical transmission channels, we can therefore interpret that a rising financial uncertainty, materialized by a higher information asymmetry in the market, undermines investor sentiment and slashes corporate financial performance. These two conduits, over a second phase, impact stock returns and increase asset volatility. In addition to being economically consistent, we can also state that these results are statistically significant since 0 is far from being included in the 90% HPDI of all uncertainty regression coefficients. In contrast, the regression effects of the lagged return (x_2) and the leverage effect variable (x_3) are not always statistically significant. However, when they are, the sign of the regression coefficient makes economic sense. Indeed, $\beta_2 < 0$ will result in higher conditional volatility when $x_2 < 0$ and lower conditional volatility when $x_2 > 0$. On the other hand, $\beta_3 > 0$ further stimulates asset volatility.

Another noteworthy observation is that the magnitude of financial uncertainty's impact is not uniform across asset types and economic sectors. Notably, financial uncertainty appears to exert a more substantial influence on high-yield bonds (JNK), US light crude oil (WTI), and real estate (JNK). Conversely, the US dollar value (USDX) exhibits the lowest sensitivity to financial uncertainty, while stocks display a moderate degree of sensitivity. More precisely, within the stock category, the energy industry (XLE), as well as the consumer discretionary (XLY) goods sector, seems to be the most directly affected. In contrast, the healthcare (XLV), consumer staples (XLP), and industrial sectors (XLI) seem to be less sensitive to financial uncertainty. From these observations, we can derive two plausible conclusions. First, assets that display lower sensitivity might either be less well explained by financial uncertainty, in which case other factors should be considered to explain their volatility, or they might inherently have lower volatility during uncertain periods. In this context, we are more inclined towards the latter scenario, which is more aligned with the findings of Baek et al. (2020). This perspective suggests that the varying degrees of responsiveness to fluctuations in financial uncertainty could stem from asset or sector-specific characteristics that drive heterogeneous investor behaviours.

Lastly, a major finding is the better robustness of the SVX model compared to the classic SV model during periods of high market stress. This phenomenon is particularly evident in the conditional volatility plots provided in Appendix 7.3. Interestingly, the SVX model demonstrates a more rapid and pronounced adjustment in its volatility modelling, especially in response to sudden and extreme tail risk events like the outbreak of the Covid-19 pandemic in March 2020. This improved adaptability of the SVX model during times of market turmoil is a particularly important feature of this model, and it showcases the effectiveness of taking into account information emanating from financial uncertainty proxies to model market volatility. This graph observation is confirmed by a formal out-of-sample model performance comparison during the initial Covid-19 pandemic wave, as detailed in Table 5.4. Throughout this period of heightened market stress, our out-of-sample performance metrics show that our SVX model is performing significantly better than the SV model, which experienced a significant performance decline.

5.5 Model's performance and comparison

As we did not directly evaluate the likelihood function of our model via our MCMC and Gaussian mixture approach, we cannot rely on likelihood-based criteria like the Bayes factor or Deviance Information Criteria (DIC) for analyzing our model's performance (see Chan and Grant (2016)). Although it would have been possible to employ estimation methods that approximate the intractable likelihood function through simulation algorithms such as particle filtering, we leave these more complex techniques for future research.

We decided to benchmark the performance of our SVX model containing 3 additional covariates (Model 3.4) against a simple AR(1) stochastic volatility (SV) model without any regression effects (Model 3.1). To begin, we assessed the in-sample performance of our SVX

Table 5.3

Posterior mean estimates and HPDI for the SVX and SV models parameters

SVX Sample (2017-11-15 - 2023-05-01)

	Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
β_1	Mean	0.4449	0.3856	0.343	0.3967	0.359	0.393	0.3579	0.4129	0.4159	0.4452	0.572	0.2739
	HPDI	[0.3867; 0.5041]	[0.3139; 0.4553]	[0.2685; 0.4144]	[0.3122; 0.4769]	[0.2758;0.4474]	[0.3208; 0.4626]	[0.2705; 0.4482]	[0.3028; 0.5127]	[0.3298; 0.4898]	[0.3879; 0.5042]	[0.4402; 0.6839]	[0.1708; 0.3718]
β_2	Mean	-0.1759	-0.1243	-0.0874	-0.1625	-0.1819	-0.1694	-0.273	-0.1953	-0.1344	-0.0519	-0.1173	-0.2311
	HPDI	[-0.2351; -0.1162]	[-0.2102; -0.0419]	[-0.1861; 0.0101]	[-0.2379; -0.0869]	[-0.3549; -0.0204]	[0.2998; -0.0466]	[-0.4271; -0.1323]	[-0.2952; -0.0959]	[-0.2431; -0.0272]	[-0.0932; -0.0110]	[-0.5770; 0.3256]	[-1.0161; 0.4414]
β_3	Mean	0.3233	0.1527	0.1072	0.2783	0.2446	0.2377	0.4481	0.4093	0.1953	0.0862	0.2264	0.0602
	HPDI	[0.2169; 0.4300]	[-0.080; 0.3143]	[-0.0743; 0.2872]	[0.1368; 0.4189]	[-0.0609; 0.5794]	[0.0113; 0.4783]	[0.1858; 0.7362]	[0.2311; 0.5865]	[0.0038; 0.3932]	[0.0195; 0.1516]	[-0.6435; 1.2075]	[-1.1109; 1.4812]
ω_h^2	Mean	0.5321	0.1896	0.1458	0.1622	0.2313	0.1706	0.1934	0.1736	0.1406	0.8039	0.2939	0.2625
	HPDI	[0.4199; 0.6548]	[0.1247; 0.2719]	[0.0935; 0.2227]	[0.1134; 0.2240]	[0.0825; 0.6537]	[0.0684; 0.4144]	[0.0823; 0.4817]	[0.1008; 0.2744]	[0.0770; 0.2766]	[0.6656; 0.9582]	[0.0795; 0.8966]	[0.0620; 0.8055]
ϕ	Mean	0.6069	0.8166	0.8439	0.8847	0.8428	0.8001	0.87	0.8988	0.8312	0.4776	0.7775	0.7297
	HPDI	[0.5180; 0.6877]	[0.7462; 0.8817]	[0.7575; 0.9275]	[0.8346; 0.9323]	[0.6268; 0.9750]	[0.6071; 0.9591]	[0.7004; 0.9739]	[0.8302; 0.9611]	[0.6957; 0.9416]	[0.3826; 0.5648]	[0.5817; 0.9792]	[0.5185; 0.9729]
μ_h	Mean	0.4602	0.0653	0.0257	0.0875	-0.1253	-0.0212	-0.0444	0.0406	0.0084	0.9144	-0.3756	-0.3688
	HPDI	[0.3433; 0.5735]	[-0.0833; 0.2107]	[-0.1864; 0.2233]	[-0.1730; 0.3334]	[-0.6287; 0.3381]	[-0.2812; 0.2605]	[-0.5097; 0.4170]	[-0.3304; 0.3901]	[-0.2270; 0.2305]	[0.7785; 1.0496]	[-1.3020; 0.1667]	[-1.3042; 0.1082]

AR(1) SV Sample (2017-11-15 - 2023-05-01)

	Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
ω_h^2	Mean	0.0676	0.0365	0.03	0.0305	0.0215	0.0217	0.0196	0.0344	0.032	0.4011	0.026	0.0121
	HPDI	[0.0382; 0.11]	[0.0253; 0.0511]	[0.0184; 0.0414]	[0.0205; 0.0414]	[0.0133; 0.0302]	[0.0117; 0.0389]	[0.0115; 0.0277]	[0.0159; 0.0792]	[0.0144; 0.0611]	[0.3342; 0.4722]	[0.0098; 0.0659]	[0.0074; 0.0188]
ϕ	Mean	0.8935	0.9376	0.9433	0.9527	0.9591	0.9612	0.9601	0.9626	0.9569	0.5576	0.9826	0.9644
	HPDI	[0.8279; 0.9448]	[0.9068; 0.9675]	[0.9125; 0.9747]	[0.9260; 0.9797]	[0.9317; 0.9875]	[0.9287; 0.9912]	[0.9323; 0.9885]	[0.9111; 0.9937]	[0.9098; 0.9912]	[0.4884; 0.6275]	[0.9502; 0.9991]	[0.9312; 0.9955]
μ_h	Mean	0.1001	0.0315	0.017	0.0512	-0.0403	0.0022	-0.0124	0.0307	0.0272	0.1217	-0.1679	-0.1687
	HPDI	[-0.0597; 0.2667]	[-0.1749; 0.2402]	[-0.1899; 0.2267]	[-0.2143; 0.3085]	[-3253; 0.2293]	[-0.3044; 0.3032]	[-0.2862; 0.2379]	[-0.4262; 0.4682]	[-0.3224; 0.3664]	[0.0294; 0.2128]	[-1.4005; 1.0993]	[-0.5012; 0.1596]

Notes: Posterior mean estimate is computed as the sample mean over the sample drawn from the posterior distribution. 90% High Posterior Density Intervals (HPDI) are written in brackets. They represent the narrowest interval that contains 90% percent of the posterior probability distribution of the parameter.

