A Heuristic Approach to the Home Healthcare Districting Problem in the Province of Liège

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Glossary

1. **Basic unit:** It refers to the smallest geographical or administrative entity used as a building block for creating larger districts.
2. **Care load:** This is related to the time spent by nurses giving care to patients.
3. **Case load:** It represents different service requirements for various patient categories.
4. **Compatibility:** This criterion makes some basic units of each district belong to the same district or, in case of incompatibility, makes it possible to avoid having incompatible basic units in the same district.
5. **Contiguity:** This criterion means that each district should be connected and should contain no enclaves.
6. **District:** A district is a defined geographical area or region that is established for various administrative, electoral, or organizational purposes.
7. **HHC:** Home Healthcare.
8. **Homogeneity:** This criterion categorizes patients by age, pathology, etc.
9. **Integrality:** Districts are constituted by groups of indivisible basic units that must be sufficiently small to allow enough flexibility in the solution design but not too small to keep the problem size manageable. A basic unit cannot be divided and belongs to only one district.
10. **Mobility:** This criterion refers to the transportation mode used by home healthcare staff while visiting patients.
11. **Territory compactness:** This criterion refers to areas formed by basic units that are as close as possible to each other, affecting transportation within the district. This is achieved by minimising a dispersion function, using distance limits, or routing cost approximations.
12. **Time windows:** This criterion is commonly associated with scheduling and logistics, often used in vehicle routing problems, such as the traveling salesman problem with time windows or the vehicle routing problem with time windows. These problems involve finding optimal routes for vehicles that have time constraints when visiting certain locations.
13. **Travel load:** This is related to the time spent by nurses going to visit their patients.
14. **Unique assignment criterion:** This means that each basic unit must be assigned to a single district.
15. **Workload equilibrium:** One important consideration when designing healthcare districts is to ensure that the total workload (expressed in hours per year), composed of care and travel load, of each district is roughly the same.
16. **LA:** Location-allocation model.
17. **SP:** Set-partitioning model.
18. **TSP:** Travelling Salesman Problem.
19. **VRP:** Vehicle Routing Problem.

1. Introduction

1.1 Preamble

The global population is currently witnessing a significant increase in both the size and the proportion of elderly people, resulting in a longer life expectancy worldwide. Presently, there are approximately 1 billion individuals aged 60 and above. However, the latest projections indicate that this number will rise to 1.4 billion by 2030 and double by 2050, reaching 2.1 billion. Initially, the aging trend was predominantly observed in high-income countries like Japan, where 30% of the population is already over 60 years old. Nevertheless, by 2050, two-thirds of the global population aged 60 and above will reside in low-income and middle-income countries (*Ageing and Health*, s. d.).
As the population age and life expectancy increases, there is a growing prevalence of chronic diseases such as diabetes and dementia, as well as functional impairments like mobility difficulties and challenges in managing household tasks. Consequently, the healthcare industry is confronted with numerous challenges, including the promotion of independence in impaired individuals and the delivery of efficient health services that can cope with the growing care demands while dealing with the reduced number of beds in hospitals. However, these challenges are further compounded by constraints related to time and budgetary limitations (Kadushin, 2004).
Under these circumstances, home healthcare services represent an attractive opportunity to optimise the performance of healthcare systems. They grant an efficient use of limited resources and contribute to the reduction of overall costs in the healthcare sector. They encompass the provision of medical, therapeutic, and other essential services at the patient's residence, delivered by skilled practitioners under the guidance of a physician. The primary objectives of HHC include maintaining and restoring health, maximising patients' independence and well-being, minimising the effects of disability and illness, and avoiding unnecessary hospitalisations or long-term care admissions (Bashir et al., 2012).
Referrals for home healthcare services can originate from healthcare professionals, family members, or even patients themselves. There are several reasons why HHC is becoming an increasingly important field. Firstly, managing chronic cases at patients' homes has proven to be more efficient. Additionally, with a significant portion of the population residing in urban areas, HHC offers a viable solution as healthcare professionals are more readily accessible in such settings. Moreover, many seniors express a preference for receiving care in the comfort of their own homes. Consequently, several governments, including those of Hong Kong, Canada, and France, have been actively promoting HHC by offering reimbursements and implementing policies that support its development. This is primarily driven by the need for public healthcare services to adopt effective tools that can provide the required level of care at minimal cost (Lin et al., 2017).
Despite the increasing demand for HHC services, resources remain limited and the industry faces challenges such as a shortage of nursing professionals, near-zero profit margins, and requirements for automobiles, medical supplies, equipment, office space, and administrative staff (Milburn, 2012). Consequently, effective resource utilisation and optimisation of activities are vital to meet the ever-growing demand for HHC. Planning plays a crucial role in addressing these challenges and it encompasses various levels and difficulties. Although the problems encountered in HHC are not fundamentally different from those in healthcare facilities, additional considerations, such as patient territory distribution, must be required. Therefore, researchers have an opportunity to enhance the ability of HHC to meet patient demands by addressing the limited existing studies on home health care.
The existing literature related to HHC primarily focuses on two levels of decision-making. At the tactical/strategic level, attention is given to effectively districting the demand points. Meanwhile, at

the operational level, problems related to routing and scheduling are addressed. The operational level problems are built upon the districting problem and involve determining the sequence and assigning nurses to conduct daily visits to patients. This thesis will concentrate on the districting problem in HHC. The districting problem within the domain of home healthcare is a logistical decision faced by healthcare providers when designing networks of services to ensure coordinated medical care delivery to patients' homes. The districting problem, also known as territory design, re-districting, or territory alignment, falls under the umbrella of discrete optimisation and focuses on partitioning decisions. Its primary objective is to best serve customers distributed across a territory by dividing the geographic area into smaller regions known as "districts." These districts represent the units of service delivery and are managed by dedicated teams. The division of territories must adhere to various planning requirements and satisfy specific context-dependent criteria (Milburn, 2012). Although districting problems share common constraints, such as compactness, uniqueness of assignment, balance, and contiguity, the nature of each problem differs based on its application area and context (Ríos-Mercado, 2020).

Furthermore, districting problems are predominantly classified as NP-hard, and as a result, most of the proposed solutions rely on heuristic and metaheuristic approaches. However, certain districting problems possess unique properties that make them suitable for exact optimisation schemes (Ríos-Mercado, 2020).

1.2 Objectives and Research Question

The objectives of this master's thesis are to review the current state of the art in districting and, in particular, districting problems in home healthcare and to propose a tailored districting configuration for a specific city by incorporating new constraints to enhance the management of home healthcare resources. Additionally, this thesis aims to develop an efficient method to solve the proposed model, considering various components that may impact the quality of the solution.

The contribution of this master's thesis lies in the advancement of districting models for home healthcare, focusing on their effectiveness and efficiency. By designing districts that account for care load and travel load, the management of home healthcare resources can be improved, leading to an enhanced patient care quality.

In order to achieve these objectives, the following research question will be addressed:

"What specific factors should be considered when adapting the home healthcare districting problem in the Province of Liège to better align with the real-world scenario? Furthermore, how can these factors be effectively integrated into a solution method to obtain districts that improve home healthcare resource management? "

1.3 Overview

This section provides an overview of the structure of this thesis, outlining the organisation and flow of the paper.
Chapter 1 serves as the introduction, setting the stage for the study. It outlines the problem and highlights the significance of developing more effective and efficient districting models for home healthcare. Additionally, the research question is introduced, emphasizing the necessity of considering specific factors in order to adapt the home healthcare districting problem in the Province of Liège to the real-world scenario.
In Chapter 2, which focuses on the literature review, an in-depth analysis of existing research and knowledge related to districting problems in home healthcare is conducted. This comprehensive review is crucial for understanding the current state of the field and the methods employed in similar studies. It provides a solid foundation upon which the research question can be addressed.
Moving forward to Chapter 3, the methodology used to answer the research question and formulate the problem is explained. The steps taken to address the problem are detailed, ensuring a clear and systematic approach to the study. This chapter helps establish the framework for the subsequent analysis and implementation.
Chapter 4 provides insights into the sources of data used in the study and the methods employed to collect relevant information. Additionally, the chapter describes the implementation of the proposed algorithm, ensuring the transparency and reproducibility of the study.
In Chapter 5, the obtained results from the implemented algorithm are presented, providing a platform for their analysis and discussion.
Finally, Chapter 6 serves as the conclusion and limitations section of the thesis. It offers a concise summary of the study, highlighting the key findings and their significance. Moreover, this chapter suggests potential areas for future research and improvements to the districting models and methods proposed in the thesis.

2. Literature Review

This literature review aims to provide a thorough examination of the research landscape surrounding districting problems, with an emphasis on home healthcare districting. Over recent years, remarkable advancements have been made in this area. Researchers have employed mathematical programming models, heuristic methods, and various other techniques to address objectives like workload balance and compactness.

Our goal is to dissect salient studies, appraise the strengths and limitations of different methodologies, and identify research gaps. This will pave the way for new research areas and ground our research ambitions. While delving into this topic, we will examine models, methodologies, and techniques tailored to the home health care districting problem, aiming to glean best practices and identify future avenues of research.

This review will enhance the existing body of knowledge by offering a detailed look at the current research landscape. With insights from prior studies, we aim to design an innovative districting approach that increases care delivery, optimises resource allocation, and improves patient well-being in the Province of Liège.

2.1. Scope of the Applications of Districting and Solution Techniques

The districting problem is multifaceted, with complexities that are contingent upon specific requirements. Typical criteria include compactness, unique assignment, balance, and contiguity. This problem is relevant to various domains, including electoral districts, schools, businesses, police jurisdictions, power supply, and home healthcare. Among these, the balance criterion is ubiquitous. It may refer to volume or customer count in commercial contexts, population equity in political arenas, or nurse workloads in home healthcare. The techniques and solutions to tackle these issues are diverse and we will explore this diversity in this section.

Kalcsics (2015) outlined a range of districting criteria and demonstrated that modelling approaches vary according to the context.

D'Amico et al. (2002) focused on police districting with the aim of optimising patrol car utilisation and ensuring an equilibrium in officer workload. They treated the issue as a graph partitioning problem and proposed a solution approach using simulated annealing.

Bozkaya et al. (2003) delved into political districting, emphasising population equity and socio-economic homogeneity. They formulated a multicriteria function and solved it using a tabu search combined with an adaptive memory heuristic, illustrating their approach with a case study based on Edmonton data.

Ríos-Mercado et al. (2007) introduced a model for commercial territory districting with objectives such as minimising territory dispersion and ensuring balance and contiguity. They utilised a GRASP approach as their solution.

Salazar-Aguilar et al. (2011) dealt with a commercial districting problem that had balance and connectivity constraints. The problem could be viewed as either a p-median or a p-center problem, depending on the objective function. They formulated the problem using a linear integer programming model and provided a quadratic integer programming formulation, which involved fewer variables than the linear one. Additionally, they introduced an exact solution framework for this problem, which was based on branch and bound, and complemented by a cut generation strategy.

Clearly, the distinct nature of each problem influences its objectives, constraints and, subsequently, the chosen solution methods. Since the problem has been identified as NP-hard (Ozturk et al., 2022), most of the solution methods employed are heuristics.

2.2. Districting in Healthcare

The hierarchical structure of health care services in many nations starts with a widespread primary care network helmed by general practitioners, succeeded by secondary care primarily located in specialised hospitals. Strategic healthcare organization entails decisions about facility placement, service regions, capacity calculations, resource allocation, and workforce scheduling. The crux of this planning lies in demarcating health care service zones, equivalently referred to as the districting of health care services (Ríos-Mercado, 2020) .

Mahar et al. (2011) suggested a location-allocation model to ascertain the optimal number and type of hospitals within a network offering specialised services. Their multi-objective model tackled capacity-demand matches and the problem is solved through optimisation.

Datta et al. (2013) tackled primary health care districting system operated by general practitioners. They proffered a multi-objective model and utilised the NSGA-II, a multi-objective genetic algorithm, for solving it.

Jia et al. (2014) leveraged a p-median model with a centric capacity objective for districting, replacing traditional distance measurements with travel time, and considering speed limits. Their solution approach was a simulated annealing.

In conclusion, districting in healthcare is a dynamic field that integrates various modelling approaches and solution techniques to optimise the delivery and accessibility to health services. The diverse methods employed by researchers emphasise the multifaceted nature of this challenge and the necessity for continuous innovation in response to evolving healthcare needs.

2.3. Home Healthcare Districting

As previously mentioned, the districting problem in the domain of home healthcare is a logistical challenge that healthcare providers encounter when designing service networks to ensure coordinated medical care delivery to patients' homes. While HHC delivers care at home, primary care focuses on maintaining general health whereas secondary care tackles specific medical issues. In this discussion, we will delve deeply into various studies that are focused on HHC.

Blais et al. (2003) conducted a study on a real-case districting problem for the Cote-Des-Neiges local community health clinic in Montreal. The objective was to divide a territory into six districts by grouping basic units while satisfying five criteria: integrality, respect for borough boundaries by incorporating a compatibility constraint, contiguity, visiting personnel mobility, and care load equilibrium. The mobility criterion aimed to capture staff mobility using public transport and walking, which is applicable to the home health care context.

Instead of employing a mathematical programming approach, the researchers considered clustering heuristics as the most efficient solution technique for combining basic units into feasible districts while minimising a multi-criteria objective function. This approach effectively combined the considerations of visiting personnel mobility and care load equilibrium into a single multi-objective function.

In order to identify near-optimal clusters, the researchers utilised the tabu search technique developed by Bozkaya et al. (2003) for political districting. This heuristic method began with an initial solution, such as a manually crafted solution, or a solution iteratively constructed by using seed basic units. The algorithm then iteratively moved from one solution to another within its neighbourhood, employing two types of moves: moving a basic unit from its current district to an adjacent district or swapping two basic units at the border of contiguous districts. The problem was efficiently solved and the satisfaction of the clinic was confirmed during a 2-year period.

