A revisit to low volatility anomaly

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ABSTRACT

The relation between risk and return has been debated for a long period. Apart from the debates on the relation being negative or positive, academics have reached the consensus that the security market line is much flatter than as implied by CAPM. Recently an increasing amount of empirical studies have provided evidences to support the existence of low volatility anomaly, arguing that bearing additional risk would give rise to lower expected returns. Some even move one step further, documenting that volatility earns significant risk premium therefore volatility should be incorporated into the asset pricing models to explain the cross section of stock returns. Different opinions strike back by concluding that after controlling for well-known anomalous factors, volatility factor becomes insignificant. In this thesis, total volatility and idiosyncratic volatility are examined separately. Evidences show that on the basis of risk-adjusted returns, low volatility anomaly strongly persists. Different weighting approaches on portfolio formation should also be taken into consideration since value-weighted approach tends to generate better results supporting low volatility anomaly. Volatility factors are also constructed based on the 2x3 portfolios after the double sorting. Results from Fama-MacBeth two-step regression presents that over the period from February 1900 to December 2015 (311 months), neither total volatility factor nor idiosyncratic volatility factor earns significant risk premium. Regression results from Carhart four-factor model prove that large, value and momentum lie behind low volatility anomaly. Robust profitability and conservative investment are also captured by Fama French five-factor model. But persistent variables independent of either Carhart or FF framework maintain explanatory power on low volatility anomaly.

Keyword: Low Volatility Anomaly, Asset Pricing, Volatility, Idiosyncratic Volatility.

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1. INTRODUCTION

1.1 Background

One of the most fundamental principles of Capital asset pricing model (hereafter CAPM) by Sharpe (1964) and Lintner (1965) is that the only approach to earn higher returns is through bearing higher risk. It is reasonable to believe that bearing additional risk should be compensated with higher return under the assumption of rational market. Thus the relation between risk and return should logically be positive. However, a growing number of empirical tests on CAPM have suggested that the relation between risk and return is at least much flatter than predicted by CAPM. Black and Scholes (1972) finds that low beta stocks have average higher returns than it should have while high beta stocks generate lower than expected returns. Frazzini and Pedersen (2014) also documents the relative flatness of security market line.

Some empirical studies even arrive to the opposite conclusion that the relation between risk and return is actually negative. Van Vliet et al (2011) claims that the relation between past volatility and expected return is negative. The low volatility portfolio outperforms the high volatility counterpart by 3.7% on average. Haugen and Heins (1975) argues that in the long run, low volatility portfolios actually outperform high volatility portfolios. Baker, Bradley and Wrugler (2011) also concludes that results on portfolios sorted based on volatility show that high volatility portfolios have consistently underperformed low volatility portfolios. Blitz and Van Vliet (2007) provides evidence that low volatility stocks outperform high volatility stocks in terms of risk-adjusted return. Ang, Hodrick et al. (2006) examines the relation between idiosyncratic volatility and cross section of expected returns, arguing that after controllong other effects, idiosyncratic volatility earns a significant risk premium of -1% per annum. Ang, Hodrick et al. (2009) again concludes that high past idiosyncratic volatility will be accompanied by low future return. Baker and Haugen (2012) adpots a

24-month rolling window analysis on past volatility, and they conclude that bearing additional risk in the equity markets around the world will result in lower average return. Ang (2014) defines the negative covariation of expected asset returns and voaltility which cannot be explained by CAPM as low volatility anomaly.

Meanwhile, some academics also provide strong evidence against the low volatility anomaly. Martellini (2008) provides further evidence that the relation between expected returns and total volatility would be positive if non-surviving stocks were excluded from the sample. Fu (2009) argues that using past volatility as a risk measurement is inherently wrong as idiosyncratic volatility is time-varying. He adopts an exponential GARCH model to estimate the future idiosyncratic volatility, and further conclude that the relation between estimated idiosyncratic volatility and expected returns is positive. Bali and Cakici (2008) demonstrates that several methodolgical choices, including portfolio weighting approaches, data frequency to measure volatility and the breakpoints to sort deciles, can affect the robustness of idiosyncratic volatility effect.