Table 5.4

Comparative out-of-sample performance assessment during first wave in Covid-19 pandemic

Sample Covid-19 (2020-01-30 - 2020-06-30)				
	HMSE SVX	HMSE SV	HMAE SVX	HMAE SV
XLE	0.6162	2.7666	0.5429	1.2684
XLF	0.2588	1.1514	0.4052	0.778
XLI	0.2599	1.2665	0.3957	0.7771
XLK	0.2851	0.8503	0.399	0.6558
XLP	0.1345	0.2674	0.2928	0.3981
XLU	0.1877	0.6253	0.3549	0.5911
XLV	0.1826	0.451	0.3569	0.5060
XLY	0.4311	0.8577	0.4735	0.6609
IYR	0.2516	0.9769	0.4045	0.7347
WTI	0.7832	8.6887	0.618	2.3903
JNK	0.5969	0.3962	0.4068	0.4999
USDX	0.3351	0.4068	0.5677	0.6300

Notes: Out-of-sample model performance is assessed with heteroscedastic adjusted mean squared error (HMSE) and mean absolute error (HMAE) of Bollerslev and Ghysels (1996). SVX model clearly outperforms the SV model in this period of economic turmoil marked by a high level of financial uncertainty.

model by evaluating its ability to accurately capture the conditional heteroscedasticity of the asset returns (i.e., the ARCH effect). The most effective model is the one that decreases the autocorrelation present in the squared residuals the most. To get this information, we conducted the Ljung-Box autocorrelation test on the squared residuals, specifying the lag length according to the rule of Tsay (2005) (i.e., $l = \log(T)$, where T is the sample size). The test results are presented in Table 5.5. Despite the rejection of the null hypothesis of non-autocorrelation, the SVX model still significantly reduces ARCH effects compared to the classical SV model. This suggests that including explanatory variables, and in particular the lag financial uncertainty level, improves the model in-sample performance. Besides, we also looked at the residual distribution. Although the residuals for both model configurations are not normally distributed as they should be, the SVX model better captures non-linear behaviours in the asset returns. Notably, for most asset series, the skewness and kurtosis values of the SVX model appear closer to a normal distribution compared to the SV model. It further supports the use of SVX at the expense of the simple SV.

Regarding the out-of-sample performance analysis, it was realized in a pseudo-out-of-sample setting since we relied on the one-day ahead conditional volatility prediction returned by the Kalman filter, which requires only information up to time $t-1$ to forecast volatility at time t. As a starting point, we asserted whether the forecasted volatility is an effective predictor of a true measure of the volatility and whether our model does not produce systematic bias in the forecasts. This was carried out with a Mincer-Zarnowitz regression test (Mincer & Zarnowitz, 1969), which regresses the true volatility measure on the forecasted volatility based

Table 5.5

Residuals diagnostic statistics for SVX and SV models

Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
Model	SVX model (15/11/2017 - 01/05/2023)											
a. Normality												
Skewness	-0.068	-0.084	-0.203	-0.168	-0.247	-0.190	-0.182	-0.210	-0.201	0.144	-0.313	0.003
Kurtosis	1.933	2.321	2.483	2.222	2.634	2.479	2.505	2.331	2.521	1.912	3.208	2.770
b. Serial Correlation												
$Q^2(7)$	60.535	35.890	37.908	65.815	24.578	30.189	33.820	55.762	29.672	63.713	2.627	11.957
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917***	0.102***
Model	AR(1) SV model (15/11/2017 - 01/05/2023)											
a. Normality												
Skewness	-0.144	-0.139	-0.247	-0.228	-0.349	-0.166	-0.265	-0.303	-0.333	-0.409	-0.113	-0.016
Kurtosis	3.634	3.772	3.762	3.433	4.488	4.006	3.883	3.659	4.529	5.224	6.569	3.286
b. Serial Correlation												
$Q^2(7)$	202.993	306.363	238.297	146.757	275.402	452.577	222.896	149.650	264.736	568.754	289.318	79.155
P-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Notes: $Q^2(7)$ denotes the Ljung-Box statistics of the squared residuals with a lag length of 7. Statistical significance levels are given by *p<0.1; **p<0.05; ***p<0.01.

on the following linear model:

$$\sigma_{t+1} = a + b\,\hat{\sigma}_{t+1} + \epsilon_{t+1}, \qquad \epsilon_{t+1} \overset{\text{iid}}{\sim} N(0,\sigma_\epsilon^2) \tag{5.4}$$

As a reminder, we employed the daily realized volatility, computed from hourly returns (see Section 4.3), as a proxy for the true volatility σ_{t+1}. Based on this regression model, we tested the joint hypothesis $H_0 : a = 0, b = 1$. If the coefficients are not statistically different from these values, it indicates that our model produces unbiased forecasts. Conversely, a non-zero constant term (a) suggests that our model consistently produces over or underestimated forecasts, while a slope (b) different from 1 implies a non-proportional relationship between forecast and realized values. Furthermore, as suggested by Bauwens et al. (2012), the regression R^2 serves as an indicator of the percentage of variance in σ_{t+1} explained by $\hat{\sigma}_{t+1}$ and thus reflects forecast performance. The regression estimated parameters and the Mincer-Zarnowitz F-test results are presented in Table 5.6. Despite the rejection of the null hypothesis, indicating constant bias in our SVX model forecasts, it significantly reduces the F-test value when compared with the SV model.

To continue the out-of-sample assessment, we utilized performance metrics based on loss functions. Following the recommendation of Bollerslev and Ghysels (1996), we employed heteroscedastic-adjusted loss functions as we are in the presence of heteroscedastic data. For both model specifications, we computed the heteroscedastic mean squared errors (HMSE) and heteroscedastic mean absolute error (HMAE), as shown in Table 5.7. Their mathematical expression is given by:

$$\text{HMSE} = \frac{1}{n}\sum_{t=1}^{n}(1-\sigma_t/\hat{\sigma}_t)^2, \tag{5.5}$$

$$\text{HMAE} = \frac{1}{n}\sum_{t=1}^{n}|1-\sigma_t/\hat{\sigma}_t|^2 \tag{5.6}$$

Table 5.6

Mincer-Zarnowitz regression test results

	SVX						AR(1) - SV					
	α		β		Wald	R^2	α		β		Wald	R^2
XLE	0.442		1.107		273.27	0.552	-2.254		3.699		891.20	0.575
	(10.858)	***	(41.050)	***			(-23.347)	***	(43.019)	***		
XLF	0.432		0.769		108.58	0.706	-1.782		2.911		529.99	0.604
	(14.736)	***	(37.233)	***			(-25.903)	***	(45.673)	***		
XLI	0.240		0.869		32.60	0.670	-1.972		3.030		451.97	0.625
	(8.015)	***	(38.319)	***			(-26.706)	***	(43.515)	***		
XLK	0.378		0.889		132.54	0.626	-1.525		2.676		501.23	0.586
	(13.088)	***	(39.560)	***			(-23.036)	***	(44.046)	***		
XLP	0.254		0.597		464.13	0.387	-1.203		2.016		448.58	0.611
	(14.732)	***	(41.250)	***			(-26.901)	***	(46.365)	***		
XLU	0.353		0.698		173.35	0.511	-1.567		2.551		480.19	0.654
	(16.424)	***	(42.114)	***			(-29.642)	***	(50.873)	***		
XLV	0.277		0.721		109.61	0.468	-1.385		2.300		302.20	0.568
	(12.750)	***	(38.040)	***			(-24.582)	***	(42.380)	***		
XLY	0.239		0.995		95.97	0.622	-1.423		2.509		407.08	0.587
	(8.109)	***	(41.188)	***			(-22.750)	***	(43.961)	***		
IYR	0.379		0.696		140.17	0.506	-1.711		2.710		445.50	0.616
	(14.50)	***	(37.477)	***			(-27.750)	***	(46.847)	***		
WTI	0.415		1.188		270.79	0.694	-2.608		4.523		1097.75	0.576
	(8.38)	***	(55.720)	***			(-21.149)	***	(43.126)	***		
JNK	0.329		0.163		163.82	0.008	-0.647		1.121		1747.05	0.537
	(3.51)	***	(3.320)	***			(-23.061)	***	(39.797)	***		
USDX	0.101		0.326		6764.14	0.372	-0.600		1.073		8428.97	0.527
	(8.47)	***	(26.119)	***			(-21.522)	***	(35.715)	***		

Notes: Each Mincer-Zarnowitz regression is estimated with OLS and t-statistics are corrected with Newey-West heteroscedasticity-robust estimators. The dependent variable σ_t is the realized volatility, while the independent variable $\hat{\sigma}_t$ is the forecasted conditional volatility. Wald statistics jointly test $\alpha = 0 \cap \beta = 1$. Since we are in a large sample condition ($T = 1370$), Wald statistics are asymptotically distributed according to a standard normal distribution. T-statistics are reported between parentheses. Finally, *** represents the following statistical significance level: p<0.01.