However, Lahrichi et al. (2006) conducted an analysis of the operational data from the Montreal health clinic three years after the implementation of the districting solution developed by Blais et al. (2003) . They identified care load imbalance as a result of fluctuations in the patient census of the home health agency over time. Resolving the districting problem periodically became necessary to counterbalance these demand fluctuations. However, this approach was to face challenges in terms of time, resource consumption and potential impact on patient satisfaction.

In order to address these challenges, Lahrichi et al. (2006) had two propositions. Firstly, they suggested a dynamic patient-to-nurse assignment approach that allowed nurses to be assigned to different districts instead of a fixed district. Secondly, they proposed to divide nurses into two groups: one group had to be assigned to a fixed district and another group was allowed to work flexibly in some or all districts.

The researchers developed a model that assigned clients to nurses based on geographic location and workload considerations. The workload was divided into three components: travel load, care load and case load, representing different service requirements for various patient categories. Patient locations were explicitly used instead of basic units for districting. The model employed a weighted sum objective function similar to Blais et al. (2003) but with a focus on balancing the three workload components.

A tabu search algorithm was used as the solution method, which facilitated the reduction of visit and caseload for nurses who accepted assignments in different districts.

In a study of the home healthcare districting problem, Bennett (2010) observed that measuring nurse workload solely based on the number of visits performed was not accurate. It was noted that larger districts required longer travel times compared to densely populated districts. Consequently, Bennett (2010) developed a method to approximate the expected daily travel time in each district. The workload measure considered both the time spent during patient care and the expected time spent traveling between patients' visits. The objective was to use the workload as constraint to generate feasible districts using a set-partitioning model.

The considered constraints allowed for the creation of contiguous, compact, and balanced districts, while minimising the expected operational routing and scheduling costs. The model assumed that the demand information was available at the zip code level but was assumed to be independent and uniformly distributed throughout the zip code.

The authors developed a solution method that combined principles from column generation and heuristic local search methods. In summary, this method made it possible to obtain a subset of feasible districts through a clustering heuristic. Subsequently, the linear relaxation of the set partitioning model was solved and the dual variable values associated with the linear relaxation solutions were used to improve the columns added to the subset of feasible districts. The construction of new columns utilised local search moves described in the work of Blais et al. (2003). This method gave solutions with fewer nurse travels compared to non-hybrid methods.

A few years later, the author published another work (Milburn, 2012) which described operations research applications in the home health industry. The author surveyed relevant literature and provided possible extensions to inspire the scientific community. These three works were considered as the only studies presenting computational results for the districting problem in home healthcare. In particular, this work focused on the tactical decision-making aspect of the home health nurse districting problem.

The authors claimed that two formulations were suitable for the districting problem in HHC. Firstly, they mentioned the location-allocation approach developed by Hess et al. (1965) for the political districting . Secondly, the set partitioning model was considered.

In a review of methods in HHC logistics management problems, Holguin et al. (2013) presented a three-dimensional framework. For each dimension, they discussed the available literature and models. The first dimension addressed the duration and impact of planning decisions across strategic, tactical and operational planning horizons. The second-dimension divided logistics functions into four decision units: network design, transportation management, staff management and inventory management. The third dimension described the set of activities involved in delivering HHC services to patients.

In their study, Benzarti et al. (2013) introduced two mixed-integer programming models to address the HHC districting problem. The first model aimed to minimize travel distance while ensuring care load equity. Emphasizing compactness, it sought to reduce overall travel distance for nurse visits and distribute the workload evenly among districts. The second model integrated compactness and care load equity objectives to improve caregiver reactivity by addressing travel distance and care load distribution simultaneously.

The study made several assumptions, including patients having the same problem, patients in a given basic unit having different profiles and knowledge of demand and capacity. They also assumed that nurses possessed multiple skills and that the number of districts was known, and they used the network distance in the modelling process. The authors assumed that only nurses assigned to a specific district provided care to patients in that district and that care time depended on the patient's profile.

The formulated constraints included indivisibility of basic units, compactness, care load balance and compatibility. They implicitly considered the ease of access within the districts as a compatibility criterion. The impact of different patient exigencies on workload and ease of access within a district was implicitly considered. This approach enabled them to assess the parameters' impacts, analyse the duality between models and demonstrate improvements in patient's and caregiver's satisfaction.

Gutiérrez et al. (2015) addressed the HHC districting problem in a rapidly growing city in Colombia. They proposed a variation of the location-allocation model, named facility-facility location, which did not require a predetermined number of districts. Their bi-objective model minimised travel load and total care load equity. They did not consider contiguity constraints, allowing nurses to travel among non-contiguous districts. The constraints considered included unique assignment, compactness, continuity of care, integrality, and socio-economic factors.

In order to model the problem, they incorporated features from the location-allocation formulation and the model proposed by Blais et al. (2003). They used a lexicographic approach for a solution, allowing decision-makers to incorporate preferences interactively. They used data from an HHC provider in Colombia for an instance and randomly generated datasets to assess model robustness. The solution analysis revealed that small travel workload deteriorations led to significant improvements in workload deviations and better districting configurations.

Lin et al. (2017) focused on a specific home health care districting problem involving meal distribution services. They employed a location-allocation approach to minimise the number of districts, considering capacity, time window limitation, maximal travel duration, accessibility, compactness and integrality. Care workers were treated as homogeneous in this study. The problem was solved using a greedy heuristic method, which provided comparable results to the optimal one but in less time.

Cortés et al. (2018) propose a mixed integer linear programming model based on the p-regions formulation proposed by Duque et al. (2011), incorporating contiguity to minimise workload imbalance. The constraints considered were unique assignment of districts and contiguity. They used a greedy randomised adaptive search to solve large-scale instances and presented a case study of an HHC provider in Colombia. The model achieved 44% optimal solutions for instances with less than 60 basic units, while the GRASP approach reached optimal solutions in 74% of the cases and improved solutions in instances with more than 60 basic units.

Ríos-Mercado (2020) summarised the literature on health care districting problems and proposed future extensions in their book. They categorised health care districting into home care services, primary and secondary health care services, and emergency health care services. Each category's characteristics, modelling approaches and solution strategies were defined.

Ozturk et al. (2022) demonstrated the NP-completeness of the districting problem and revisited the care load balancing districting problem proposed by Benzarti et al. (2013). The revisited model modifies the objective function proposed by Benzarti et al. (2013). In the previous model, the maximum gap is measured by comparing the workload of each district, but this measurement does not yield a balance with certainty. Consequently, Ozturk et al. (2022) define the maximum gap as the difference of care loads among districts. Also, they propose a three-phased heuristic method. The first phase is a pre-assignment procedure (PAP) which finds the most incompatible BU since there are incompatible basic units in the problem. The second phase assigns remaining locations to obtain a feasible solution. The last phase is an iterative process that improves the initial solution.

The heuristic finds the optimal solution in most instances and provides a good solution when there are more than 10 districts and 500 basic units. The analysis shows that the PAP procedure can halve the solution time when the number of districts is low.

Table 1 summarises the characteristics of the different models studied in HHC districting, with 'O' indicating that the characteristic is considered as an objective and 'X' indicating that it is considered as a constraint.

Constraints	Blais et al. (2003)	Lahrichi et al. (2006)	Bennett (2010)	Benzarti et al. (2013)	Benzarti et al. (2013)	Gutiérrez et al. (2015)	Lin et al. (2017)	Cortés et al. (2018)	Ozturk et al. (2022)
Time Windows							X		
Mobility / Accessibility	O		X				X		
Care Load Equity	O	O		O	X	O		O	O
Patient homogeneity		O							
Travel Load Equity		O	O			O	X		
Integrality	X			X	X	X	X	X	X
Compactness			X	X	O		X		X
Compatibility	X			X	X				X
Contiguity	X		X					X	
Capacity							X		
Number of Districts							O		
Model	Clustering	LA	SP	SP	LA	LA	LA	P-region	SP
Method	Tabu Search Heuristic	Tabu Search Heuristic	Column Generation and Tabu			Lexicographic Algorithm	Greedy Heuristic	GRASP	Three-phased Heuristic Method

Table 1 : Comparative Analysis of Home Health Care Districting Approaches

2.4. Workload balancing

Balancing workload is a pivotal component in districting problems. This section aims to shed light on the techniques used in load balancing, providing an overview of methods adopted in both districting and vehicle routing domains. It is essential to recognise the significance of vehicle routing in this context. Operations in home health care divide into two major time-consuming processes: the care load and the travel load.

Benzarti et al. (2013) considered only the care load. They presented two scenarios: one wherein each team's care workload across districts closely mirrors the average care workload and another that introduces a tolerance interval, ensuring that district workloads only marginally deviate from the average. The last one is also used in the model proposed by Blais et al. (2003) because perfect equity cannot be achieved.

Oyola et al. (2014) investigated a capacitated vehicle routing problem in which route length balance was incorporated as an objective. This balance was defined as the difference between the lengths of the longest and shortest routes, leading to the formulation known as the vehicle routing problem with route balancing. This approach bears similarities to the care workload balancing definition proposed by Ozturk et al. (2022) which is also referred to as the range formulation. While this formulation remains straightforward to implement and interpret, with an optimal value of 0, it doesn't adequately capture the absolute levels of the outcomes (*Workload Equity in Vehicle Routing Problems*, s. d.).

Another widely recognised formulation is the Min-Max VRP, striving to minimise the maximum value. This model bears its own set of limitations, primarily its inability to discern distributions with identical worst outcomes (*Workload Equity in Vehicle Routing Problems*, s. d.). Other definitions of workload balance are the mean absolute difference between each outcome and the mean, or standard deviation. While these capture the entire distribution, they pose challenges in terms of computational complexity and intuitiveness, potentially causing difficulties in real-life implementations.

In conclusion, even though equity is a fundamental principle, a consensus on its measurement and evaluation in the logistics field remains out of reach. Each problem, with its unique characteristics, might demand a specialised approach. It is crucial to understand that more complex equity measures do not necessarily result in better trade-off decisions (*Workload Equity in Vehicle Routing Problems*, s. d.).

3. Methodology

This chapter discusses the methodology employed in this thesis. As a reminder, one of the aims of this research is to find an efficient solution to the districting problem in a specific area. In order to ensure that we can easily collect information from the field, and due to feasibility and potential data availability considerations, we have decided to focus on the Province of Liège. The first aim of this master's thesis is to identify the needs of HHC districting in the Province of Liège and to propose a solution method. This solution strategy is an important component of this thesis. The components that can influence the goodness of a solution will be analysed to find the most appropriate solution.

To begin with, a comprehensive literature review has been conducted to identify the prevailing trends, extensions and issues associated with the application of the districting problem in HHC. Multiple databases have been utilised to ensure the thoroughness and inclusiveness of the review.

Following the literature study, interviews have been conducted to gain a deeper understanding of the current state and specific requirements of HHC within the Province. This section provides a description of the interviews and presents the findings obtained from these interviews. By comparing these findings with the existing literature, any disparities or gaps between theoretical knowledge and practical needs within the realm of HHC in the Province are identified.

Furthermore, this chapter explains the specific problem within the HHC domain in the Province of Liège that has been chosen as the central focus of this thesis. It also outlines the selected articles that form the basis for the proposed solution methodology.

In addition, the problem formulation is presented in such a way as to establish a coherent framework for addressing the identified problem.

Finally, the chosen solution method is presented.

3.1 Interviews

One of the aims of this research is to find an efficient solution to the districting problem in a specific area. In order to ensure easy information collection from the field and consider feasibility and data availability, the focus has been directed towards the Province of Liège. Conducting semi-structured interviews with team leaders in Home Healthcare became essential to understand the practices and challenges associated with HHC and HHC districting. It was important to survey multiple individuals to obtain a comprehensive approach. Therefore, people in charge of scheduling and assigning nurses, who also had experience in nursing activities within these HHC structures, were chosen as survey participants. This approach has permitted a concrete understanding of the situation and the needs of both nurses and management.

Due to time constraints, the interviews were only conducted in two companies. However, these companies represent a significant portion of the demand in the region under study. An interview guide tailored to the profile of the participants was developed for the semi-structured interviews. This guide helped structure the interview and provide necessary information to the participants. The interviews were conducted in French, recorded and transcribed. The first interview lasted approximately 1 hour and 30 minutes, while the second interview lasted approximately 30 minutes. The interview guide can be found in *Appendix.8*.

3.2 Identified Needs for Districting in HHC in the Province of Liège

After conducting interviews, we summarised and analysed the information collected from the field. From these analyses, we identified the problems and needs faced by HHC agencies. Here's a brief overview:

HHC services are crucial to Belgium's healthcare system. These services are requested by patients, hospitals, doctors and family members. A call centre assesses these demands and forwards them to the district manager in charge of the patient's location.

With Belgium grappling with high inflation in 2023, it is imperative to cut costs. Despite rising expenses, revenues and reimbursements for care aren't commensurated. This necessitates cost savings without sacrificing the quality of care provided by HHC agencies.

In the Province of Liège, while HHC agencies predominantly use cars for transportation, they have yet to explore greener alternatives, recognising their potential benefits. Equitable workload distribution is crucial, ensuring that nurses across districts can manage a similar patient load. Although nurses are multiskilled and can treat any patient, those who aren't are offered additional training. Nonetheless, nurse satisfaction remains a concern due to significant workloads and prevalent burnout.

HHC services are provided in three shifts, with patients visited at specific times of the day based on their medical conditions. The morning shift has a higher number of patients, primarily long-term patients who are diabetics or those who require care each morning and are not autonomous.

Nurses assigned to a district can also travel to contiguous districts to provide care, which allows them to have a workload balance and to reduce travel time, as the end of one district may be closer to a contiguous district.