Ang, Hodrick et al. (2006) claims that idiosyncratic volatility can explain the cross section of expected return by earning a significant risk premium, suggesting volatility as a factor be incorporated into asset pricing models. Hou, Xue, and Zhang (2017) identifies and examines more than 400 anomalies, concluding that most of them are not significant factors. Cochrane, J. H. (2011) explicitly expresses his concerns that now we have a zoo of new factors. Beveratos et al (2017) presents that after controlling for common factors, low volatility anomaly performs insignificantly. Attention moves to potential interaction between low volatility anomaly and common anomalous factors.

The debate on low volatility anomaly would continue infinitely. The hottest topic in this debate would eventually turn into the question of whether it is just an asset pricing anomaly or is it a priced risk factor that earns significant risk premium? The most convincing answer to this question might be that it is an anomaly for sure, but may not be a priced factor that explains the cross section of expected return.

1.2 Research questions

This thesis are dedicated to providing answers to the following questions:

i) Does the low volatility anomaly exist? Total volatility or idiosyncratic volatility?

ii) What is the impact of different portfolio weighting approaches?

iii) Is it just an asset pricing anomaly, or also a priced risk factor that earns significant risk premium?

iv) What is the relation between low volatility anomaly and other anomalous factors?

This thesis separately examines the low volatility anomaly both on total volatility and idiosyncratic volatility. It follows the similar methodology from Ang, Hodrick et al. (2006). Stocks are ranked at the end of each month and then sorted into volatility deciles. Both total volatility and idisoyncratic volatility are calculated in a 1-month window, and monthly rebalancing is adopted to form volatility deciles at the end of each month. Inspired by Bali and Cakici (2008), three different portfolio weighting approaches, including equally, value and volatility weighted approaches are implemented. The holding period return of stocks is the subsequent 1-month return ofter each ranking. Return of volatility deciles are calculated with the above-mentioned three weighting approaches. The time series of volatility deciles are then fitted into CAPM, Carhart four-factor (Carhart (1997)) and Fama French five-factor (hereafter FF-5) model (Fama and French (2015) to examine their performance.

Asness et al (2014) introduce the approach of double sorting on size and then quality to construct a "Quality Minus Junk" (QMJ) factor, the factor are formed based on the intersection of the 2x3 portfolios. This thesis mimics the same approach to double sort stocks on size and then volatility, and construct the total volatility (VOL) factor and idiosyncratic volatility (IVOL) factor based on the intersection of the 2x3 portfolios. After obtaining these volatility factors, Fama-MacBeth two-step (Fama and MacBeth (1973)) regression is utilized to calculate the factor risk premium. Volatility factors will also be regressed against other anomalous factors to detect the potential interaction.

1.3 Main findings

In terms of risk-adjusted return, low volatility deciles are generally outperforming high volatility deciles, this may provide evidence to support the existence of low volatility anomaly. In the value-weighted approach, the spread portfolio (long the lowest decile and short the highest decile) has significant Carhart alpha, which also supports the low volatility anomaly. Apart from these evidence, no other positive evidence supporting the significant existence of low volatility anomaly can be extracted from the analysis on the volatility deciles.

Analysis on different weighting approaches supports the argument that it is way easier to find positive signals in favor of the potential existence of low volatility anomaly in value weighted approach than equally or volatility weighted approach. This finding contributes new insight to current studies in this field since the majority of research publications on low volatility anomaly simply take value-weighted approach for granted.

The risk premiums of total volatility (VOL) and idiosyncratic volatility (IVOL) factors calculated by the Fama-MacBeth two-step regression are all statistically insignificant, which provides evidence that volatility is not a priced factor that earns significant risk premium.

Analysis of volatility deciles sorted on total volatility or idiosyncratic volatility show that large, value and momentum may lie behind the low volatility anomaly. Results from the FF-5 regression add robust profitability and potentially conservative investment. The regression between volatility factors (VOL and IVOL) and Carhart four factors confirms the findings that apart from large, value and momentum, other factors are consistently needed to explain volatility factors. Regression results under the FF-5 framework suggest that value and robust profitability are the biggest reasons behind low volatility anomaly.

1.4 limitations of this study

The biggest limitation of this study lies in the data sample, which only contains 1000 US listed companies covering a period from January 1990 to December 2015 (312 months in total). It may be not convincing enough to use a sample of 1000 companies to represent the whole market.

Secondly as implied by Bali and Cakici (2008), data frequency to measure volatility can also be a source of potential limitations of this thesis. In this thesis, the volatility is measured within each month for a length of one month. Other window lengths or even the application of rolling window analysis might lead to different conclusion on low volatility anomaly. Also, the holding period return is settled into one month, different holding periods might present different results.