The results indicate that the SVX model yields better quality forecasts with lower loss function values for 10 out of 12 financial assets. However, for technology (XLK) and consumer discretionary stocks, our SVX model underperforms the SV model. To ascertain the statistical significance of this superior forecasting ability, we conducted a modified Diebold and Mariano (2002) (DM) test applied to the HMSE and HMAE heteroscedastic-adjusted loss functions. This test enables us to determine if one model significantly outperforms another by comparing their forecast errors. Considering forecast errors as $e_{i,t} = \hat{\sigma}_{i,t} - \sigma_{i,t}$, with $i \in \{1,2\}$ denoting the model specification, we define the loss differential as

$$d_t = g(e_{1,t}) - g(e_{2,t}) \tag{5.7}$$

where $g(.)$ is a specific loss function defined on $\mathbb{R}^+$ such that for larger forecast errors it returns larger losses. In practice, the square or absolute value are two commonly employed loss functions, but we will make use of their heteroscedastic-adjusted versions detailed above. Under H_0, two forecasts have the same accuracy if $E(d_t = 0)$, and the DM statistic follows a standard normal distribution. Otherwise, forecasts are considered to have different levels of accuracy. For one-period ahead forecasts, the DM statistic comparing forecasts of model i and j is computed as follows:

$$\text{DM}_{i,j} = \frac{\overline{d}}{\sqrt{\frac{Var(d)}{T}}} \rightarrow N(0,1) \tag{5.8}$$

The DM test results are presented in Table 5.7, confirming that in almost all cases where the SVX loss function values were lower, the superiority of the SVX model's forecasts is statistically significant at the 90% level. Overall, all things considered together, we can reasonably conclude that considering lagged financial uncertainty level as an additional factor model effectively improves the in-sample and out-of-sample performances of our stochastic volatility model. This conclusion is rather generalized for all categories of financial assets, apart from technology and consumer discretionary stocks.

5.6 Robustness checks

In order to validate the credibility and robustness of our research findings, we modified the SVX model specification to include additional covariates that capture various sources of economic uncertainty beyond the financial domain. By doing so, we aim to assert whether financial uncertainty would hold its statistical significance as a predictor of asset volatility when compared to other economic uncertainty proxies. Therefore, on top of the three covariates already included in our basic SVX model, we introduced three more lagged variables: economic policy uncertainty (x_4) from S. R. Baker et al. (2016), geopolitical uncertainty (x_5) from Caldara and Iacoviello (2022), and macroeconomic uncertainty (x_6) from Bekaert et al.

Table 5.7

Forecasting performance and predictive ability of SVX and SV models

	HMSE			HMAE		
h = 1	SVX	SV	DM statistics	SVX	SV	DM statistics
XLE	0.608	0.8375	-4.6739***	0.5718	0.6415	-5.5883***
XLF	0.2916	0.3363	3.1828***	0.3987	0.4206	-2.6335***
XLI	0.213	0.2967	-3.4441***	0.3469	0.3944	-6.4316***
XLK	0.3316	0.3184	0.7109	0.4243	0.4119	1.3503
XLP	0.1274	0.1758	-9.7017***	0.2867	0.3548	-13.7258***
XLU	0.1647	0.1838	-1.8745*	0.3048	0.326	-3.1956***
XLV	0.1676	0.191	-2.4952**	0.3071	0.3481	-3.1956***
XLY	0.345	0.3084	2.0578**	0.4272	0.407	2.1780**
IYR	0.2047	0.2596	-3.6209***	0.3219	0.3607	-5.3529***
WTI	0.5551	2.008	-11.656***	0.5563	1.0725	-28.3441***
JNK	0.3616	0.4529	-12.949***	0.5687	0.6376	-17.5153***
USDX	0.3377	0.3791	-16.4947***	0.5613	0.5962	-14.0203***

Notes: The equal predictive ability of the SVX and SV models are compared at a one-period ahead forecast horizon ($h = 1$). DM denotes the Diebold-Mariano test statistics given in Equation 5.8. Under the null, forecasts of the two models have the same power and accuracy. HMSE and HMAE denote the heteroscedastic-adjusted mean squared error and mean absolute error loss functions of Bollerslev and Ghysels (1996). Statistical significance levels are given by *p<0.1; **p<0.05; ***p<0.01.

(2022). Under this modified specification, our SVX model now contains six explanatory factors and we estimated it on the full sample. It represents model specification 3 in Table 3.3.

From the posterior mean estimates and HPDI reported in Appendix 7.4, it appears that the coefficients β_1 associated with financial uncertainty remain statistically significant and exhibit consistent magnitudes across all asset series. A similar observation can be made for coefficients β_2 and β_3, although they might not be consistently different from 0 for some series. In contrast, the coefficients β_4, β_5, and β_6, which correspond to other uncertainty predictors lack statistical significance in many cases. However, they might exhibit nonzero values and achieve significance for specific asset series. For example, it is specifically the case for economic policy uncertainty (EPU) (β_4) regarding technology (XLK), consumer staples (XLP) and healthcare stocks (XLV). These specific industries might be more particularly sensitive to the uncertainty arising from economic policies. As a result, this observation brings a more nuanced view on the debate about the predictive capability of EPU for stock volatility, by confirming the findings of both sides. We, therefore, reasonably conclude that EPU could potentially serve as an effective predictor of stock volatility for certain industries (as S. R. Baker et al., 2016; Liu & Zhang, 2015; Pástor & Veronesi, 2012), but it is not uniform across all asset classes and economic sectors (as Asgharian et al., 2023). Turning to geopolitical uncertainty (GRI), it seems to impact only the volatility of stocks in the industrial sector (XLI). This sector, by its nature and its reliance on international raw materials, is more susceptible to geopolitical conflicts such as the Russia-Ukraine war and other similar issues. Lastly, macroeconomic uncertainty (MU) consistently drives the volatility of oil (WTI), the US dollar value index (USDX), and consumer discretionary stocks (XLY). In the case of the exchange rate, the close link between exchange rate volatility and both macroeconomic policies and interest rate differentials makes this connection unsurprising (Ulm & Hambuckers, 2022).

From a more general point of view, we can nevertheless conclude that financial uncertainty maintains its standing as a consistent predictor of asset volatility in comparison to other uncertainty measures. Whereas other sources of uncertainty impact the volatility of certain assets in isolation, financial uncertainty uniformly influences the volatility of all assets, without exception. This underlines the robustness of our results and the appropriateness of financial uncertainty as a predictor of volatility.

5.7 Value-at-Risk application

In this last section, we consider an empirical application of our SVX model by constructing a parametric Value-at-Risk (VaR). We believe that this constitutes the right experiment to test the reliability of our model in more real-life conditions.

The VaR is a well-established risk metric, whose purpose is to estimate the maximum potential loss, expressed as either a specific dollar amount or a percentage, that an investor or financial institution could encounter in a financial position within a given probability and over a specified time frame. They notably play a pivotal role in calculating regulatory capital requirements for operational, market, and credit risks. Financial institutions and investors utilize VaR models to assess potential losses stemming from these risk

Mathematically, given a confidence level $\alpha \in [0,1]$, the VaR is defined as the value l for which the probability of the loss L exceeding l is less than or equal to (1 - α):

$$VaR_\alpha = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} = \inf\{l \in F_L(l) \geq \alpha\} \tag{5.9}$$

where $F_L(l)$ represents the cumulative distribution function of the profit and loss. In other words, VaR corresponds to the (1-α) quantile of the loss distribution below which only α percent of possible loss outcomes lie (Tsay, 2005).

Parametric VaR is a common calculation approach that assumes the returns or changes in the value of the portfolio follow a known parametric distribution, such as the normal distribution or the Student's t-distribution. While this method offers simplicity and straightforward calculations, it also assumes a fixed distribution that may not capture the true dynamics and tail risk events of asset returns. Despite this limitation, the parametric VaR method aligns well with advanced volatility models like our SVX. By incorporating the forecasted conditional volatility $\hat{\sigma}_t$ from our SVX model, we are able to estimate the α-percentage VaR at time t for a k-day horizon:

$$VaR_{\alpha,t} = \mu_t + \hat{\sigma}_t \sqrt{k}\, q_\alpha(F) \tag{5.10}$$

where μ_t is the mean of the returns F assumed parametric distribution and $q_\alpha(F)$ is the quantile at level α of the assumed distribution.

In our empirical application, we considered the Gaussian parametric distributions. Using the one-day ahead forecasted conditional volatility of our SVX model, we constructed one-day-ahead parametric VaR predictions at levels and 99%. We, therefore, estimated our VaR in an out-of-sample context and we end up with two parametric VaR models per asset series.

To assess the performance and the reliability of the constructed VaR models, we back-tested them with the unconditional coverage test of Kupiec (1995) (POF) and the test of independence of Christoffersen (1998) (CCI). The former is a variation of the binomial test and examines whether the proportion of VaR failures (i.e. when the realized loss exceeds the predicted VaR) matches the expected proportion of failures for a chosen confidence level $p \in [0,1]$. If the observed number of violations significantly deviates from what would be expected from the VaR level, the test statistic may be greater than the critical value, indicating that the VaR model is not adequately capturing the risk. On the other hand, the latter tests whether a VaR violation on a given day influences the likelihood of observing a violation on a subsequent day. The independence between two exceedances is therefore the object of this test. Ideally, a well-designed VaR model should have unrelated violations, occurring irrespectively from a particular time frame. These two tests are based on likelihood ratio tests, and their combination allows us to form a conditional coverage (CC) test such that it jointly tests the unconditional coverage and independence assumptions, as suggested by Christoffersen (1998).

$$LR_{CC} = LR_{POF} + LR_{CCI} \tag{5.11}$$

One-day ahead 99% VaR plots against asset returns for both SVX and SV models are displayed in Figure 7.6 and 7.7. VaR calculations and the results of backtesting tests are summarized in Table 5.8. Examining the backtesting results, we observe that the Gaussian 99% VaR from the SVX model clearly outperforms that from the simple SV model, which almost consistently underestimates risk for most financial assets. Notably, the incorporation of past financial uncertainty levels significantly reduces VaR failures for all financial asset categories. This improvement is particularly evident in the case of the US crude oil petrol (WTI), for which the number of violations exceeds more than 9 times the critical value under the SV VaR, whereas it is fully compliant under the SVX VaR. Among the backtesting tests, SVX VaR was accepted for 6 of the 12 financial assets. Three of the 6 rejections were due to VaR overestimating risk, implying that it was too restrictive. This indicates that while in some instances SVX VaR leads to fewer breaches than the expected level, it also implies a more resource-intensive approach to regulatory capital allocation, which could be suboptimal for financial institutions. Furthermore, a look at VaR plots (see Figure 7.6 and 7.7) shows that the main difference between the SVX VaR and SV VaR lies during periods of market turmoils. Indeed, the SVX model significantly better captures tail events and market crises, as highlighted by the almost absence of VaR breaches during the Covid-19 crisis period. In contrast, the SV model struggles to account for the rapid market crash and prolonged volatility during the same period, and returns a large number of serious exceedances during this tough period.