The objective of HHC agencies is to provide care to each patient, but resources are sometimes insufficient. While rejecting patient demands is seen as a failure by nurses, they may have to transfer some patients to freelance nurses. Rejecting patient demands is also related to cost-effectiveness, as travelling long distances for poorly paid care is not profitable. Consequently, nurses prioritise nearby patients or those with well-reimbursed health issues. Palliative patients, for example, are never refused as it is the best-reimbursed care category.

The time spent during HHC visits varies based on patient needs and it has been rising. Reducing travel time remains a pivotal goal to increase efficiency and enhance patient interactions. From a logistical standpoint, nurses visit patients directly, relying on them for specific medical supplies.

Overall, the demand for HHC services remains stable throughout the year without any noticeable seasonality. This study emphasises the importance of reducing costs while maintaining care quality. It also highlights the need for workload equity, nurse satisfaction, and continuity of care to achieve positive outcomes and to increase patients' satisfaction.

In order to address the specific needs of HHC structures in Liège, the districting model should carefully consider multiple factors, including travel time and workload equity. Achieving workload balance is crucial, as it ensures that each district has a similar number of patients and workload, thus preventing caregiver burnout and ensuring the delivery of high-quality patient care.

Furthermore, taking into account the conformity to administrative boundaries can be a relevant factor, as demonstrated in the study of Benzarti et al. (2013). This consideration provides help in the effective management of healthcare delivery procedures and fosters the establishment of long-term relationships between caregivers and patients, as it prevents interference in caregivers' responsibilities.

In terms of travel time and costs, an effective approach to reduce them involves minimising the maximum distance between two basic units assigned to the same district. Additionally, Bennett (2010) highlights the significance of contiguity in reducing travel time.

In summary, the proposed districting solution for Liège should promote efficient, equitable and high-quality patient care, while addressing the needs of both patients and healthcare providers. These districts should exhibit equal workloads, be contiguous, indivisible and consider travel distances.

3.3 Link with Literature

Drawing from the identified needs of HHC districting in the Province of Liège, pertinent literature has been explored, providing insights into similar situations. Two notable papers, Benzarti et al. (2013) and Ozturk et al. (2022), offer valuable contributions to this thesis. In this section, we will summarise these papers and establish their relevance to our problem.

3.3.1. Benzarti et al. (2013)

We will focus on the first model presented by the authors, which aims to balance the care workload across districts. As mentioned in the literature review section, the assumptions are that the decision-maker knows the number of districts to design, all the basic units are covered and nurses are multi-skilled.

Sets	
$I = 1,\ldots N$ indexed by i, k	Set of basic units
$H=1,\ldots,\lvert H\rvert$ indexed by h	Set of patient profiles
$J=1,\ldots,N$ indexed by j	Set of districts
E	Set of basic units' pairs (i, k) where $(i,k)\in E$ if and only if $e_{ik}=0$
D	Set of basic units' pairs (i, k) where $(i,k)\in D$ if and only if $d_{ik} > d_{max}$

Parameters	
N	Number of basic units
M	Number of districts to design
H	Number of patients 'profile considered
b_h , $h \in H$	Number of visits required by a patient profile *h*
T_h ,$h \in H$	Average duration of a visit relative to the profile *h*
P_{ih} ,$i \in I, h \in H$	Number of patients living in the basic unit *i* and having the profile *h*
d_{ik} , $i, k \in I$	Distance between the basic units *i* and *k*
d_{max}	Maximum distance allowed between two basic units that can be assigned to the same district
$\overline{wd}$	Average care workload among all districts
e_{ik}	Compatibility index, 1 if the basic units *i* and *k* are compatible.

Variables	
x_{ij} , $i \in I$, $j \in J$	1 if basic unit *i* is assigned to district *j*
wd_j , $j \in J$	Total care workload of district *j*
gap_{max}	Maximum deviation between the care workload associated to each district and the average care workload among all districts.

Model

Minimise gap_{max}

S.t

$$wd_j = \sum_{i=1}^{N} \sum_{h=1}^{H} P_{ih}\, b_h\, T_h x_{ij} \qquad \forall\, i, \forall\, j, \forall\, h \qquad (1)$$

$$\overline{wd} = \frac{\sum_{i=1}^{N} \sum_{h=1}^{H} P_{ih} b_h T_h}{M} \qquad \forall\, i, \forall\, h \qquad (2)$$

$$gap_{max} \geq \frac{wd_j - \overline{wd}}{\overline{wd}} \qquad \forall\, j \qquad (3)$$

$$gap_{max} \geq \frac{\overline{wd} - wd_j}{\overline{wd}} \qquad \forall\, j \qquad (4)$$

$$\sum_{j=1}^{M} x_{ij} = 1 \;\; \forall i = 1\ldots, N \qquad \forall\, i, \forall\, j \qquad (5)$$

$$x_{ij} + x_{kj} \leq 1 \qquad \forall\, (i,k) \in E, \forall\, j \qquad (6)$$

$$x_{ij} + x_{kj} \leq 1 \qquad \forall\, (i,k) \in D, \forall\, j \qquad (7)$$

$$x_{ij} \in \{0,1\} \qquad \forall\, i, \forall\, j \qquad (8)$$

The objective function coupled with constraints (3) and (4) guarantees the minimisation of the maximum deviation of the care workload from the average care workload among all districts. Constraints (1) and (2) define the care workload for each district and the average care workload across all districts respectively. Constraint (5), in conjunction with constraint (8), ensures that each basic unit is assigned to one district only. Constraint (6) guarantees compatibility between assigned units and districts. Lastly, constraint (7) relates to the compactness requirement, bounding the distance between two basic units in the same district by a maximum distance, d_{max}. This upper bound ensures that travel time within each district remains manageable.

However, the author does not propose an alternative solution method other than solving the MILP using a solver. Moreover, for a fixed value of N, the feasibility of the model decreases as the number of districts to be designed, M, increases, leading to a higher mean gap_{max} and a lower mean distance. Hence, this model is best suited for moderately sized instances. It is essential to understand that the incompatibility of locations introduces a feasible assignment problem that is as important as the optimal assignment of locations. Finally, as the travel load is not considered in the objective function, in the context of the Province of Liège it might be beneficial to include it to ensure that districts have a balanced total load, which would encompass both the care and travel loads.

3.3.2. *Ozturk et al. (2022)*

The authors address the same problem as in the work of Benzarti et al. (2013), maintaining the same assumptions and constraints. However, they propose a modification to the objective function to ensure the minimisation of workload differences among basic units. While the model presented by Benzarti et al. (2013) measures the maximum gap by comparing the workload of each basic unit to the average workload, it does not always guarantee balance across different districts. Therefore, the authors propose equation (9) as a replacement for constraints (3) and (4), defining the maximum gap as the difference in workloads between basic units.

$$Minimize\ gap_{max}$$

$$gap_{max} \geq |\ wd_j - wd_k| \qquad \forall\ j \neq\ k \qquad (9)$$

Furthermore, Ozturk et al. (2022) introduced a heuristic approach to address larger-sized problems. This heuristic features a pre-assignment procedure (PAP) that begins the assignment of the most incompatible basic units without compromising the optimal value of the objective function. However, it is important to note that this solution method does not consider the travel load generated both within and between basic units. The algorithm also neglects network distance, and incompatibility is not well-defined within the algorithm. The papers treat compatibility as pre-determined data, exploring the issue from a conceptual standpoint. We need to define compatibility by considering data on which basic units are contiguous, the maximum allowable distance between them, and their accessibility.

3.4 Problem Statement for HHC Districting in the Province of Liège

We will now state the home healthcare districting problem that we aim to address for the Province of Liège. For this study, we work under the assumption that demand information is available at the "commune" level, a sub-regional unit within the province. Notably, our proposed approach can be adapted to various levels of aggregation, depending on data availability and the distinct needs of the districting challenge. We have chosen this level of aggregation in order to have conformity to existing administrative boundaries.

There are N basic units, referred to as "communes", with each basic unit possessing an area A_i . Each of these units also has a care load, denoted as wc_i, which is measured by the total care time required in that basic unit, represented in minutes. This care load is determined by the number of patients num_i multiplied by the visit time per patient, T_i . The locations of the patients are not known; they are assumed to be independent and uniformly distributed throughout the basic unit.

The objective of the problem is to create M contiguous districts, where M is a predetermined number based on factors such as population density, geographical constraints, or administrative considerations. These districts should be designed to achieve a balanced distribution of the daily total workload.

The workload consists of two kinds of load. Firstly, there is the care load, which is the time the nurse spends providing care to the patient. The care load of a district, wd_j , corresponds to the sum of the care load of the basic units, wc_i ,within this district. Secondly, there is the travel load, wt_i, which is the time the nurse spends travelling by car from patient to patient at a speed S. This travelling time depends on the distance between the patients' homes, but since we do not know the exact locations, we will consider it to depend on the area of the basic unit and the distance between basic units.

Workload balance is achieved when each district has a similar workload. Our solution method aims to achieve total workload equity between districts by modifying the care load balance definition (9) proposed by Ozturk et al. (2022) in order to incorporate the travel load and achieve total workload equilibrium.

3.5 Solution Method

Our research problem constitutes an extension of the one addressed by Ozturk et al. (2022), with several distinctions. Notably, our approach considers the total workload, a single patient profile, and contiguity. Nevertheless, we have designed our methodology to be adaptable for accommodating multiple patient profiles. Given that we work with aggregated data from the Province of Liège, devoid of specific patient information, we have opted for a single patient profile.

Our objective is to develop a solution method by refining and customizing the heuristic approach proposed by Ozturk et al. (2022), renowned for its efficacy in handling large-scale problems. Our intention is to improve this heuristic by incorporating the concept of travel load into the objective function, thereby producing more precise solutions. Additionally, we aim to include the contiguity constraint, which will contribute to reduce the travel load (Kalcsics & Ríos-Mercado, 2019a) .

In order to accomplish our task, we have devised two scenarios for defining the travel load, namely scenario 1 and scenario 2. In scenario 1, we allow flexibility in the order of patients' visits within a district, enabling nurses to attend to patients in different basic units without necessarily completing all patients in a specific basic unit before moving on to the next one. Scenario 1 is particularly suitable when patients are uniformly distributed within a district, and the distances between patients within a basic unit are smaller than those between locations in different basic units. It produces good results using a TSP (Traveling Salesman Problem) estimator that is based on the combined areas of the basic units and the total number of patients within them, without taking network distances into account.

In contrast, scenario 2 incorporates network distances into the solution. Here, the nurse visits all the patients within a basic unit before proceeding to the next basic unit within the district. We calculate the intra basic unit travel load within each basic unit using the TSP estimator and incorporate the minimum network distance between any two basic units within the same district. Scenario 2 takes both travel distances within basic units and network distances between them into account, offering a more comprehensive perspective. A detailed definition of these scenarios will be presented later in our study.

In order to enhance the robustness and efficiency of the solution, the Simulated Annealing (SA) metaheuristic is incorporated. SA is well-suited for complex optimisation problems like home healthcare districting, as it can explore the solution space extensively and avoid local optima. By introducing SA, the study aims to evaluate whether superior results can be obtained compared to the adapted method of Ozturk et al. (2022).

Overall, the proposed solution method seeks to provide a comprehensive approach that efficiently manages the travel load, leading to more balanced and efficient districting plans for home healthcare in the Province of Liège.

As previously mentioned, our objective is to minimise the overall workload imbalance. Therefore, it is crucial to establish a clear definition of the total workload. This involves determining the travel load within our chosen framework.

Total workload of district *j* = $\sum_{i \in j} wc_i + wt_j$

As previously explained, in order to define the travel load, we have explored two distinct approaches that we will define in this section. However, before delving into these approaches, let's first introduce an estimator that will be used in both. We have N basic units, each containing num_i patients that are uniformly distributed and need to be visited. The task of visiting these patients can be seen as a Travelling Salesman Problem (TSP).

In the routing literature (Kwon et al., 1995), Beardwood proposed an estimator for the length of a TSP route serving X points uniformly and independently distributed in a planar region of area A :

$Length \approx 0{,}75\sqrt{XA}$.

By employing this estimator, we can obtain an approximation of the travel distance required to visit all the patients in the system.

3.5.1 *Scenario 1*

In the first scenario, the travel load of district j is assumed to be equal to the TSP estimator. We calculate the travel load as follows:

$$wt_j = S\,0{,}75\sqrt{\sum_{i \in j} A_i \sum_{i \in j} num_i}$$

Here, we sum up the areas of all the basic units in district j and multiply it by the sum of the patients in those basic units. Multiplying by the speed factor gives us the time in minutes rather than the distance.

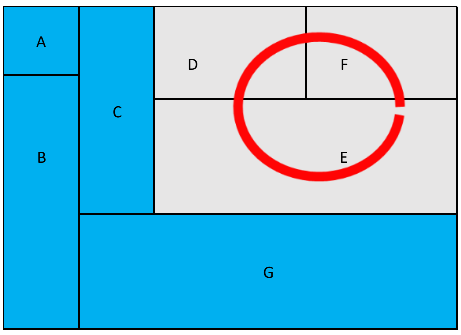

Figure 1 : Schematic Representation of Scenario One

For instance, in Figure 1, let's consider the grey district comprised of basic units D, E, and F. The travel route within this district is denoted by the red circle. This formulation considers the travel load of the entire district and assumes that we can visit one patient in basic unit D, then go to visit a patient in basic unit F without having visited all the patients in the basic unit D. This approach can be accurate if the centres of the basic units are not too far apart and if the next patient to visit in F is closer to the nurse's location than another patient in D in real life. However, in reality, populations often cluster around the centre of each basic unit, making the distances between patient locations within the same unit smaller than those between different units. Additionally, this method neglects considerations such as network distances and accessibility concerns.