Finally in this thesis, the potential influence of interest rates on low volatility anomaly has not been examined, adding the interest rates into this study may provide better explanation on low volatility anomaly.

1.5 Thesis structure

Here are a brief outline of the thesis structure: Chapter 1 reports the background, research questions, main findings and limitations of this study. Chapter 2 presents the literature review on the historical researches within the scope of low volatility anomaly. Chapter 3 describes the data and introduces the methodology. Chapter 4 presents all the empirical results and Chapter 5 draws the conclusion.

2. LITERATURE REVIEW

2.1 Relationship of risk and return

From efficient market hypothesis to CAPM, relation of risk and returns are better understood by the market. Bearing excess risk more than the average market risk should be compensated with higher than market average returns. But recent studies argue that higher risk actually earns lower returns, low volatility anomaly arises.

Fama (1970) came up with the efficient market hypothesis (hereafter EMH), which is one of the founding theory of modern finance. He argues that stock prices must fully reflect all available information that affect the performance of stocks in the market. A direct implication of the theory is that under the assumption of rational market, no matter which stocks that investors purchase in any stock markets, investors can only earn risk-adjusted market average return. Also mispricing of stocks is non-existent and all stocks are traded at fair value. Thus it is impossible to beat the market through market timing and stock selection. The only way to higher average return is to buy riskier assets. The weak form of EMH argues that all publicly available information in the past should be reflected in the asset prices. The semi-strong form of EMH adds that asset prices reflect not only all publicly available information of the past but also the future new information instantly. The strong form of EMH claims that either public or private information is fully reflected in the asset price, even in the case of "insider information". Criticism of EMH has mainly focused on its fundamental assumption of rational market, disputing that financial markets are actually fully of imperfections. Information barrier, transaction cost and behavior bias of market participants like overconfidence, overreaction, representative bias t just name a few, are the major underlying factors that greatly undermine the explanatory power of EMH.

Markowitz (1959) tries to explain the relation of expected asset returns and its variance, which brought up the modern portfolio theory. It claims that investors will seek to optimize the mean-variance tradeoff to maximize their expected asset returns.

According to modern portfolio theory, market portfolio is the optimal portfolio on the efficient frontier which lies on the tangent line going through the risk-free rate. Investors proportionately adjust their weights on market portfolio based on their preference of risk, which make the combination of all capital allocation precisely forms a capital market line. The slope of capital market line means the reward to risk, also known as the Sharpe ratio, which is denoted by one unit of return over one unit of risk.

Sharpe (1964) and Lintner (1965) provides further evidence for the mean-variance tradeoff in their publication on the CAPM, and came up with the idea of beta coefficient to measure the sensitivity to market factor. The theory also claims that the market risk-free rate has a beta of zero and a beta of one for the market portfolio, the discovery of linear relation between expected returns and beta paves way for the further development in asset pricing theories.

The CAPM is defined as follows：

$$\mathrm{E}[R_i] - R_f = \beta_i \times \left(E[R_m] - R_f\right) \tag{1}$$

Where $\mathrm{E}[R_i]$ denotes the expected asset returns， $E[R_m]$ is the market average return, R_f is the risk-free rate, $E[R_m] - R_f$ is the market risk premium, and β_i is the sensitivity to market risk, also called beta coefficient.

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2}$$

Where β_i is calculated by the covariance of asset returns and market returns divided by the variance of market returns.

Under the CAPM framework, the beta coefficient of a stock explicitly measures the co-movement of the stock with the market, thus a higher beta means a higher degree of risk relative to the market. The market risk, or systematic risk is the risk that cannot be eliminated through portfolio diversification. The linear relation of expected asset returns and beta coefficients means that expected asset returns comes directly from bearing market risk, which coincides with the direct implication of EMH that no one can consistently outperform the market by earning excess return, the only way to earn higher average return is to purchase riskier assets. Therefore, it is reasonable to observe that the most commonly accepted rule in the market is that higher risk should be compensated with higher returns. Volatility of stock returns is usually adopted as the measurement of risk. The more volatile a stock is, the riskier it will be. Also volatility is positively related to beta, and it echoes the fundamental concept that higher risk should be compensated with higher expected returns.