In summary, this application showcased the practical utility of our SVX model in risk management, specifically its superior ability in capturing market stress events and associated tail risks. It further supports our previous conclusion that incorporating past information from financial uncertainty levels really improves the forecasting of asset volatility and related risk management metrics. Obviously, this can constitute an important consideration for risk managers and investors in the design of market crisis-resilient VaR metrics.

Table 5.8

Gaussian parametric VaR backtesting

Asset	Obs.	Failures	Prop. Failures	LRatioCC	CC	LRatioPOF	POF	LRatioCCI	CCI
				SVX (2017-11-15 - 2023-05-01)					
XLE	1370	35	2.55%	24.47	Reject	23.39	Reject	1.08***	Accept
XLF	1370	20	1.46%	3.15***	Accept	2.56***	Accept	0.59***	Accept
XLI	1370	25	1.82%	8.49*	Accept	7.56	Reject	0.93***	Accept
XLK	1370	38	2.77%	31.54	Reject	29.37	Reject	2.17***	Accept
XLP	1370	4	0.29%	9.64	Reject	9.62	Reject	0.02***	Accept
XLU	1370	17	1.24%	1.17***	Accept	0.74***	Accept	0.42***	Accept
XLV	1370	9	0.66%	1.97***	Accept	1.85***	Accept	0.12***	Accept
XLY	1360	35	2.57%	25.56	Reject	23.71	Reject	1.85***	Accept
IYR	1370	20	1.46%	3.15***	Accept	2.56***	Accept	0.59***	Accept
WTI	1370	13	0.95%	0.28***	Accept	0.03***	Accept	0.25***	Accept
JNK	1370	0	0.00%	27.53	Reject	27.54	Reject	0.00***	Accept
USDX	1149	0	0.00%	23.10	Reject	23.10	Reject	0.00***	Accept
				AR(1)-SV (2017-11-15 - 2023-05-01)					
XLE	1370	96	7.01%	215.12	Reject	214.31	Reject	0.81***	Accept
XLF	1370	42	3.07%	38.46	Reject	38.10	Reject	0.36***	Accept
XLI	1370	32	2.34%	18.02	Reject	17.94	Reject	0.08***	Accept
XLK	1370	52	3.80%	63.84	Reject	63.21	Reject	0.63***	Accept
XLP	1370	11	0.80%	0.75***	Accept	0.58***	Accept	0.18***	Accept
XLU	1370	27	1.97%	12.69	Reject	10.17	Reject	2.53***	Accept
XLV	1370	19	1.39%	2.38***	Accept	1.85***	Accept	0.53***	Accept
XLY	1360	47	3.46%	50.69	Reject	50.60	Reject	0.09***	Accept
IYR	1370	28	2.04%	11.85	Reject	11.58	Reject	0.27***	Accept
WTI	1370	130	9.49%	362.97	Reject	362.71	Reject	0.26***	Accept
JNK	1370	1	0.07%	20.29	Reject	20.28	Reject	0.00***	Accept
USDX	1149	0	0.00%	23.10	Reject	23.10	Reject	0.00***	Accept

Notes: CC, POF and CCI denote Christoffersen's Conditional Coverage test, Kupiec's POF test and Christoffersen's Interval Forecast test, respectively. "Reject" means that the VaR model is not a valid risk measure. "Accept" means the opposite. Statistical significance levels are given by *p<0.1; **p<0.05; ***p<0.01.

6 Conclusion

This research work aimed at exploring the intricate relationships between financial uncertainty and asset volatility. Our primary objective was to inspect whether financial uncertainty effectively causes some effects on asset volatility and whether these effects are heterogeneous depending on asset classes. In addition, we also wanted to answer if considering financial uncertainty can enhance the modelling and forecasting performance of asset volatility. To this end, we relied on the extended stochastic volatility (SVX) model of Ulm and Hambuckers (2022) and introduced lagged financial uncertainty as an additional covariate to assess its impact on model performance. The model estimation was carried out using a Bayesian approach involving a Gibbs sampler and a Kalman filter following the estimation procedure of Kim et al. (1998) and Omori et al. (2007). This model was applied to a diverse set of 12 financial assets encompassing stocks, bonds, exchange rates, oil, and real estate.

In our study, we chose to proxy financial uncertainty in the United States using our own metric. Based on a principal component analysis (PCA), we constructed a synthetic financial uncertainty index as the first principal component of various well-established financial uncertainty indicators. This approach effectively addressed the limitations of relying on a single proxy or measurement technique, resulting in a more robust measure of financial uncertainty. In addition, our index is available at a daily frequency facilitating real-time monitoring of financial uncertainty, unlike the other indexes currently available in the literature.

Our empirical application reveals that financial uncertainty can be consistently regarded as a statistically significant driver of volatility across all asset classes. Consequently, a positive variation in lagged financial uncertainty positively affects the contemporaneous level of conditional volatility. Yet, the magnitude of responsiveness to fluctuations in financial uncertainty depends on asset classes or economic sectors. On the one hand, we saw that US light crude oil, real estate, and stocks from the energy and consumer discretionary sectors are the most sensitive assets to a rise in financial uncertainty. On the other hand, the US dollar and stocks from the industrial, healthcare and consumer staples industries are less responsive to shock in financial uncertainty. This suggests that these sectors might demonstrate more defensive characteristics and be less volatile during periods of heightened uncertainty. Such insights could guide uncertainty-averse investors towards these sectors that potentially offer more stability during uncertain market conditions.

Throughout our literature review, we identified investor sentiment and corporate financial performance as the main transmission channels between financial uncertainty and asset volatility. We further support the existence of the investor sentiment channel through a regression analysis, which enables us to empirically conclude that a higher financial uncertainty level can erode investor sentiment. Drawing from studies such as M. Baker and Wurgler (2007), Berardi (2022), and Birru and Young (2022), it is apparent that negative subjective beliefs held by investors may in turn impact asset valuations and stock market volatility. The recognition of these transmission channels has implications not only for investors but also for financial regulators and policymakers. This information may also be of interest to financial regulators and policymakers to act on these transmission channels to limit the propagation effect of financial uncertainty. By understanding how financial uncertainty affects investor sentiment and corporate financial performance, regulators can effectively design proper policy measures to mitigate the cascading effects of financial uncertainty and enhance market resilience.

Our assessment of model performance and its comparison against a simple autoregressive stochastic volatility (SV) model also reveals that integrating financial uncertainty substantially enhances the modelling and prediction of asset volatility. This improvement becomes especially notable during periods of elevated market stress when uncertainty tends to rise sharply. This enhancement is also perceptible in the construction of appropriate risk metrics. In particular, the performance of a Gaussian parametric Value-at-Risk (VaR) is considerably improved when financial uncertainty is taken into account, surpassing the performance of the VaR constructed from the simple SV model. Practitioners in risk management might be particularly interested in this application of our model. By leveraging the information brought by financial uncertainty, they can construct more robust and effective risk metrics.

However, a degree of caution should still be kept when interpreting our findings, and there remains room for improvement in our methodology. Firstly, it would be relevant to consider other economic periods and countries in our analysis to confirm whether our conclusions would hold when viewed through a different temporal and geographical lens. Secondly, it could prove beneficial to relax the normality assumption underpinning our SVX model, as we saw that our residuals are not Gaussian (see Table 5.5). An alternative approach could involve adopting a distribution with heavier tails for the model's innovations, such as the stochastic volatility with student's errors, as outlined by Omori et al. (2007). Thirdly, an interesting avenue would involve a formal comparative assessment of our SVX model's performance against other established volatility models, including the HAR or GARCH model. Such a comparative analysis would provide valuable insights into the relative strengths and weaknesses of each model for this application.

Finally, our work could also motivate new areas of investigation. For instance, we recommend a comprehensive exploration of the transmission channels that link financial uncertainty and volatility. Gaining insight into not only the effects of uncertainty but also the mechanisms through which it spreads is crucial for effectively mitigating its potential impacts on the broader economy. More specifically, the relationship between financial uncertainty and investor sentiment merits further investigation using more sophisticated models. Indeed, the linear regression model we utilized, while informative, has limitations in terms of establishing the direction of causality between variables. Specifically, to address this limitation, more advanced techniques such as Vector Autoregressive (VAR) or Vector Error Correction Model (VECM) approaches could be more suitable.