3.5.2 Scenario 2

In the second scenario, we consider a different approach that incorporates network distances. Here is the mathematical formulation for this scenario:

$$wt_j = S\ \sum_{i\in j}(\underbrace{(0{,}75\sqrt{A_i num_i})}_{\text{Intra Basic Unit Load}} + \underbrace{min_{ik\in j, i\neq k}(d_{ik})}_{\text{Inter Basic Units Load}})$$

In order to calculate the total travel load for each district, we take into account two types of travel load, which we then sum up together:

1. Intra Basic Unit Travel Load: This represents the travel load generated when visiting patients within the same basic unit. We calculate this using the TSP estimator, as defined previously.
2. Inter Basic Units Travel Load: This represents the travel load generated when moving between basic units within the same district. In order to compute this, we identify the two basic units *i* and *k* with the minimum distance d_{ik} , where *i* and *k* are different basic units, both belonging to district *j*. We then sum these minimum distances.

By including the Inter Basic Units Travel Load in the overall calculation, we ensure that the estimation considers the network distance between any two different basic units within the district. This adjustment allows for a more precise estimation of the travel load by considering the actual distances between the basic units that require visits while taking into account the accessibility factors.

Finally, we multiply the total travel load by the speed factor to obtain the time in minutes rather than just the distance.

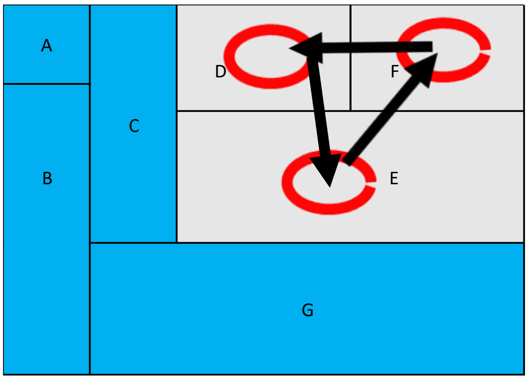

Figure 2 : Schematic Representation of Scenario Two

It is essential to acknowledge that Scenario 2, which considers network distances, has some limitations. As depicted in Figure 2, the network distance is calculated between the centres of each basic unit (as indicated by the black arrows). Thus, when the next patient to be visited is located near the border of a basic unit, the distance estimation might be off. In such instances, Scenario 1 could be more appropriate. Furthermore, our current method of choosing the shortest distance between basic units does not ensure the most optimal routing solution. In order to simplify the implementation and sidestep the intricacies of the precise routing problem, we have decided to exclude it from our solution approach. This choice enables us to concentrate on other pivotal facets of the study, leading to a more streamlined yet effective solution. Given our goal of establishing efficient districts, a general approximation of the travel load suffices.

In summary, compared to Scenario 1, Scenario 2 integrates network distances and a sequential patient-visiting strategy within each basic unit. This may enhance the accuracy of the travel load estimation but still has some limitations, particularly when patient locations are closer to the borders of different basic units.

4. Implementation

In this section, we will present the methodology that has been followed to collect data and construct the dataset. Then, we will explain the algorithm that has been developed to address the objectives of the problem.

4.1 Data Collection

Our objective is to focus on the HHC districting problem within the Province of Liège. As such, we have collected pertinent data specific to this region. In order to ensure an accurate problem-solving with the most relevant data, a substantial portion of this thesis has been dedicated to the data collection process.

The section detailing the data collection process holds great importance as it lays the groundwork for the subsequent utilisation of this data within our heuristic. Several key aspects have been considered in this process.

Firstly, in HHC districting, workload is a pivotal factor requiring precise estimation. Consequently, it's vital to discern how demographic and socio-economic variables influence the care load. For example, districts with a dense elderly population are likely to necessitate more extensive workload allocation. Therefore, it is necessary to collect pertinent data so that our model can incorporate and account for these factors in a comprehensive way.

In order to deal with it, comprehensive research has been conducted to obtain relevant data on population demographics and socio-economic indicators for each basic unit (commune) in the Province of Liège. Population distribution based on varying age brackets and gender has been determined. We have identified three age categories: under 20, between 20-64, and those aged 65 and above.

Another important aspect we have taken into consideration has been how to determine the percentage of each age category utilising HHC services. While detailed data on HHC service utilisation by age and gender was not accessible, informed estimates have been derived from research findings, interviews, and hospitalization data to provide a fair representation of demand. Interviews with HHC agencies have revealed that individuals aged 65 and above formed a significant portion of their clientele. Hospitalisation data have highlighted that elderly woman (65 and above) and men between 20-64 were hospitalised more frequently. Furthermore, discussions with nursing professionals have indicated a minimal percentage of individuals under 20 receiving HHC. Given these findings, we have estimated the proportion of individuals utilising HHC services by age and gender.

Demand distribution of HHC according to age					
<20 Men	20-64 Men	>65 Men	<20 Women	20-64 Wome	>65 Women
2%	23%	25%	1%	18%	30%

Table 2: Demand Distribution of HHC According to Age

For an accurate estimation of the care workload, we have made several assumptions and calculations. We have assumed that each patient was given 20-minute care and that HHC services catered to 3000 patients daily. The number of potential patients from each basic unit has been determined by multiplying the count for each age/gender category in each basic unit by the distribution of HHC demand according to age and gender. In order to account for the capacity limitation of 3000 patients, the potential number of patients from each basic unit has been added up to obtain the total potential number of patients. The total has been divided by 3000, which yielded a value of 68.85.

Through a thorough analysis of different age and gender segments and after aligning them with available data and expert insights, we have tried to ensure that the care workload truly mirrored the needs and demands of each basic unit.

The patient count for each basic unit has been multiplied by the 20-minute care time to obtain the care load. Moreover, the potential number of patients from each basic unit has been divided by 68.85 to determine the expected number of patients from each basic unit, ensuring the fair incorporation of demographic characteristics based on the total capacity. Our estimation process aimed to achieve equity by accounting for age and gender distribution and their HHC service utilisation patterns. By analysing the proportions of different age and sex groups and aligning them with available data and expert opinions, the goal was to ensure that the care load accurately reflects the needs and demands of each basic unit in a fair manner.

However, it is important to acknowledge that the estimation process relies on assumptions and available data, which may introduce some limitations and potential biases. In order to enhance the accuracy and equity of the estimation, obtaining more specific data on HHC service utilisation, considering additional factors influencing demand and involving a diverse range of healthcare professionals' perspectives would be beneficial.

Secondly, another part of the workload is related to travel load, which considers distances between different patients' locations. Additionally, travelling between basic units within each district may create additional workload. In order to incorporate data related to these issues into the model, network distances have been computed using the "Bonnes Routes" online tool where we have input the coordinates of the city, which provides the exact network distance. Moreover, we have determined the cities to which the different basic units (communes) of the Province of Liège belong, namely Huy, Waremme, Liège, and Verviers, and we have acquired data on the area of each basic unit to further refine the calculations.

Thirdly, while existing literature often assumes compatibility as given data, a data-driven approach needed to be pursued to define compatibility. Factors such as contiguity between basic units and maximum allowable distances between them have been considered to achieve a more reliable compatibility between different basic units. As for contiguous districts, it was essential to obtain data concerning the contiguity between different basic units. As there were not available data based on the map, we have checked for each basic unit its contiguous basic unit meticulously from maps and compiled them into an Excel file.

The link to the dataset can be found in *Appendix.1*.

4.2 Algorithm

We have developed two specific algorithms, each one tailored to one of the previously defined scenarios. Both algorithms consist of four steps, with the initial two steps laying the groundwork for creating initial solutions. The only variation between these algorithms is the computation of the travel load.

The initial step, referred to as the *Pre-Assignment Procedure* (PAP) step, focuses on allocating the most incompatible M basic units to M different districts. This step is crucial for setting the groundwork for the subsequent phases. The second step, named the *Workload Balancing Heuristic* (WBH), helps creating an initial solution, which serves as the starting point for further improvements.

The third phase, denoted as *Solution improvement using descent heuristic*, employs a descent algorithm to enhance the initial solution iteratively. This process involves exploring neighbouring solutions and selecting the best improvements at each step to optimize the objective function.

In the final phase, we have introduced a simulated annealing heuristic, inspired by the annealing process in metallurgy. Simulated annealing allows for more extensive exploration of the solution space by accepting "worse" solutions with a decreasing probability over time. This approach helps avoiding local optima and facilitates the discovery of more globally optimal solutions.

The key distinction between the two scenarios lies in the computation of the total workload, as defined in the solution method phase. The total workload includes the cumulative intra basic unit travel load, inter basic unit travel load, and care load. The first method aligns with the first definition of travel load, while the second method corresponds to the second definition.

Our approach builds upon the foundational work of Ozturk et al. (2022) and draws inspiration from their heuristic approach. However, we have made significant adaptations to suit our specific research objectives, and we have refined the computation of the total workload to accurately capture the intricacies of the home healthcare districting problem.

For implementing our algorithms, we have chosen to use the Julia programming language. Julia offers a powerful combination of performance, expressiveness, and productivity and it is specifically tailored for numerical and scientific computing purposes. Leveraging Julia will allow us to efficiently handle the complex computations involved in the districting problem.

For the execution of the algorithm, computations were carried out on an ASUS laptop. The system is equipped with an Intel® Core™ i5-10210U CPU, operating at a base frequency of 1.60GHz with turbo speeds up to 2.11GHz. The computer runs on the Windows 10 operating system.

4.2.1 PHASE 1: Pre-assignment procedure for incompatible locations

As stated previously, in this step, our goal is to identify the M basic units with the highest incompatibility and allocate them to different districts. Incompatibility refers to the idea that certain basic units should not be grouped together in the same district due to factors such as geographic distance, socioeconomic considerations or other elements that may impact the provision of healthcare services.

In order to define incompatibility, we have considered three main factors: contiguity, network distance and the city of each basic unit. Two basic units are considered incompatible if they are not contiguous, if they have a substantial network distance between them or are not in the same city.

In order to incorporate these factors into the heuristic approach, we have assigned a compatibility score to each pair of basic units based on their contiguity, network distance and the city. This score guides the grouping of basic units into districts, ensuring that compatible units are clustered together while minimising the maximum workload difference between districts.

We have introduced three weights, denoted as *w1, w2,* and *w3,* for the factors distance, contiguity, and city, respectively. These weights allow us to adjust the relative importance of each factor in the overall incompatibility index calculation.

The incompatibility index, S_{ij}, between two basic units *i* and *j*, can be computed using the following function:

$S_{ij} = (w2 * (1 - e_{ij})) + (w3 * (1 - city_{ij})) + (w1 * d_{ij})$

The function yields a lower value when there is contiguity between basic units *i* and *j*, when the two basic units are in the same city and when the network distance between them is small. Lower values of the incompatibility index indicate higher compatibility between the basic units.

We have defined the following weights and parameters:

Weights	
w1=0.1	Distance weight
w2=2	Contiguity weight
w3=1	City weight
Parameters	
e_{ik}	Contiguity index, 1 if the basic units *i* and *j* are contiguous.
$city_{ij}$	City index, 1 if the basic units *i* and *j* are compatible.
d_{ij}	Distance between basic units *i* and *j*

A matrix called *MatrixRelation* has been created to compute S_{ij} for all basic units. Based on *MatrixRelation*, another matrix named *MatrixI* has been constructed. If the incompatibility score is greater than 4, a score of 1 is assigned, indicating incompatibility between basic units *i* and *j*. For example, two basic units are incompatible if they are not in the same city, are not contiguous and have a distance between them greater than 10km. If they are in the same city, are not contiguous, and have a distance greater than 30km, they are also considered incompatible.

Subsequently, a vector named *scoreIncompa* has been introduced, which means that for each basic unit *i*, the sum of all its incompatibility indices ($\sum_{j=1}^{N} S_{ij}$, $\forall i$) is computed.

The total incompatibility score represents the total number of pairs of basic units that are incompatible with each other. The scores have then been sorted in decreasing order. The basic unit with the highest incompatibility score has been assigned to the first district. The other most incompatible basic units have been assigned to different districts if they are incompatible with the previously assigned basic units. Incompatibility with previous assignments is determined by considering whether the network distance between the assigned units and the current unit exceeds 22km. If the selected unit does not have a network distance greater than 22km, the next basic unit with the highest incompatibility score is chosen. The minimum distance of 22km has been selected based on the premise that the weighted average distance between basic units within a city is approximately 22.7 kilometres.

We will consider the 4 cities of the province of Liège for the average inter-city distances to determine incompatibility between basic units. Using the average inter-city distance allows us to incorporate the regional characteristics and spatial distribution of healthcare services within the province of Liège.

Average inter-city distance	Number of basic units	Ratio	Weight
21,51470588	18	0,214286	3,857143
20,93383743	24	0,285714	6,857143
34,58743842	29	0,345238	10,0119
16,96153846	13	0,154762	2,011905
Total	84	1	22,7381

Table 3 : Distance Estimation

The link to the code can be found in *Appendix.1*.

The pseudocode is presented herein:

Algorithm 1: Pseudo-code of the proposed PAP procedure

Sort the basic units based on their incompatibility scores in descending order (Matrix "$sorted\_incompa$").
Set $i = 1$ (Current district index).
Set $t = 1$ (Current index in $sorted\_incompa$).
While $i \leq M$ (number of districts):
Select the most incompatible basic unit index from $sorted\_incompa$: $sub1 = sorted\_incompa[t]$.
IF $i = 1$ (For the first district):
Add $sub1$ to $matrices[1]$ (First district's basic unit list).
$t = t + 1$; $i = i + 1$.
ELSE
Set $noncontig = 0$ (Counter to ensure that the network distance between the assigned units and the selected basic unit exceeds 22km).
Set $x = 1$ (Previous district index).
$a = i - 1$ (Number of previous districts).
END
While $x \leq a$:
IF $d[matrices[x][1], sub1] \geq 22$ (We will check the distance of the basic unit to all the assigned ones):

$noncontig = noncontig + 1$
IF $noncontig == a \; and \; x == a$:
Add $sub1$ to $matrices[i]$(Current district's basic unit list).
$t = t + 1$.
$i = i + 1$.
Break.
ELSE
$x = x + 1$.
END
Else
$t = t + 1$ (We will select the second most incompatible).
Break.
END
END
END

4.2.2 PHASE 2: Workload balancing heuristic / Initial solution

Based on the results obtained in phase 1, we will proceed to allocate the remaining basic units to different districts using a heuristic methodology. The primary objective of this methodology is to minimize the disparity in workload distribution among districts, with a specific focus on reducing the workload differences between the least and most heavily loaded districts.