7 Appendix

7.1 Synthetic financial uncertainty index constituents

Table 7.1

Uncertainty proxies description and sources

Financial uncertainty proxies	Abbreviation	Period	Source
CBOE S&P500 Implied Volatility	VIX	2000-01-03 - 2023-05-01	Eikon - Refinitiv
OFR Financial Stress Indicator	FSI	2000-01-03 - 2023-05-01	Office of Financial Research
Cross-sectional Firm Uncertainty index	IV_FIRM	2000-01-03 - 2020-12-31	Dew-Becker and Giglio (2023)
Equity Market Volatility	EMV	2000-01-03 - 2023-05-01	Bekaert et al. (2022)
S&P500 Trading Volume	VOLUME	2000-01-03 - 2023-05-01	Yahoo Finance
ICE BofAML MOVE Index	BOND_MOVE	2003-08-01 - 2023-05-01	Eikon - Refinitiv
CBOE Crude Oil Implied Volatility Index	OVX	2008-06-06 - 2023-05-01	Eikon - Refinitiv
Cboe Gold Implied Volatility Index	GVZ	2008-06-06 - 2023-05-01	Eikon - Refinitiv

7.2 Regression analysis

Table 7.2

Regression results - Investor sentiment as a function of Financial Uncertainty

	α_{FU}	Std.	t-stat	β_{FU}	Std.	t-stat	T
log(IS)	2.2415	0.3549	6.315***	0.0286	0.1756	0.1632	268
log(NIS)	2.4444	0.0302	81.086***	-0.0619	0.0139	-4.4706***	268
log(CS)	5.3921	0.2954	18.249***	-0.4236	0.1403	-3.0199***	268

Notes: Log-log linear regression between each investor sentiment proxy (IS, NIS, CS) and lagged financial uncertainty (FU). The logarithm of both dependent and independent variables is taken to get more linearity in parameters. Standard errors are estimated with Newey-West heteroscedastic robust estimators. Statistical significance is given by *p<0.1; **p<0.05; ***p<0.01.

Table 7.3

Augmented Dickey-Fuller unit root test results

Variable	Test statistics	P-value	Lag order	H1
FU	-3.1107	0.1081	6	Stationary
IS	-3.2065	0.08722*	6	Stationary
CS	-2.805	0.2369	6	Stationary
NIS	-1.8354	0.6453	6	Stationary

Notes: Lag order is determined automatically using the rule $k = \lfloor (T-1)^{1/3} \rfloor$ where T is the sample length of the tested series and $\lfloor . \rfloor$ is the floor function. Statistical significance level is given by *p<0.1; **p<0.05; ***p<0.01.

Table 7.4

Johansen's cointegration test results

	Null hypothesis: r = 0		Null hypothesis: r <= 1	
Variable	Test statistics	Critical val. (5%)	Test statistics	Critical val. (5%)
IS	24.54***	15.67	3.52	9.24
CS	23.63***	15.67	7.45	9.24
NIS	26.39***	15.67	15.53***	9.24

Notes: Cointegration test results of Johansen (1991). Cointegration order is denoted r. Lag length of 1 was assumed. Statistical significance is given by *p<0.1; **p<0.05; ***p<0.01.

Figure 7.1

Linear regression plot: Consumer Sentiment (CS) as a function of lagged Financial Uncertainty (FU)

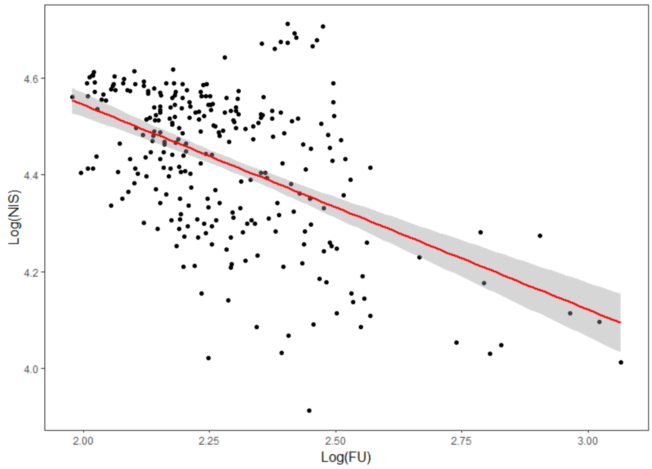


Figure 7.2

Linear regression plot: Investor Sentiment (IS) as a function of lagged Financial Uncertainty (FU)

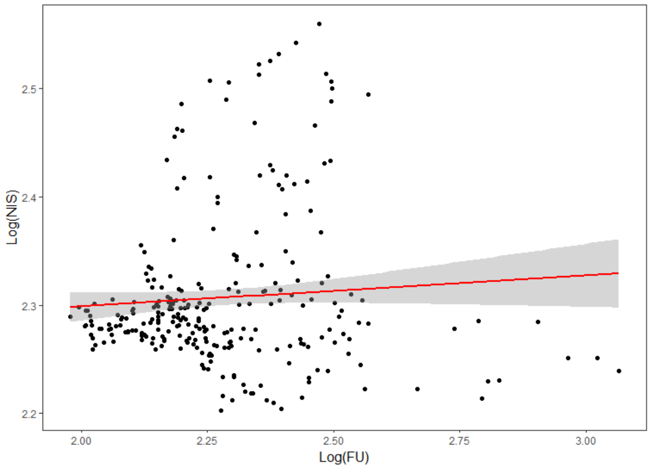
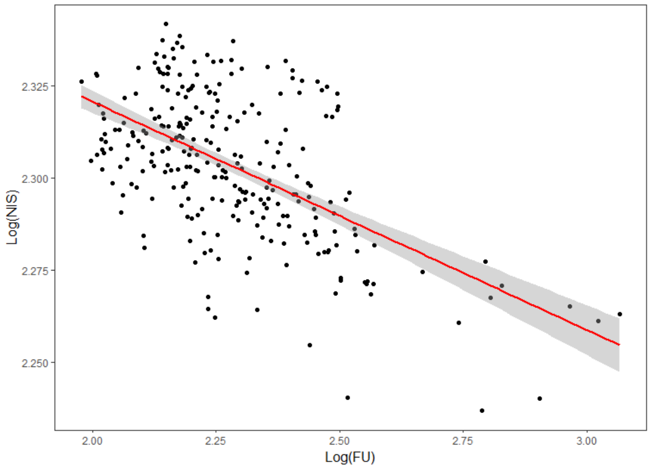


Figure 7.3

Linear regression plot: News Investor Sentiment (NIS) as a function of lagged Financial Uncertainty (FU)

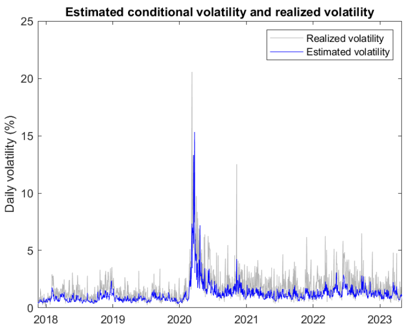


7.3 Estimated volatility plots

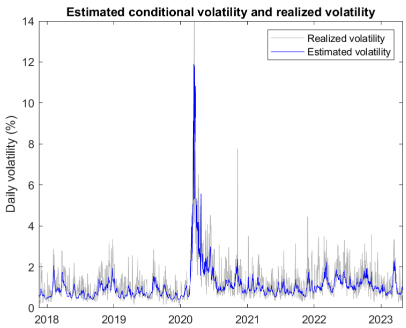
Figure 7.4

Plot of the SVX estimated conditional volatility against realized volatility

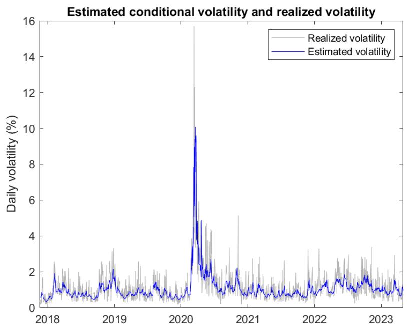
(a) *XLE*



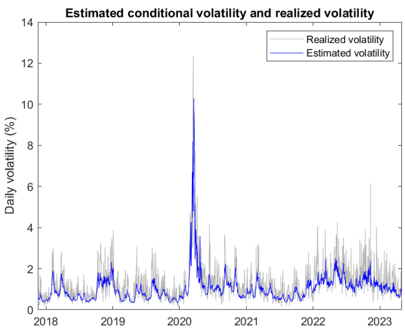
(b) *XLF*



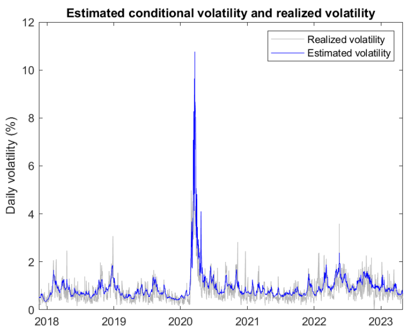
(c) *XLI*



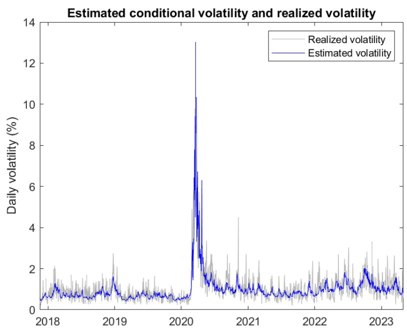
(d) *XLK*



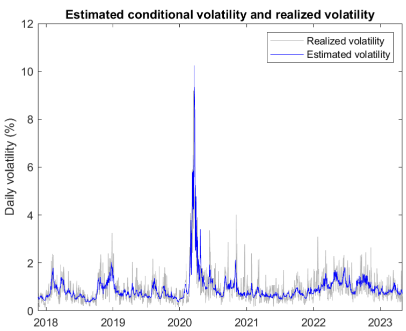
(e) *XLP*



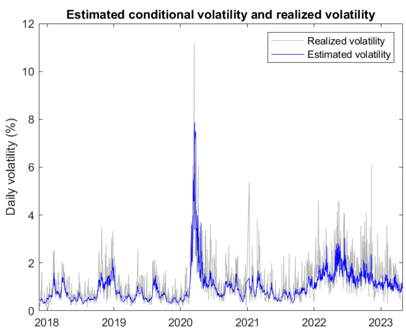
(f) *XLU*



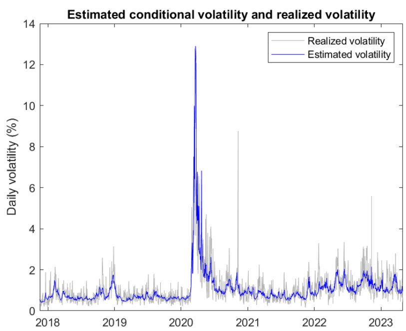
(g) *XLV*



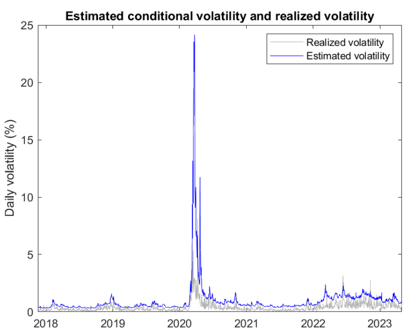
(h) *XLY*



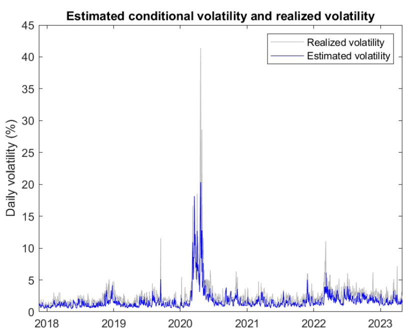
(i) *IYR*



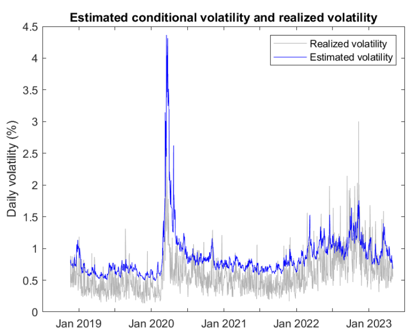
(j) *JNK*



(k) *WTI*



(l) *USDX*

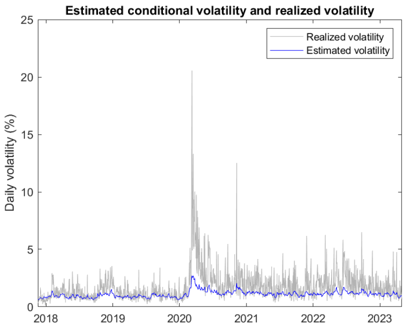


7.4 Value-at-Risk plots

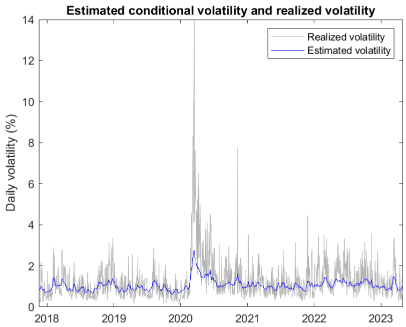
Figure 7.5

Plot of the AR(1)-SV estimated conditional volatility against realized volatility

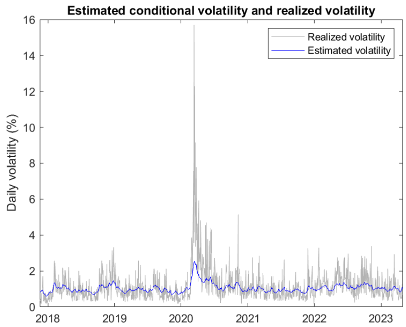
(a) *XLE*



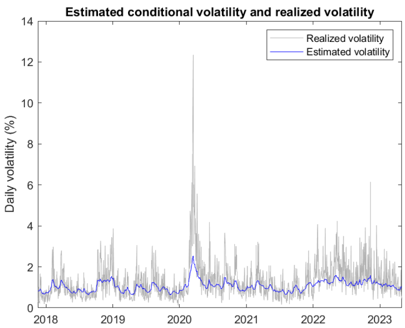
(b) *XLF*



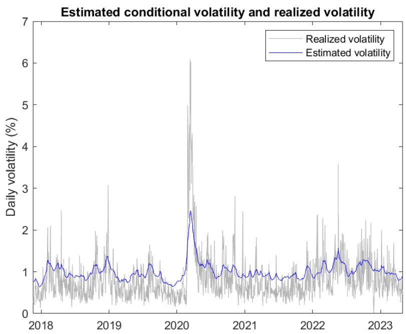
(c) *XLI*



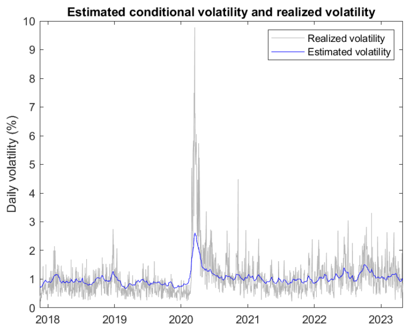
(d) *XLK*



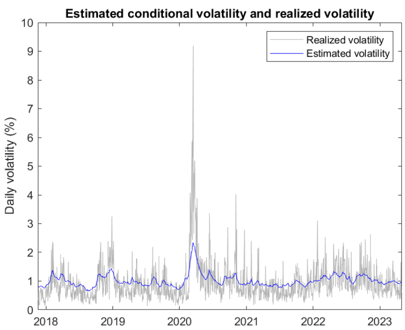
(e) *XLP*



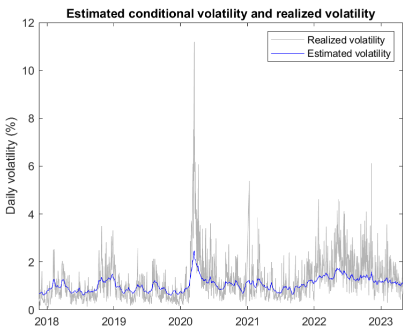
(f) *XLU*



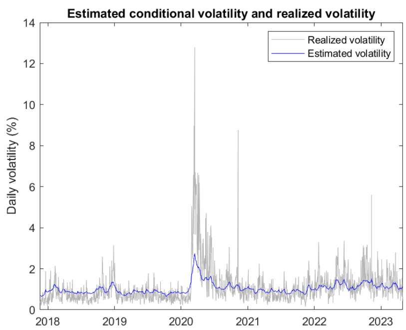
(g) *XLV*



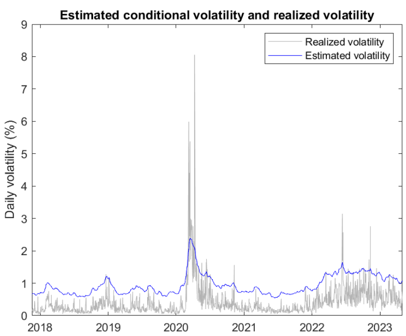
(h) *XLY*



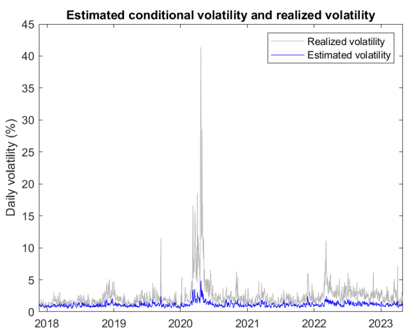
(i) *IYR*



(j) *JNK*



(k) *WTI*



(l) *USDX*

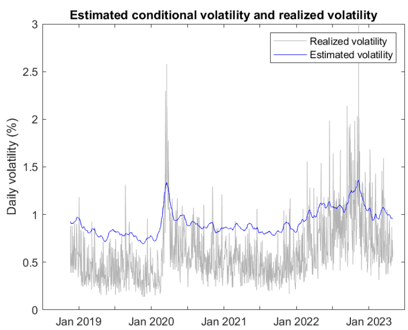
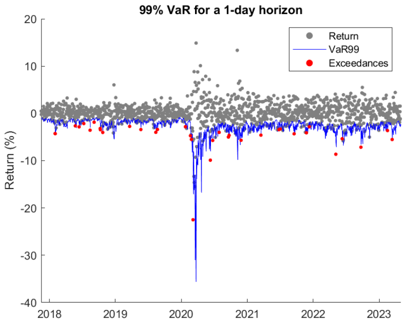