The procedure begins with a pre-assignment phase, which means that the districts are arranged in ascending order based on their total workloads. This step helps identify the least loaded districts at the beginning of the allocation process. Next, the non-assigned basic units from the pre-assignment phase are sorted in descending order according to their individual workloads.

The workload of the basic units is computed according to the care load and the travel load definitions provided by Kwon et al. (1995) to integrate the travel load from the start.

The workload of each district is computed thanks to the workload definition corresponding to the scenario we have taken into account.

In order to start the allocation process, we will take the first basic unit from the sorted list and attempt to assign it to the least loaded district, but only if the basic unit is contiguous with at least one already assigned basic unit within that district. The algorithm iterates through the remaining basic units in the sorted list, attempting to allocate each one in a similar manner.

The objective of this algorithm is to assign basic units to districts in a way that minimizes the workload difference between them. In other words, it aims to construct an initial solution to the problem.

The link to the code can be found in *Appendix.1 .*

Here is the pseudocode for the algorithm:

Algorithm 2: Pseudo-code of the proposed Workload Balancing Heuristic

```
Sort the basic units in decreasing order of workloads (Matrix named "index_decreasing").
Remove basic units assigned during the PAP phase from index_decreasing.
Compute the initial workload of each district.
Set a = 1 .
While index_decreasing == empty:
    assigned == false.
    i = 1.
    While i ≤ M:
        Sort the districts in ascending order of workload (Matrix "sorted_district").
        Least_index = sorted_district[i].
        found_assignement = false.
        j = 1.
        While j ≤ size(matrices[least_index]):
                Check if there is contiguity between index_decreasing and elements of
                matrices[least_index][j] .
                IF contiguity exists:
                        Assign the element  adid = index_decreasing[a] to the least loaded
                        district (matrices[least_index]).
                        Calculate the new workload of the district.
                        Remove the assigned basic unit from index_decreasing.
                        found_assignement = false.
                        assigned = true.
                ELSE
                        j = j + 1 (We will check the next element of matrices[least_index]).
                END
        END
        IF found_assignement == true:
                a = 1.
                Break.
        ELSE
                i = i + 1.
        END
    END
    IF assigned == false:
        a = a + 1 (We will check the next element of index_decreasing).
    END
END
```

4.2.3 PHASE 3: Solution improvement using descent heuristic

After obtaining an initial solution in phase 2, a subsequent phase 3 is executed to further improve the solution by employing a descent algorithm. Each iteration of the algorithm involves a systematic approach to determine the least loaded district and the most loaded district.

In this process, the algorithm identifies the district with the lowest workload and the district with the highest workload. The goal is to improve the overall balance by redistributing the basic units. Therefore, a basic unit is carefully selected from the most heavily loaded district in a manner that the total workload of the chosen basic unit is closest to the difference between the workloads of the least loaded and most loaded districts.

If there is at least one element in the least loaded district that is contiguous to the selected basic unit, the algorithm considers the assignment of the basic unit to the least loaded district. However, this assignment is only accepted if it leads to an improvement in accordance with the objective function. The objective function serves as a measure of the solution's quality and evaluates the overall distribution of workload and is computed according to the scenario we have chosen.

In the event that there is no contiguity between the selected basic unit and any element in the least loaded district, or if the assignment does not improve the objective function, the algorithm seeks an alternative basic unit from the most loaded district. This iterative selection process aims to find a basic unit with a workload difference that is closer to the disparity between the workloads of the least loaded and most loaded districts, ultimately striving for a more balanced distribution.

The procedure continues with iterations until either an improvement is achieved, or the maximum limit of 1000 iterations is reached, ensuring the algorithm does not get stuck in a loop. The final output of the algorithm is a well-balanced distribution of basic units among the districts.

However, it should be noted that the algorithm does not provide information regarding whether the obtained solution is a local optimum or a global optimum. In order to assess the optimality of the result, additional analyses or evaluations may be necessary beyond the scope of this algorithm. Nonetheless, by using the described iterative process and the objective function, the algorithm efficiently approaches a more equitable distribution of workloads among the districts.

The link to the code can be found in *Appendix.1* .

Here is the pseudocode for the algorithm:

Algorithm 3: Pseudo-code of the proposed Descent Heuristic

Set $iter = 0$.
Set $limit = 1000$.
While $iter < limit$:

1.Sort the districts in descending order of total workloads.
2.Select the districts with the highest and lowest workloads: $most\_index$ and $least\_index$.
3.Create a copy of $most\_index$ named $copy$.
4.Calculate the workload difference ($difference$) between $most\_index$ and $least\_index$.
5.Find the basic unit in $copy$ which has the workload closest to $difference$ and that has not been selected during the last 2 iterations.
6.Check for contiguity between the selected element and any element of the least loaded districts.

IF there is contiguity and the objective function is improved:
Assign $closest$ to $least\_index$.
Update the workloads of the involved districts.
ELSE
$iter = iter + 1$.
Delete $closest$ from $copy$.
IF length ($copy$) > 1:
Select the next element of $copy$ that has the closest load to $difference$ and that has not been selected during the last 2 iterations.
Name it $closest$.
Go to step 6.
ELSEIF $least\_index + 1 < most\_index$:
$least\_index = least\_index + 1$ (We select the next least loaded district)
Go to step 3.
END
END
END

4.2.4 PHASE 4 : Simulated annealing

In order to improve the solution search process and escape local optima, a Simulated Annealing algorithm has been chosen to complete the phase 3. Simulated Annealing is a metaheuristic that allows exploration of a wider search space and has a non-zero probability of accepting worse solutions early in the optimisation process, making it more robust in finding global or near-global optima.

Before starting the Simulated Annealing process, several critical parameters need to be set: the initial temperature (*T*), the cooling factor (*alpha*), the length of a temperature plateau (L), the number of temperature plateaus (*K2*) and the stopping parameter (*Epsilon*). These parameters play a vital role in guiding the algorithm's behaviour throughout the optimisation process.

The algorithm then proceeds with a series of steps, each contributing to the overall efficiency and effectiveness of the Simulated Annealing approach. The first step is to find a new solution. In each iteration, the districts are sorted in descending order as for their total workloads. Following that, the districts with the highest and lowest workloads are selected. This selection process helps evaluate potential improvements and introduces necessary diversity in the search process. A basic unit is randomly chosen from the district with the highest workload, further promoting exploration. Then the algorithm checks whether the selected basic unit has been chosen in the last two iterations to ensure diversity and prevent the algorithm from being trapped in local optima. If not, the algorithm selects another element randomly, broadening its search space and considering more possibilities.

Once a suitable basic unit has been selected, the algorithm checks for contiguity and computes the new objective function value according to the workload definition of the scenario. Next, the algorithm assesses whether there is an improvement in the objective function value. If so, it increments the variable *acceptedmove_within_plateau*. If no improvement is observed, the algorithm compares a randomly and uniformly distributed number *u* against the transition probability $p_k = e^{-\Delta F/T}$, where *ΔF* is the change in the objective function value and *T* is the current temperature. If *u* is less than or equal to p_k, the algorithm still accepts the move. This acceptance of worse solutions early on allows the algorithm to escape local optima.

Throughout the optimisation process, the algorithm keeps track of the number of accepted moves within the current plateau. This information is valuable in decision-making and controlling the annealing process.

After evaluating the current iteration, the algorithm determines whether to continue in the same plateau or move to the next one. This decision is based on the number of iterations since the last temperature decreases. If this number is less than *L*, the algorithm remains in the current plateau and continues iterating.

Once the number of iterations has reached *L* or higher since the last temperature decrease, the algorithm checks for two conditions to terminate the optimisation process. Firstly, if there has been no improvement in the current iteration and the percentage of accepted moves falls below the specified threshold, the algorithm terminates.

Otherwise, if the stopping conditions are not met, the algorithm reduces the temperature by a factor of *Alpha*. This reduction in temperature directly impacts the exploration-exploitation trade-off within the Simulated Annealing process. At higher temperatures, the algorithm accepts worse solutions more frequently, enhancing exploration and preventing premature convergence to local optima. As *T* decreases, the acceptance probability decreases, leading the algorithm to transition from exploration to exploitation. This transition allows the algorithm to converge towards better solutions, improving the likelihood of finding global or near-global optima.

In conclusion, by employing Simulated Annealing and its adaptive threshold mechanism, the algorithm becomes more effective at navigating through the solution space, making it more likely to discover better global or near-global optima and avoid getting stuck in local optima.

The link to the code can be found in *Appendix.1.*

Here is the pseudocode for the algorithm:

Algorithm 4: Pseudo-code of the proposed SA

Set initial temperature T, $p_k = e^{-\Delta F/T}$.
Set the number of plateaus $K2$.
Set the length of a plateau L.
Set the % of accepted moves threshold $Epsilon$.
Set the cooling factor $Alpha$.
Set $terminate = false$.

While $terminate == false$:

1.Sort the districts in descending order of total workloads.

2.Select the districts with the highest and lowest workloads ($most\_index$ and $least\_index$).

3.Create a copy of $most\_index$ named $copy$.

4.Select randomly a basic unit from $copy$ that has not been selected during the last 2 iterations.

5.Check for contiguity between the selected element and any element of $least\_index$.

IF there is contiguity:

Update the workloads of the involved districts.

Calculate the new objective function value.

IF there is improvement in the objective function:

acceptedmove_within_plateau=acceptedmove_within_plateau+1.

Assign the basic unit to least_index.

Improve=true.

ELSEIF $u \leq p_k$

acceptedmove_within_plateau=acceptedmove_within_plateau+1.

Assign the basic unit to least_index.

Improve=false.

ELSE

Improve=false.

END

ELSE

Delete the selected element from $copy$.

IF $lenght(copy) > 1$:

Select another element of $copy$ that has not been selected during the last 2 iterations.

Go to step 5.

ELSEIF *least_index+1<most_index:*

least_index=least_index+1.

Go to step 5.

END

END

IF number of iterations since the last temperature decrease < L :

$terminate = false$.

ELSE

IF *Improve=false* and $the\ \%\ of\ accepted\ moves\ < Epsilon\ for\ the\ last\ K2\ plateaus$:

$terminate = true$.

ELSE

Decrease temperature: $T = Alpha * T$.

END

END

END

5. Results

5.1. Parameter Setting for Simulated Annealing

One of the challenges in using simulated annealing is the need of carefully tuning various parameters to achieve optimal results. These parameters include the initial temperature (*T*), the cooling factor (*Alpha*), the length of a temperature plateau (L), and the number of temperature plateaus (*K2*), all of which must be appropriately set.

The choice of the right values for these parameters depends on the specific problem instances and may vary with the number of districts involved. In order to determine suitable values, we have combined recommendations from relevant literature with insights gained from our experience in working on this problem.

Setting the initial temperature requires a delicate balance. It needs to be high enough to enable effective exploration of the search space and escape local optima, while gradually decreasing to converge towards a global minimum. Initially, we have set *T* to ensure that the percentage of accepted moves at the starting temperature (*T0*) ranged between 50% and 90%. However, in order to further refine the results, as described in D'Amico et al. (2002), we have decided to keep the temperature constant and run the procedure for a large number of iterations. For each constant temperature, we have calculated the average objective value and used this information to fine-tune the initial temperature setting.

In the context of simulated annealing, enabling the algorithm to thoroughly explore the entire neighbourhood during a temperature plateau is advantageous. In order to achieve this, we have defined the neighbourhood size as the product of the number of districts and the number of basic units. Consequently, the length of the temperature plateau (*L*) was set proportionally to the size of this neighbourhood, allowing sufficient exploration to facilitate the discovery of high-quality solutions.

Finally, we have conducted iterative runs of the code for several times, adjusting all the parameters on each occasion, to strike the right between computation time and solution quality. By carefully fine-tuning these parameters, we have managed to obtain the most favourable compromise, resulting in the best solutions within an acceptable timeframe.

The parameters can be found in *Appendix.9.*

5.2. Computational Experiments and Results

This section aims to provide a comprehensive analysis of the different components of the heuristic and to compare them to the lower bound. Before presenting the results, it is essential to note that we have applied the solution methods to real-world scenarios in the basic units of the Province of Liège. We have considered varying district numbers, specifically 4, 6, 8, 10, and 12 districts, to thoroughly assess the heuristic's performance among different district numbers.

Now, we will proceed to present the results obtained from the two scenarios, along with their corresponding lower bounds. A detailed analysis of the results will showcase how the heuristic performs in various scenarios. Additionally, we will compare our results with those obtained using

Ozturk et al. (2022)'s algorithm, which takes into consideration only the care load and assesses the impact of including travel load in the objective function.

As explained in the section "Parameter Setting," we recognise the significance of appropriately configuring the algorithm's parameters to achieve optimal results. In order to ensure robustness and accuracy, we have conducted multiple runs of the algorithm, meticulously fine-tuning the parameters to obtain the best possible outcomes. This iterative process guarantees the reliability of our findings and validates the efficacy of our heuristic in various scenarios.

In conclusion, this section provides a comprehensive account of our heuristic's performance, comparing it to the lower bound and benchmarking it against the formulation proposed by Ozturk et al. (2022).

The solutions can be found in *Appendix.4* for scenario 1 and *Appendix.5* for scenario 2.

5.2.1. Lower Bound Definition

The lower bound definition serves as a fundamental benchmark for evaluating the acceptability of our workload distribution solution and provides an approximate indication of its reasonability. In order to compute the lower bound, we assume an idealistic scenario where the basic units are divisible, relaxing the indivisibility constraints.