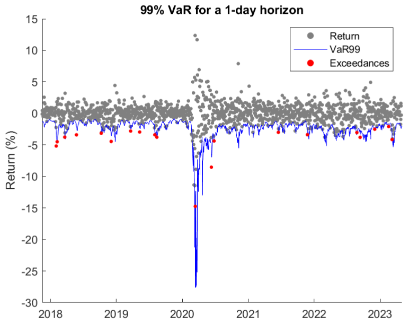
Figure 7.6

Gaussian 99% one-day VaR plots - SVX model

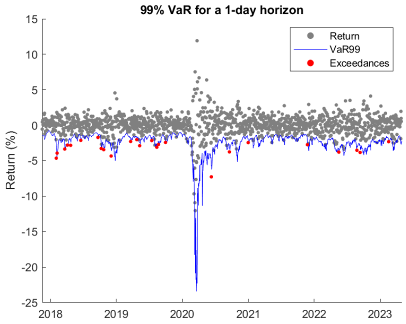
(a) *XLE*



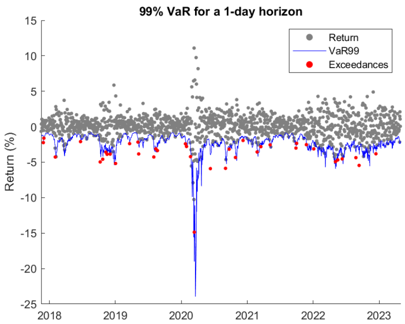
(b) *XLF*



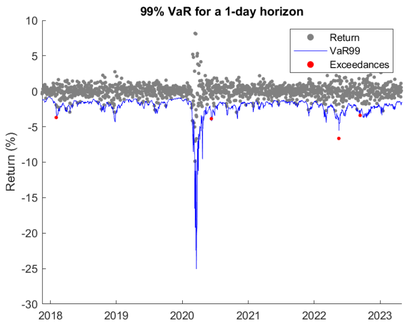
(c) *XLI*



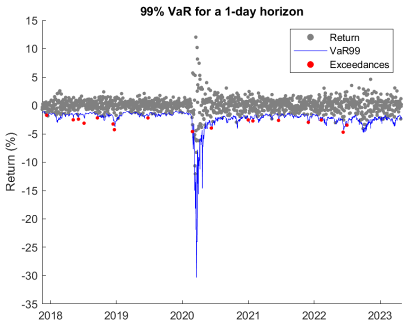
(d) *XLK*



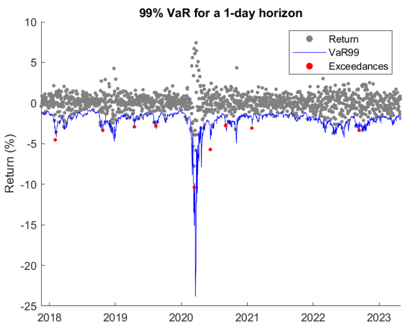
(e) *XLP*



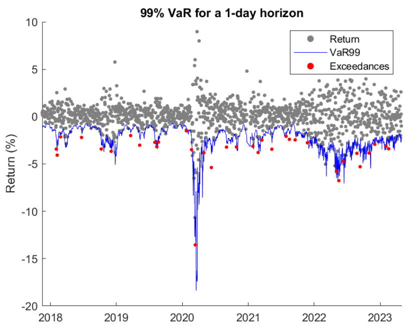
(f) *XLU*



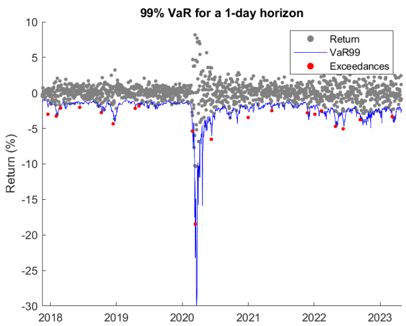
(g) *XLV*



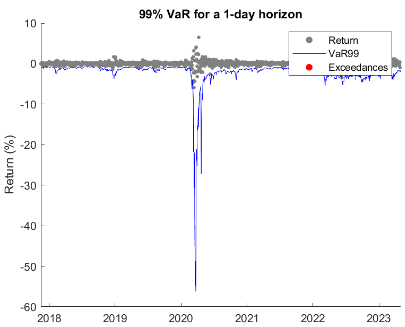
(h) *XLY*



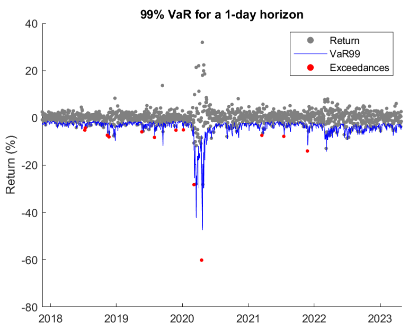
(i) *IYR*



(j) *JNK*



(k) *WTI*



(l) *USDX*

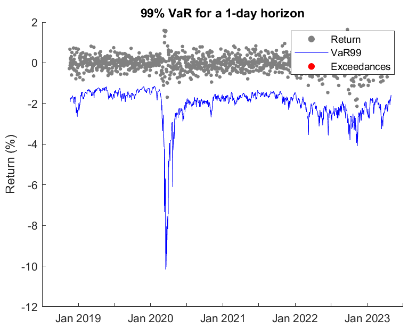
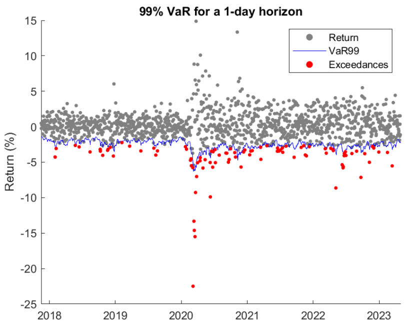


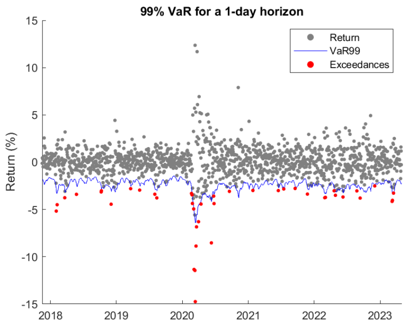
Figure 7.7

Gaussian 99% one-day VaR plots - AR(1)-SV model

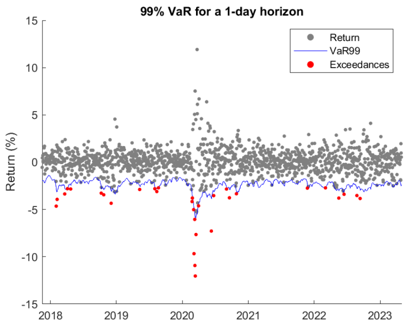
(a) *XLE*



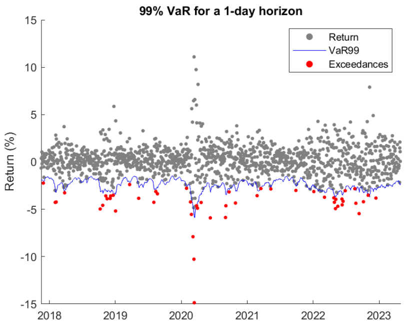
(b) *XLF*



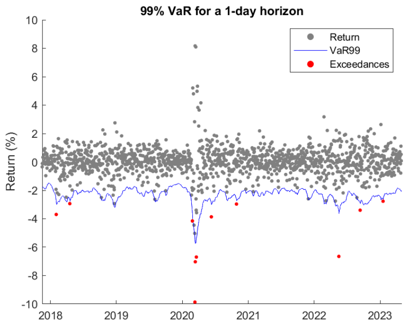
(c) *XLI*



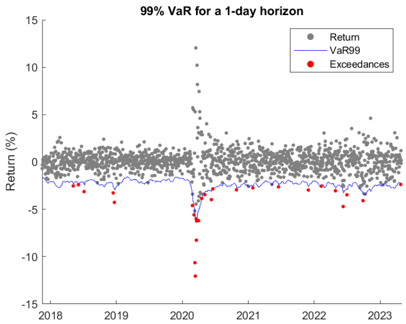
(d) *XLK*



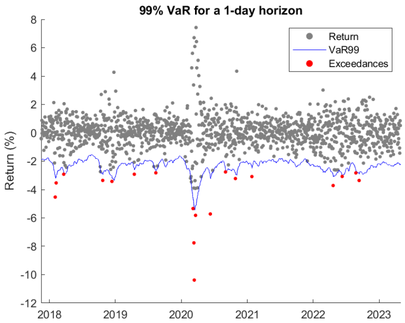
(e) *XLP*



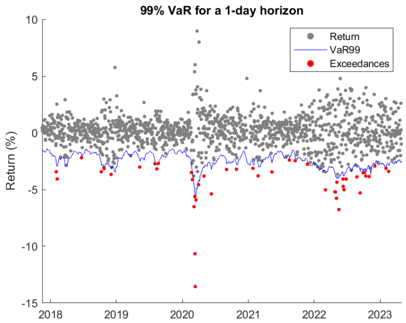
(f) *XLU*



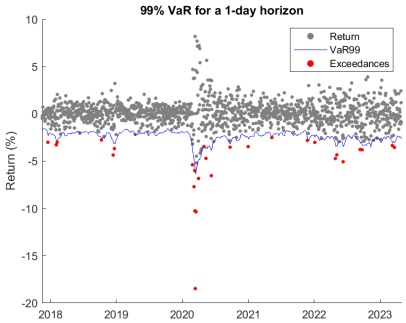
(g) *XLV*



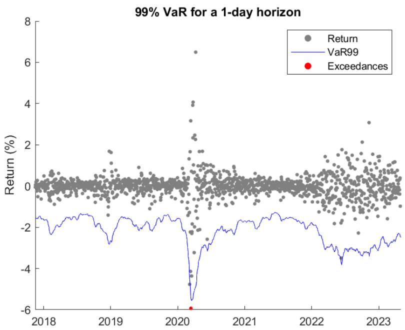
(h) *XLY*



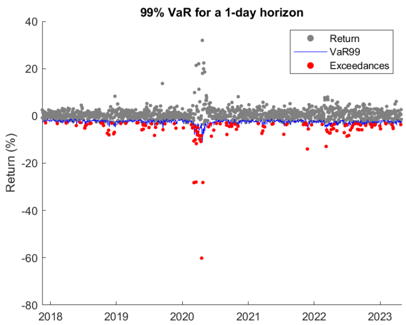
(i) *IYR*



(j) *JNK*



(k) *WTI*



(l) *USDX*

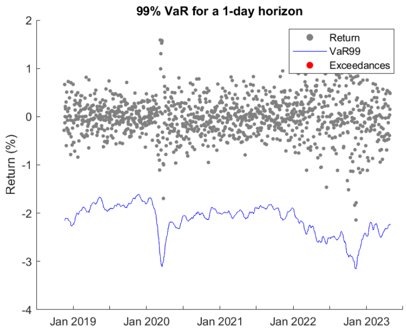


Table 7.5

Posterior mean estimates and HPDI for the robustness check SVX model specification

	SVX 6 covariates: robustness check (Sample 2017-11-15 : 2023-05-01)												
	Asset	XLE	XLF	XLI	XLK	XLP	XLU	XLV	XLY	IYR	WTI	JNK	USDX
β_1	Mean	**0.4566**	**0.4446**	**0.4006**	**0.4416**	**0.452**	**0.4048**	**0.4271**	**0.4213**	**0.413**	**0.4553**	**0.5741**	**0.2224**
	HPDI	[0.3752; 0.5415]	[0.3523; 0.5367]	[0.3016; 0.4952]	[0.3415; 0.5402]	[0.3378; 0.5574]	[0.3157; 0.4916]	[0.3100; 0.5354]	[0.3012; 0.5321]	[0.3075; 0.5099]	[0.3721; 0.5395]	[0.4332; 0.7061]	[0.1098; 0.3371]
β_2	Mean	**-0.1777**	**-0.1337**	-0.0901	**-0.1657**	**-0.1789**	**-0.177**	**-0.2635**	**-0.1896**	**-0.1375**	**-0.0589**	-0.1055	-40.75
	HPDI	[-0.2377; -0.1174]	[-0.2200; -0.0495]	[-0.1885; 0.0085]	[-0.2430; -0.0895]	[-0.3544; -0.0146]	[-0.3037; -0.0570]	[-0.4204; -0.1249]	[-0.2853; -0.0910]	[-0.2524; -0.0265]	[-0.1016; -0.0163]	[-0.5464; 0.3258]	[-1.0810; 0.2388]
β_3	Mean	**0.3314**	**0.1829**	0.1306	**0.2936**	0.2668	**0.2553**	**0.4513**	**0.4048**	**0.2009**	**0.0968**	0.2614	0.3224
	HPDI	[0.2202; 0.4402]	[0.0253; 0.3453]	[-0.0523; 0.3172]	[0.1540; 0.4366]	[-0.0353; 0.5980]	[0.0272; 0.4911]	[0.1876; 0.7383]	[0.2328; 0.5788]	[0.0092; 0.4063]	[0.0287; 0.1636]	[-0.5512; 1.1345]	[-0.7829; 1.5385]
β_4	Mean	-0.0022	-0.0876	-0.0985	**-0.1473**	**-0.2414**	0.0092	**-0.1573**	-0.1198	0.007	0.0392	-0.147	0.0162
	HPDI	[-0.1263; 0.1241]	[-0.2094; 0.0355]	[-0.2219; 0.0272]	[-0.2748; -0.0157]	[-0.3796; -0.0919]	[-0.1075; 0.1317]	[-0.3031; -0.0022]	[-0.2577; 0.0172]	[-0.1172; 0.1325]	[-0.0839; 0.1674]	[-0.3226; 0.0404]	[-0.1585; 0.1967]
β_5	Mean	-0.1483	-0.0623	**-0.1929**	-0.0681	0.0521	-0.1025	-0.003	-0.1012	-0.0342	0.1424	-0.0907	0.1824
	HPDI	[-0.2960; 0.0033]	[-0.2155; 0.0957]	[-0.3465; -0.0314]	[-0.2411; 0.1009]	[-0.1296; 0.2346]	[-0.2623; 0.0579]	[-0.1778; 0.1711]	[-0.2799; 0.0733]	[-0.1876; 0.1259]	[-0.0131; 0.2962]	[-0.3200; 0.1402]	[-0.0568; 0.4274]
β_6	Mean	-0.0282	-0.3195	-0.1872	0.1889	-0.1545	-0.1545	-0.2471	**0.9225**	0.0589	-0.3444	**0.8148**	**0.8495**
	HPDI	[-0.3669; 0.3192]	[-0.7045; 0.0748]	[-0.5966; 0.2331]	[-0.3021; 0.7029]	[-0.6747; 0.6202]	[-0.6033; 0.2646]	[-0.7864; 0.4136]	[0.3770; 1.5392]	[-0.3608; 0.4827]	[-0.6784; -0.0071]	[0.2699; 1.4636]	[0.3112; 1.3929]
ω_h^2	Mean	**0.5302**	**0.2027**	**0.1519**	**0.161**	**0.2274**	**0.1667**	**0.1883**	**0.1643**	**0.1491**	**0.7903**	**0.2524**	**0.2438**
	HPDI	[0.4181; 0.6554]	[0.1313; 0.2893]	[0.0929; 0.2449]	[0.1140; 0.2204]	[0.0802; 0.6402]	[0.0684; 0.3825]	[0.0808; 0.4570]	[0.0986; 0.2833]	[0.0781; 0.2801]	[0.6553; 0.9386]	[0.0692; 0.7559]	[0.0545; 0.7361]
ϕ_h	Mean	**0.5996**	**0.8039**	**0.8318**	**0.8818**	**0.8252**	**0.7985**	**0.8601**	**0.8849**	**0.8309**	**0.472**	**0.756**	**0.7082**
	HPDI	[0.5108; 0.6813]	[0.7252; 0.8748]	[0.7326; 0.9210]	[0.8281; 0.9312]	[0.6085; 0.9761]	[0.6074; 0.9583]	[0.6724; 0.9724]	[0.7997; 0.9533]	[0.6892; 0.9420]	[0.3773; 0.5603]	[0.5584; 0.9738]	[0.4978; 0.9715]
μ_h	Mean	**0.4635**	0.0705	0.0277	0.0918	-0.1226	-0.0181	-0.0573	0.0452	0.005	**0.9181**	-0.3538	-0.3614
	HPDI	[0.3452; 0.5763]	[-0.0731; 0.2039]	[-0.1599; 0.2142]	[-0.1593; 0.3304]	[-0.6076; 0.2917]	[-0.3069; 0.2455]	[-0.5227; 0.3431]	[-0.2752; 0.3370]	[-0.2143; 0.2183]	[0.789; 1.0467]	[-1.1756; 0.1782]	[-1.2237; 0.0283]

Notes: Posterior mean estimate is computed as the sample mean over the sample drawn from the posterior distribution. 90% High Posterior Density Intervals (HPDI) are written in brackets. They represent the narrowest interval that contains 90% percent of the posterior probability distribution of the parameter.

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Executive summary

This master's thesis[1] attempts to provide additional insights into the intricate relationship between financial uncertainty and asset volatility. Using the extended stochastic volatility model (SVX) of Ulm and Hambuckers (2022), we explore the effects of financial uncertainty on the conditional volatility of a diverse set of 12 financial assets. Our analysis is conducted over the period spanning from November 2017 to May 2023, and relies on a daily synthetic financial uncertainty index that we constructed by means of a principal component analysis. In our examination, we uncover that a higher financial uncertainty level generally reinforces volatility. However, this influence is heterogeneous in magnitude across the various categories of assets examined. Importantly, our study also unveils that a part of the effects of financial uncertainty is propagated to asset volatility through investor sentiment. Knowing that uncertainty rises sharply in times of market stress, our study also demonstrates that incorporating the financial uncertainty level substantially improves both in-sample and out-of-sample volatility modelling performance during these periods. Interestingly, this positive effect extends to normal market conditions as well, albeit to a lesser extent. This improvement also materializes in the construction of risk metrics that better capture tail events and extreme market conditions.

Keywords— Financial uncertainty, stochastic volatility, volatility modelling, volatility forecasting, investor sentiment, daily US financial uncertainty index

[1]This thesis contains approximately **18444 words**