First, we will explain how we calculate the lower bound in the first scenario. In order to compute the travel load for each district, we follow these steps:

- Divide the total number of patients by the number of districts, resulting in the average number of patients per district.
- Estimate the area allocated to each district by dividing the total area of the Province by the number of districts.
- Apply the formula proposed by Bearwood to calculate the travel load.

For the care load of each district, we aggregate the loads of all the basic units and divide the sum by the number of districts. The total workload of each district is determined by summing the care load and the travel load.

In an ideal scenario with divisible basic units, we would expect equal workloads in each district, leading to a load difference of 0. However, due to the indivisibility of the basic units, this equality is not achieved. Consequently, we define the lower bound as the difference between the care load of the most heavily loaded basic unit and the computed total workload of the district. If the care load of the most heavily loaded basic unit exceeds the total workload, the lower bound is set to this difference. Otherwise, it is set to zero.

Moving to the second scenario, we calculate the lower bound using a different approach. In order to obtain the travel load for each district in an ideal situation with divisible basic units, we will follow these steps:

- Compute the average distance between contiguous cities for the inter-basic unit travel load.
- Carefully calculate the average number of patients and the average area of each basic unit and apply Bearwood's function to estimate the intra-basic unit travel load.

- Multiply the inter and intra travel load by the number of basic units and divide the result by the number of districts, which enables us to obtain the travel load for each district.

For the care load, we will follow a similar approach as in the first scenario:

- Divide the sum of the care loads of the basic units by the number of districts to compute the care load of each district.
- The sum of the care load and travel load represents the total workload of each district.

The lower bound is defined as the difference between the care load of the most heavily loaded basic unit and the computed total workload of the district. If the care load of the most heavily loaded basic unit exceeds the total workload, the lower bound is set to this difference. Otherwise, it is set to zero.

5.2.2. Results

Comparison of the two scenarios:

In the following discussion, we will present a comparative study between the two workload definitions, each one derived from the results attained at various stages of our algorithmic implementation.

M	Lower Bound	Initial Solution	Running Time	A.Descent	Running Time	SA	Running Time	SA-LB
4	0	55968	2s	206	1,9s	82	14s	82
6	0	51917	1,6s	543	1,68s	240	98s	240
8	2149	43413	1,5s	2771	1,84s	2678	40s	529
10	3728	42918	1,5s	4755	10s	4461	110s	733
12	4781	30932	1s	5811	1,85s	5489	63s	708

Table 4 : Comparison of Objective Function Values and Computation Time for Scenario 1

M	Lower Bound	Initial Solution	Running Time	Descent	Running Time	SA	Running Time	SA-LB
4	0	55942	4s	692	2s	20	40s	20
6	0	52125	1,8s	579	2s	240	120s	240
8	2013	43529	1,7s	2893	1,9s	2601	30s	588
10	3619	43029	1,25s	4796	1,9s	4349	79s	730
12	4690	26917	1,8s	5489	1,9s	5403	100s	713

Table 5 : Comparison of Objective Function Values and Computation Time for Scenario 2

M	Initial-SA	Initial-Descent	Descent-SA
4	-99,85%	-99,63%	-60,19%
6	-99,54%	-98,95%	-55,80%
8	-93,83%	-93,62%	-3,36%
10	-89,61%	-88,92%	-6,18%
12	-82,25%	-81,21%	-5,54%
Mean	-93,02%	-92,47%	-26,22%

Table 6 : Percentage Improvement Scenario 1

M	Initial-SA	Initial-Descent	Descent-SA
4	-99,96%	-98,76%	-97,11%
6	-99,54%	-98,89%	-58,55%
8	-94,02%	-93,35%	-10,09%
10	-89,89%	-88,85%	-9,32%
12	-79,93%	-79,61%	-1,57%
Mean	-92,67%	-91,89%	-35,33%

Table 7 : Percentage Improvement Scenario 2

As Tables 4 and 5 illustrate, each scenario presents a remarkably high initial solution across all instances. In other words, the objective function here quantifies the difference between districts with the highest and lowest loads. After employing the descent algorithm, a remarkable average improvement of 92% in solution quality was observed in both scenarios (Tables 6 and 7). Moreover, both scenarios show short computation times, remaining extremely low even in instances with a larger number of districts. Furthermore, in both cases, the difference between the lower bound and the best solution found is quite similar. The optimal solution appears slightly lower and better in the second case. However, it should be noted that the improvement diminishes as the number of districts increases, a point that will be explored further later.

Upon applying the Simulated Annealing (SA) algorithm to outcomes from the descent algorithm, there is a consistent decrease in the objective value, albeit less pronounced than during the descent phase. This suggests that the descent algorithm successfully achieves acceptable solutions. As for the descent algorithm, using the SA algorithm reveals a decreasing rate of improvement when the number of districts increases, resulting in a larger difference between the lower bound and the final solution in instances with a greater number of districts.

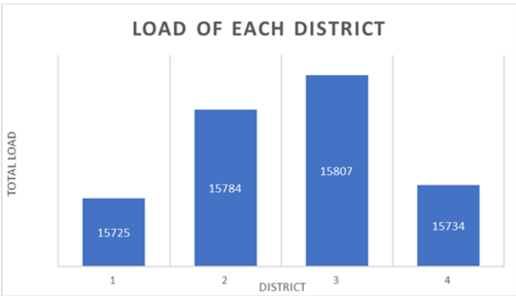

Figure 3 : District Load Distribution in Scenario One with M=2

Generally, it can be concluded that the results have been satisfactory. For instance, in Scenario 1 with four districts (Figure 3), it seems challenging to achieve better results since the least loaded basic unit has a load of 148. Transferring this basic unit to any other district would still yield a difference.

The lesser improvement observed in instances with a high number of districts compared to those with fewer districts stems from the fact that finding a contiguous basic unit is simpler when each district has a larger number of basic units. Therefore, as the number of districts decreases, the restriction related to contiguity lessens, leading to more significant improvements.

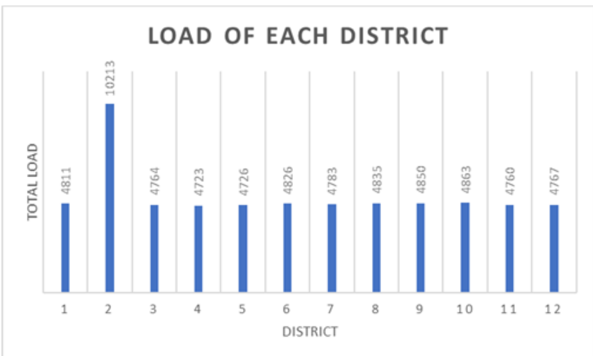

Figure 4 : Load Distribution in Scenario 1 M=12

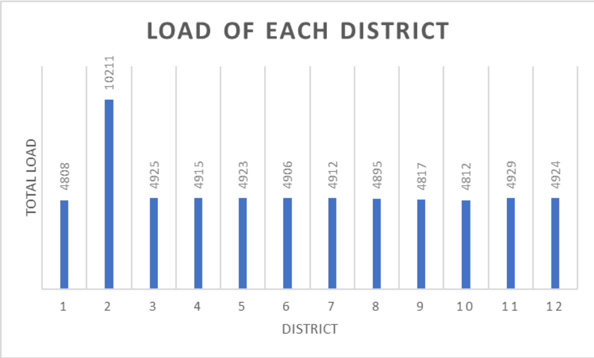

Figure 5 : Load Distribution in Scenario 2 M=12

Regarding workload disparities within districts, as opposed to just between the districts bearing the highest and lowest loads (as defined by the objective function), the workload appears to be well-distributed.

M	MEAN	STD	MEAN WITHOUT	STD WITHOUT
4	15762,5	39,41657858	15747,66667	31,78574104
6	10511,83333	25,325218	10506,4	24,08941676
8	7875,875	944,3604695	7542	6,531972647
10	6300,9	1377,373382	5866,222222	93,13132902
12	5286,727273	1565,69238	4791,636364	48,37411028

Table 8 : Comparison of Mean and Standard Deviation with and without Liège in Scenario 1

M	MEAN	STD	MEAN WITHOUT	STD WITHOUT
4	15946,75	9,322910847	15944,66667	10,21436896
6	10656,33333	86,9566942	10628,4	59,99833331
8	7977,875	902,90981	7658,857143	35,28185828
10	6376,3	1348,021435	5950,222222	44,23171311
12	5331,416667	1537,393748	4887,818182	49,46274999

Table 9 : Comparison of Mean and Standard Deviation with and without Liège in Scenario 2

Instances with M=4 and M=6 showcase a relatively low standard deviation, as demonstrated in Table 8 and 9. However, instances featuring a larger number of districts display a higher standard deviation, primarily attributable to the basic unit Liège, which bears the highest total workload of 10 213 minutes. When the mean total workload of the districts in a given instance is higher than this value, the workload seems to be more evenly distributed among districts. However, when the mean total workload is lower than the load of the most loaded basic unit, the standard deviation is higher because we have a district that will have a load that cannot be divided. Even if the basic unit of Liège is alone in a given district, its workload will be higher than the others due to the indivisibility of basic units, resulting in a higher standard deviation. By computing the mean and standard deviation without considering the city of Liège, more reasonable values are obtained. Therefore, we can conclude that the workload is fairly distributed if we do not consider the special case of the city of Liège, as illustrated in the Figure 4 and 5.

By comparing Scenario 1 and Scenario 2, Scenario 2 appears to have a slightly higher average workload. However, this difference is not substantial and is mainly due to different considerations of the travel load. In Scenario 2, it's assumed that the HHC provider will travel from the centre of one basic unit to another, visiting all the patients within each basic unit before moving on to the next one.

M	Total Load SA Scenario 1	Total Load SA Scenario 2	Travel Load SA Scenario 1	Travel Load SA Scenario 2
4	63050	63787	3006	3743
6	63071	63938	3027	3894
8	63007	63823	2963	3779
10	63009	63763	2965	3719
12	62921	63977	2877	3933

Table 10 : Comparison of Travel Load and Care Load across Configurations

Table 10 displays the total workload and total travel load for each configuration, indicating that the travel load is higher in the second scenario due to the consideration of the network and the constraint of travelling between the centres of basic units. Therefore, Scenario 2 might be more advantageous as it accounts for an additional amount of travel load that is directly linked to the network distance.

<u>Comparison of the new method with the heuristic provided by Ozturk et al. (2022) :</u>

In this section, we will provide a comprehensive and rigorous comparison between our solution approach and the algorithm proposed by Ozturk et al. (2022). The purpose of this comparison is to examine the effects of integrating travel load into the objective function meticulously.

We have conducted this comparison using our dataset from the Province of Liège because our study specifically targets the HHC districting in the Province of Liège. As the instances in Ozturk et al. (2022) were generated randomly and their definition of incompatibility differs from ours, it has been challenging for us to use the dataset from their previous work.

In order to make a comparison, we have replicated Ozturk et al. (2022)'s heuristic approach. Our rendition of the previous method remains faithful to the original, with one critical amendment: the inclusion of our strategically formulated contiguity constraint that replaces the compatibility constraints. This careful adjustment enables us to conduct a targeted study of the specific effects of travel load on the objective function. As Ozturk's method has no simulated annealing phase, our comparison will focus mainly on the first three phases of the heuristic.

Once we have got the results for the districting problem using this solution method, we get a solution where the goal is to balance the care load only. Based on the districts formed thanks to this method, we will calculate the travel load of each district using the two definitions of travel load in order to obtain the total workload.

M	Care Load Ozturk	First Scenario Ozturk	Second Scenario Ozturk	First Scenario New Method	Second Scenario New Method
4	172	501	756	206	692
6	606	703	669	543	579
8	3217	3024	2871	2771	2893
10	4850	4755	4701	4755	4796
12	5640	5607	5519	5607	5489

Table 11 : Objective Function Comparison between Ozturk's Method, Ozturk's Method with Travel Load, and Our Method

Table 11 shows that our proposed descent algorithm, which incorporates travel load, exhibits remarkable improvements compared to the previous heuristic. It consistently yields lower objective function values, highlighting its enhanced efficiency over the former method in the first scenario. However, it is essential to acknowledge that for instances where M equals 8 and 10, the objective function results were less satisfactory in the second scenario.

District	Care Load Ozturk	First Scenario Ozturk	Second Scenario Ozturk	First Scenario New Method	Second Scenario New Method
1	7118	7566	7641	7453	7846
2	10045	10213	10213	10213	10213
3	7282	7749	7840	7556	7823
4	7322	7703	7847	7441	7958
5	6845	7217	7342	7520	7318
6	7273	7668	7795	7646	7612
7	6828	7188	7343	7545	7326
8	7331	7717	7928	7647	7751
SUM	60044	63021	63949	63021	63847

Table 12 : Load Comparison for M=8 Between Ozturk, Ozturk with the Travel Load, and Our Method

However, reviewing Table 12, which compares the load of different districts where M equals 8, it is evident that the total load of the configuration is lower in our new heuristic's second scenario even if the objective function is higher.

M	First Scenario % of Travel Load Ozturk	First Scenario % of Travel Load New Method	Second Scenario % of Travel Load Ozturk	Second Scenario % of Travel Load New Method
4	4,8190%	4,7797%	6,1886%	6,0343%
6	4,8537%	4,8295%	6,0873%	6,0565%
8	4,7238%	4,7238%	6,1064%	5,9564%
10	4,7148%	4,7027%	6,4589%	6,0682%
12	4,6951%	4,6845%	6,7002%	6,3174%

Table 13 : Comparison of the Proportion of Travel Load Regarding the Total Load

For our comparative study, we have compiled an exhaustive table (Table 13) that portrays the ratio of travel load to the total load. Notably, our new descent algorithm consistently has outperformed Ozturk's method in this area in all scenarios, leading to a decrease in the travel load proportion.

In conclusion, our newly introduced descent algorithm, which incorporates travel load from its onset, has continually outperformed Ozturk et al. (2022)'s method in reducing the proportion of travel load relative to the total load. This distinction clearly indicates the superiority of the new method in optimising travel distances for more effective solutions. A comparison of the results from Ozturk et al. (2022)'s method (where we incorporated travel load) with our simulated annealing solution (Table 4 and Table 5) reveals a more optimised and superior improvement, as it was expected.

6. Conclusion and Limitations

This study introduces a heuristic approach to tackle the districting problem in home healthcare. The objective function herein integrates the travel load into its calculations. We delve into two interpretations of travel load and address them using a four-stage algorithm. This algorithm merges a descent method with a simulated annealing metaheuristic.

Our method builds upon the work of Ozturk et al. (2022) and gives good results thanks to a more thorough evaluation of workload distribution. This leads to more equitably balanced districting solutions, especially in scenarios with fewer districts. In these instances, our algorithm is extremely performant, yielding an optimal balance between computational time and solution quality. The low standard deviation of our solutions highlights robustness and even workload distribution.

The use of the descent algorithm and the simulated annealing (SA) algorithm has clearly improved initial solutions. The descent algorithm demonstrates exceptional efficacy, with an average improvement of 92%. Computation times for both scenarios remain low, with no notable increase relative to the number of districts. The optimal solution is marginally lower in the second scenario. The SA algorithm further diminishes the objective value, though not as significantly as during the descent phase. This suggests that satisfactory solutions had been already achieved using the descent algorithm.

The number of districts has a discernible effect on the quality of the solutions, with larger improvements when fewer districts are included. Cases with fewer districts display a relatively balanced workload, while those with a high number of districts exhibit higher standard deviations. This variation is primarily due to the workload of the basic unit "Liège".

In comparison, the second scenario presents a marginally higher average workload, primarily attributable to the inclusion of the travel load, which is based on the network distance between the centres of the basic units. This approach could generate more realistic and efficient solutions.

In conclusion, the analysis underscores the effectiveness of the descent algorithm and SA algorithm in improving the initial solutions for workload distribution in home healthcare. The findings furnish valuable insights for resource allocation optimisation and enhancing the overall system efficiency.

This study transcends mere theoretical advancements. The developed algorithm has practical implications for home healthcare providers seeking to boost service quality, efficiency, and patient satisfaction. By refining previous algorithms, we have presented improvements in algorithmic approaches for workload distribution optimisation. These improvements underscore the effectiveness of these algorithms in producing superior solutions, they offer valuable insights for researchers and practitioners facing similar home healthcare optimisation challenges.

Despite these encouraging findings, the study acknowledges several limitations and proposes directions for future research. In the second scenario, the algorithm does not necessarily calculate the shortest path to enhance access between basic units. Future research might consider computing the shortest path for the whole district each time a basic unit is added or explore the "close enough" travelling salesman problem for nearly optimal solutions without significant computation time increases.

Another limitation lies in the formation of basic units. Overloaded basic units can lead to disparities between districts and system inefficiencies. Future studies should focus on forming basic units based on similar care loads to promote fairness and effective load distribution.

Moreover, we have underscored the need for more research on the characteristics of Home Health Care (HHC) patient profiles in Belgium. Future research could examine variations in demand, the complexity of care among patients and comprehensive surveys on the use of HHC services among different age groups for more realistic solutions.

In summary, our heuristic, based on simulated annealing, stands out as a promising solution to the districting problem in home healthcare. It generates high-quality and optimized results, considering both care load and travel load. However, further research and refinement are needed to address the limitations and broaden the applicability of the proposed method to different home healthcare contexts. Despite its limitations, our heuristic offers valuable insights and sets the stage for future advancements in the realm of home healthcare districting.

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Appendices

Appendix.1 Different Links

Link to the General File:

https://drive.google.com/drive/folders/1Mx3Mw32LV-0CMU8grlDLKCA7CS-F72E_?usp=sharing

Link to the code for Scenario One:

https://drive.google.com/file/d/1Z5-33ym-PKcOymgwGifTWRHSzneBc_DT/view?usp=drive_link

Link to the code for Scenario Two:

https://drive.google.com/file/d/1R0y7icrlMW4HyUHLCGwISM1io7oIUyTj/view?usp=drive_link

Link to the code of Ozturk's Algorithm for Scenario One:

https://drive.google.com/file/d/1ElAdit6WPpfjwFyu3yRplB_5XGFJXwx5/view?usp=drive_link

Link to the code of Ozturk's Algorithm for Scenario Two:

https://drive.google.com/file/d/1_feUxqWaSzR09p9dHweDQm2TAiD8JhMG/view?usp=drive_link

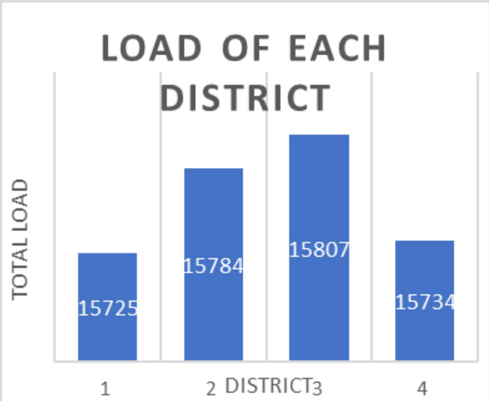
Link to the Data Utilised in the Code:

https://docs.google.com/spreadsheets/d/1hsvNeT9LMSTSd3GrE2GEvE4IXQ_zg59U/edit?usp=drive_link&ouid=103643069747819435508&rtpof=true&sd=true

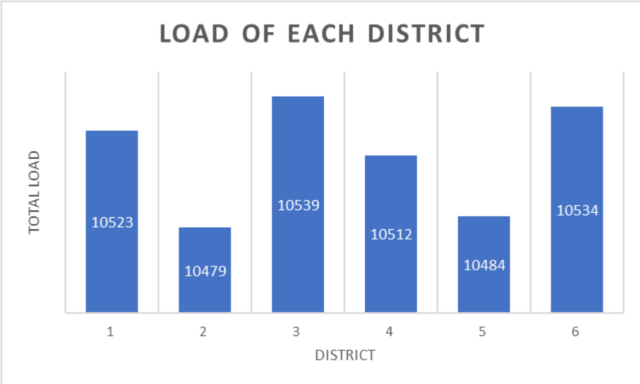
Link to the Data Construction:

https://drive.google.com/drive/folders/1KGBP5YOA1lXqBiUEd0UEgZdh91Qi2_6V?usp=sharing

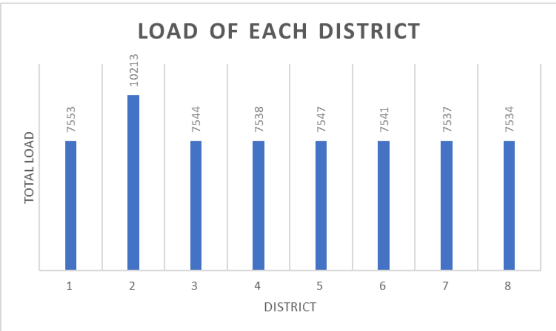
Appendix.2 Load Scenario 1 for Each Instances



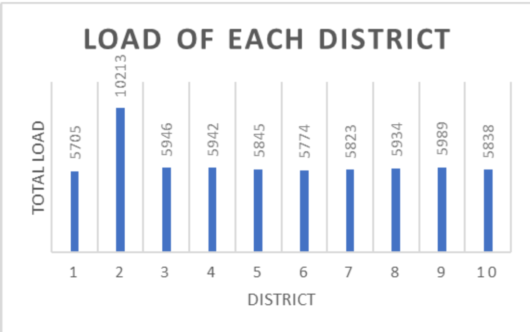
M=4



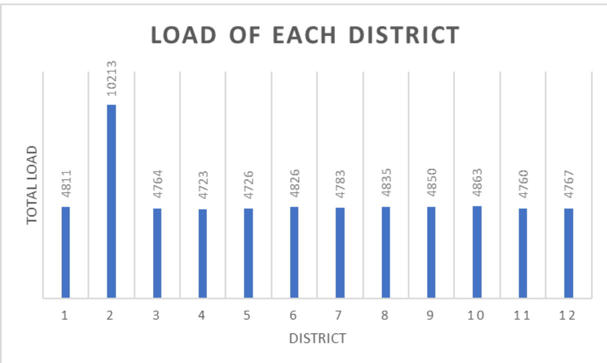
M=6



M=8

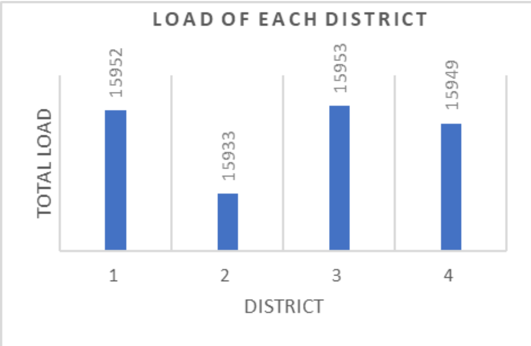


M=10

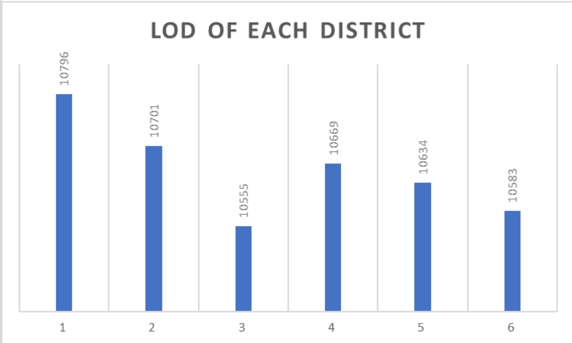


M=12

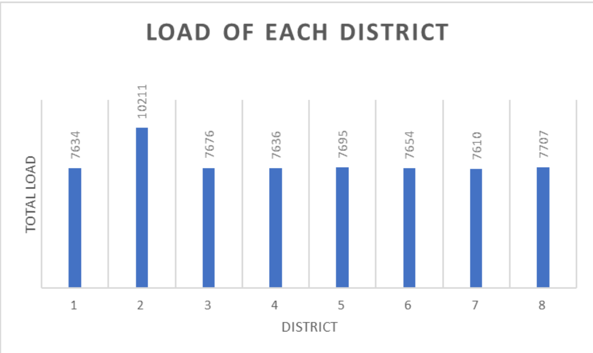
Appendix.3 Load Scenario 2 for Each Instance



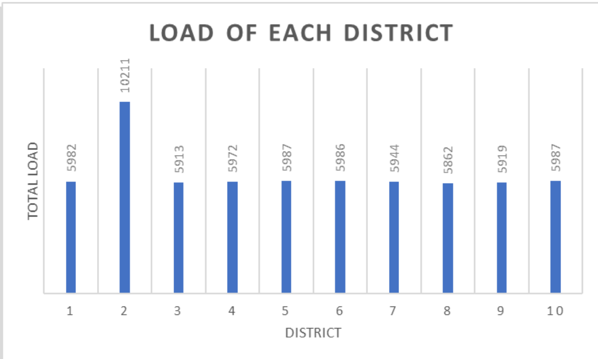
M=4



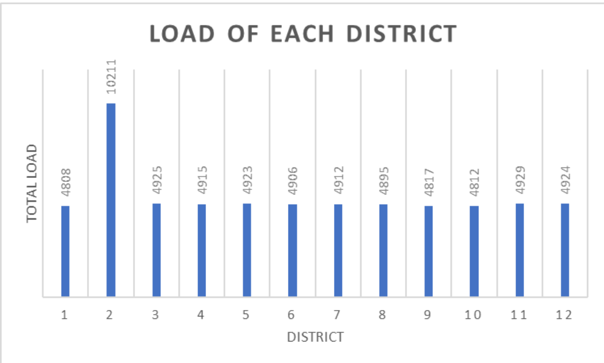
M=6



M=8



M=10



M=12

Appendix.4 Results Scenario 1 for Each Instance

M=4

Best solution found=[[68, 24, 36, 48, 18, 35, 44, 31, 64, 61, 59, 14, 32, 2], [62, 67, 30, 37, 23, 34, 47, 38, 13], [69, 51, 40, 43, 81, 52, 46, 74, 84, 9, 56, 10, 17, 75, 22, 8, 16, 70, 12, 11, 45, 41, 53, 19, 21, 58, 15, 65, 66, 57, 42, 25, 26, 4, 60, 29, 54, 50, 20, 63], [83, 77, 79, 78, 71, 6, 33, 80, 27, 39, 82, 1, 28, 49, 73, 3, 76, 55, 7, 72, 5]]

M=6

Best solution found=[[68, 59, 61, 45, 46, 27, 22, 7, 26, 33, 18, 56, 67], [62, 30], [42, 55, 65, 44, 53, 14, 48, 31, 64, 63, 50, 52, 36], [83, 77, 72, 84, 82, 80, 79, 76, 73, 75, 2, 78, 71, 74, 28, 20, 47, 49, 37, 70], [66, 69, 51, 58, 60, 29, 57, 19, 21, 13, 54, 6, 12, 43, 10, 24, 41, 1, 40, 25, 16, 11, 81, 4, 17], [3, 8, 32, 39, 34, 35, 9, 5, 38, 23, 15]]

M=8

Best solution found=[[45, 46, 68, 52, 59, 60, 40, 33, 31], [30], [7, 34, 47, 42, 55, 65, 64, 44], [83, 77, 82, 80, 79, 73, 78, 71, 74, 39, 23, 1, 27, 56], [4, 20, 57, 24, 5, 8, 63, 28, 36, 15, 61, 67, 9, 41], [32, 62, 17, 84, 13, 76, 53, 50, 37], [14, 16, 6, 12, 2, 35, 81, 11, 3, 72, 10, 25, 43, 70, 69, 58, 66, 26], [29, 21, 18, 19, 75, 22, 38, 49, 48, 54, 51]]

M=10

Best solution found=[[46, 33, 2, 45, 14, 55], [30], [42, 50, 65, 53, 44, 31, 23, 34, 54, 61], [83, 77, 72, 84, 76, 78, 71, 74, 39, 37, 27], [4, 24, 15, 5, 52, 62, 63, 47, 64], [3, 8, 9, 11, 17, 32, 10], [38, 26, 22, 40, 29, 25, 81, 13, 16, 6, 12, 19, 73], [58, 66, 70, 43, 49, 51, 69, 48, 1, 56], [60, 57, 41, 67, 68, 35, 18, 59, 7], [80, 82, 79, 75, 28, 36, 20, 21]]

M=12

Best solution found=[[68, 59, 61, 45, 46, 52, 62, 67, 23, 37], [30], [42, 55, 50, 65, 44, 7, 31], [83, 77, 72, 78, 8, 70, 49, 18], [53, 60, 63, 24, 64], [32, 9, 3], [21, 36, 29, 81, 1, 14, 13, 6, 12], [58, 48, 51, 54, 43, 35, 69, 66], [41, 39, 4, 27, 2, 5, 57, 20, 15], [73, 80, 79, 75, 74, 28, 34, 19, 56], [47, 38, 26, 22, 40, 25, 16, 10, 11, 17], [71, 82, 84, 76, 33]]

Appendix.5 Results Scenario 2 for Each Instance

M=4

Best solution found=[[68, 46, 35, 36, 53, 61, 23, 59, 55, 32, 31, 64, 52, 18], [34, 47, 44, 48, 30, 70, 37, 14, 56], [63, 51, 21, 16, 10, 8, 3, 15, 12, 9, 11, 17, 45, 84, 60, 62, 67, 24, 5, 20, 4, 25, 42, 80, 57, 26, 29, 54, 22, 13, 75, 50, 58, 66, 69, 43, 41, 65, 19, 81], [78, 40, 71, 76, 79, 83, 77, 73, 1, 27, 72, 33, 82, 39, 49, 6, 74, 7, 28, 38, 2]]

M=6

Best solution found=[[68, 18, 1, 67, 9, 63, 34, 46, 61, 33, 48, 59, 44], [52, 62, 30], [53, 23, 47, 55, 20, 27, 64, 65, 39, 50, 42, 45], [83, 77, 84, 82, 80, 79, 76, 73, 75, 2, 78, 71, 74, 28, 49, 38, 22, 35, 37], [4, 66, 69, 51, 58, 60, 40, 29, 57, 19, 21, 41, 25, 81, 13, 16, 54, 10, 6, 70, 24, 15, 12, 43, 56, 5], [3, 8, 11, 17, 31, 36, 72, 14, 32, 26, 7]]

M=8

Best solution found=[[68, 59, 61, 45, 46, 31, 40, 42, 33], [30], [55, 50, 65, 53, 44, 64, 47, 52, 37, 67], [83, 77, 84, 80, 79, 76, 73, 75, 78, 71, 74, 39, 82, 1, 23, 2, 62], [4, 56, 5, 41, 36, 20, 60, 57, 63, 21, 25, 28], [3, 8, 9, 11, 17, 7, 49, 32], [22, 29, 19, 81, 13, 16, 10, 6, 12, 14, 72, 38, 15, 35, 26, 24], [58, 48, 51, 69, 66, 54, 70, 43, 27, 34, 18]]

M=10

Best solution found=[[68, 59, 45, 46, 33, 1, 9], [30], [42, 50, 65, 53, 44, 23, 47, 31, 55], [83, 77, 72, 84, 76, 78, 71, 28, 74, 13, 36], [4, 5, 11, 8, 17, 64, 34, 49], [32, 7, 6, 2], [26, 40, 81, 16, 10, 24, 15, 3, 29, 35, 21, 12, 20], [58, 48, 51, 69, 66, 54, 70, 43, 22, 27], [60, 57, 56, 67, 61, 52, 18, 41, 63, 38, 25], [73, 19, 80, 79, 75, 39, 37, 62, 14, 82]]

M=12

Best solution found=[[68, 61, 65, 33, 46], [30], [23, 44, 45, 42, 22, 55, 39], [83, 78, 71, 52, 29, 28, 67, 77, 12], [4, 20, 24, 15, 5, 2, 64], [9, 11, 17, 32], [31, 21, 37, 19, 38, 75, 74, 73], [58, 51, 69, 66, 54, 70, 43, 50, 48], [60, 63, 41, 56, 18, 57, 27], [80, 82, 79, 62, 59, 76, 49, 25, 34], [35, 53, 16, 81, 47, 40, 36, 26], [1, 7, 14, 10, 6, 8, 3, 13, 72, 84]]

Appendix.6 Table summarizing the Numbers Assigned to each Basic Unit in the Code.

Basic Unit	Nbr	Basic Unit	Nbr	Basic Unit	Nbr	Basic Unit	Nbr	Basic Unit	Nbr
Amay	1	Aywaille	20	Neupré	40	Spa	60	Remicourt	80
Burdinne	2	Bassenge	21	Trooz	41	Stavelot	61	Saint-Georges-sur-Meuse	81
Clavier	3	Beyne-Heusay	22	Amblève	42	Stoumont	62	Waremme	82
Ferrières	4	Chaudfontaine	23	Aubel	43	Theux	63	Wasseiges	83
Hamoir	5	Comblain-au-Pont	24	Baelen	44	Verviers	64	Faimes	84
Héron	6	Dalhem	25	Bullange	45	Waimes	65		
Huy	7	Esneux	26	Butgenbach	46	Welkenraedt	66		
Marchin	8	Fléron	27	Dison	47	Trois-Ponts	67		
Modave	9	Herstal	28	Eupen	48	Burg-Reuland	68		
Nandrin	10	Juprelle	29	Herve	49	Plombières	69		
Ouffet	11	Liège	30	Jalhay	50	Thimister-Clermont	70		
Verlaine	12	Oupeye	31	La Calamine	51	Berloz	71		
Villers-le-Bouillet	13	Saint-Nicolas	32	Lierneux	52	Braives	72		
Wanze	14	Seraing	33	Limbourg	53	Crisnée	73		
Anthisnes	15	Soumagne	34	Lontzen	54	Donceel	74		
Engis	16	Sprimont	35	Malmedy	55	Fexhe-le-Haut-Clocher	75		
Tinlot	17	Visé	36	Olne	56	Geer	76		
Ans	18	Grâce-Hollogne	37	Pepinster	57	Hannut	77		
Awans	19	Blégny	38	Raeren	58	Lincent	78		
		Flémalle	39	Saint-Vith	59	Oreye	79		

Appendix.7 Load Table for the Case M=10

District	Care Load Ozturk	First Scenario Ozturk	Second Scenario Ozturk	First Scenario New Method	Second Scenario New Method
1	5195	5518	5616	5465	6159
2	10045	10213	10213	10213	10211
3	5652	6033	6112	5922	6160
4	5676	5999	6141	5942	5415
5	5695	6046	6247	6029	5855
6	5200	5457	5511	5457	5505
7	5640	5941	6099	5941	6059
8	5707	6021	6198	6038	6208
9	5609	5872	5969	5964	6183
10	5625	5915	6084	6036	6166
SUM	60044	63015	64190	63007	63921

Appendix.8 Interview Guide

<u>CONTEXT OF THE PROBLEM</u>

The topic of my thesis focuses on the districting problem, also known as the territory allocation problem in home healthcare. Let me provide you with a brief overview. The main objective of districting problems is to best serve clients scattered across a territory. This problem involves dividing a geographical area into smaller regions called "districts." Each district represents a service delivery unit and is under the responsibility of a well-defined team. This territorial division must meet various planning requirements and be considered "good" according to those requirements.

The objective of this interview is to understand the requirements, criteria and challenges involved in assigning patients to nurses and how nurse schedules are organised in your organisation. By doing so, I aim to identify improvements and potential solutions to the allocation process. Based on this information, I will need to develop a mathematical model that meets the needs of home healthcare in Belgium.

Through this interview, I hope to gain a better understanding of the factors taken into account when assigning patients to nurses, such as the patients' needs, the transport accessibility and the availability of healthcare providers. Additionally, I hope to gain insights into any problems or potential challenges encountered in the allocation process, as well as the solutions or improvements that have been implemented.

<u>QUESTIONS:</u>

1) Organizational Overview

• How many patients does your organisation currently care for in the context of home healthcare? How many nurses do you have? Are these numbers fixed or can they vary? Are there any capacity constraints?

• How is the home healthcare service currently organized (e.g., by geographic region, medical specialty, etc.)? Is there a specific territorial allocation or districting system in place? If so, how are these districts formed, and what criteria are used? Is there a capacity limit for each district?

2) Assignments of Nurses to Patients

• How are nurses assigned to patients? How are caregiver schedules planned and organized?

• What considerations are currently taken into account when assigning patients to caregivers (e.g., location, nurse's residence, medical needs, caregiver availability)? What criteria do you use to determine how patients are assigned to nurses (patient's needs, transport accessibility, nurse's availability, other - please specify)?

• After providing care, do nurses need to return to the offices to restock on supplies? Or do they proceed directly to the next patient?

• How do you ensure that the patient needs are met while optimising resource utilisation (patient needs take priority, resources are monitored and adjusted as necessary, other)?

• Are nurses versatile or do they have well-defined specialties?

• Are patients’ groups based on their medical conditions?

• How do caregivers travel to reach patients? Do they use cars or other means of transport?

• Is a patient consistently assigned to the same nurse throughout their stay with your organization? Is there any flexibility in this regard?

• Do you use any specific software or tools for patient-nurse assignment?

3) Objectives

• Are there specific objectives that your organization aims to achieve in terms of home healthcare services when assigning patients to nurses? For example, reducing travel time for caregivers, minimising the distance travelled by each caregiver.

• Do you strive for workload equity among different nurses? How is the workload currently distributed among caregivers (visit duration vs. travel duration vs. distance)?

• How are patients' care needs taken into account when distributing the workload among caregivers? Do you aim for equitable visit durations?

• How do you ensure/Is it important that patients assigned to a nurse are geographically close to each other? Is this an important consideration?

• If there are districts, is it important that the patients assigned to a nurse belong to a single district without any overlap between districts?

• What is the most important goal in your opinion? Is it equitable visit durations, minimising travel time, ensuring that the same nurse always takes care of a patient, etc.?

• Are there criteria that are more important than others?

• How do you measure the effectiveness of your activities?

• Are there any regulatory or compliance issues to consider?

• Are there any changes, challenges or improvements you would like to see implemented in the districting process?

• How is communication and coordination managed with other stakeholders such as hospitals, primary care physicians or other healthcare providers?

• How is the districting configuration reviewed and updated? How often do you perform planning? Is it on a weekly or a daily basis?

Appendix.9 Parameters for Simulated Annealing (SA)

First Scenario					
M	T	Alpha	L	K2	Epsilon
4	(ObjectiveDescent-lb)/M	0,96	N*M*2	10	5
6	(ObjectiveDescent-lb)/M	0,9	N*M	10	5
8	((ObjectiveDescent-lb)*1,45)/M	0,96	N*M*2	10	5
10	(ObjectiveDescent-lb)/M)/10	0,9	N*M	10	7
12	(ObjectiveDescent-lb)/M	0,9	N*M	10	5
Second Scenario					
M	T	Alpha	L	K2	Epsilon
4	(ObjectiveDescent-lb)/M	0,96	N*M*2	10	5
6	(ObjectiveDescent-lb)/(M*2)	0,96	N*M*2	10	5
8	((ObjectiveDescent-lb)*1,45)/M	0,96	N*M	12	5
10	(ObjectiveDescent-lb)/M)/10	0,8	N*M	15	5
12	(ObjectiveDescent-lb)/M	0,9	N*M*2	12	5

Executive Summary

In recent years, home healthcare (HHC) has gained increasing importance, particularly in developed countries, due to an aging population and changing family structures. The demand for efficient HHC districting has become crucial to control costs and optimize resources in healthcare agencies. This master thesis addresses the needs and improvements in HHC districting, focusing on the specific case of the Province of Liège.

The document commences by providing an insightful overview of the challenges associated with home healthcare and districting, emphasizing the growing significance of HHC in the healthcare landscape. It then proceeds to conduct an in-depth review of existing literature on the subject, gaining valuable insights from previous studies.

The thesis proposes an innovative extension of the solution method developed by Ozturk et al. (2022) to tackle the unique challenges faced in HHC districting within the Province of Liège. The extension incorporates travel load into the objective function through two distinct scenarios and integrates contiguity constraints that take into consideration the geographical layout and specific requirements of the region. In order to enhance the solution method's effectiveness, a simulated annealing approach is integrated into the heuristic, providing a powerful technical optimisation.

The proposed heuristic is subject to rigorous computational experiments to evaluate its performance with a focus on workload balance, computation time, and solution quality. The results demonstrate its promising efficacy, showcasing an efficient workload distribution within a remarkably short computation time. By achieving a more balanced distribution of workloads, the proposed method enables healthcare providers to optimise their resources and deliver quality care directly to patients' doorsteps.

While the thesis offers substantive contributions, it also identifies prospective avenues for future research. Specifically, further exploration is needed to calculate the shortest path between basic units, refining the formation of basic units based on similar care loads and investigating the characteristics of HHC patient profiles in Belgium.

In conclusion, this master thesis provides a comprehensive and pragmatic study on home healthcare districting. It not only identifies the challenges faced in HHC but also offers novel insights and a tailored approach specifically designed for the Province of Liège. By revolutionising the way healthcare providers manage their resources, this research contributes significantly to improve home healthcare services and ensure better patient outcomes.

Key words: home health care, tactical level, nurses, patients, districting, home healthcare districting, simulated annealing, heuristic, workload, optimisation

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