In terms of the source of risk, riskiness of a stock can be divided into two components: one is systematic risk, also known as the undiversifiable risk. Systematic risk also comes as the consequence of the impact from politics, economy as well as culture. It is the uncertainty that influences the movement of entire market, and cannot be demolished through portfolio diversification. Another is idiosyncratic risk, which is also called unsystematic risk. Idiosyncratic risk doesn't interfere with the market and basically results from the internal issues of a firm like earnings, financing and investing decisions. Since it doesn't have a direct connection with other stocks, thus investors cannot diversify away idiosyncratic risk. Regardless of the type of risk, the theoretical consensus on risk compensation within traditional CAPM framework is that volatility ought to be positively correlated with expected asset returns.

2.2 Low volatility anomaly

A number of empirical studies seemingly arrive to the opposite conclusion that expected asset returns are actually negatively correlated with volatility. There is increasing evidence showing that the slope of security market line is not as high as predicted by CAMP. Most of the criticism was put forward by the incapability of CAPM to explaining the pricing of idiosyncratic risk, since according to the CAPM formula, systematic risk is the only factor that is priced rather than idiosyncratic risk in the CAPM. While mounting publications have documented that idiosyncratic risk should be taken into consideration when pricing a stock. The relation between risk and return re-emerge as the center of debates among academics.

Shortly after the introduction of CAPM, Jensen, Black and Scholes (1972) proposes that in their empirical tests on CAPM with a sample of U.S stocks from 1931 to 1965, the security market line has a positive slope but much flatter than predicted by CAPM. They find that low beta stocks generate average higher returns than it should have, whereas high beta stocks receive average lower returns. Frazzini and Pedersen (2014) also finds relative flatness of security market line in the U.S. equities for the period from 1926 to 2012. Similar flatness is also documented in international equity markets, bond markets for corporate and treasury bonds, and future markets.

Van Vliet et al (2011) put forward that in their empirical study on 1963-2009 U.S. sample, the relation of past volatility and expected return is negative. The high volatility portfolio underperforms the low volatility counterpart by -3.7% on average. Even after excluding small caps the return spread still persists at a level of -1.7%. The spread become even larger by -3% in the case of compounding effects. Meanwhile, they claim that various results of positive relation between return and volatility can be attributed to look-ahead bias and different choices on methodologies.

Fu (2009) claims that using a historical volatility as a risk measurement is inherently wrong since idiosyncratic volatility is time-varying in nature. He subsequently adopts expected volatility rather than historical volatility as the basis to sort portfolios. An exponential GARCH model is used to estimate expected idiosyncratic volatility. He points out that the relation between estimated expected idiosyncratic volatility and expected returns is significantly positive. At the first glimpse the adoption of different approach seems to provide another useful method on examining the relation, but later several empirical tests on this method has proven that the positive relation no longer persists after the elimination of look-ahead bias.

Martellini (2008) provides further evidence that the relation between expected returns and total volatility would be positive if non-surviving stocks were excluded from the sample. An intuitive interpretation of this positive relation is that generally companies with high volatility tend to go bankrupt easier than low volatility companies, thus they will generate high return if they survive, while low return when they go bankrupt. In the case of considering only survivors, the expected return of high volatility stocks will naturally be inflated as the high volatility stocks with low returns are excluded at the very beginning.

Ang (2014) defines the negative covariation of expected asset returns and voaltility which cannot be explained by CAPM as low volatility anomaly. Haugen and Heins (1975) challenges the tradiational belief of risk compensation and claims that in the long run, low volatility portfolios actually outperform high voaltility counterparts. Baker, Bradley and Wrugler (2011) also concludes that in the sample period of 40 years (1968-2008), results on portfolios sorted based on volatility shows that the high volatilty portfolio has consistently underperformed the low volatility portfolio, drawing to the conclusion that the relation between volatility and returns should be negative.

Blitz and Van Vliet (2007) documents an explicit volatility effect: low volatility stocks actually produce higher risk-adjust return than the market portfolio, while high volatility stocks clearly underperform the low volatility stocks. Also, they claim that the volatility effect is not only restricted to U.S. stocks but also existent in regional or global markets. The alpha spread between the high volatility portfolio and low volatility portfolio is negative 12% per annum for large-cap sample during the period of 1986-2006. They also conclude that volatility effect is independent of other well-known effects: size, value and momentum.

Ang, Hodrick et al. (2009) put forward that high past idiosyncratic volatility will be accompanied by low future return, the spread between the highest and lowest quintile portfolios sorted on idiosyncratic volatility is statistically significant at -1.31% per month in 23 developed countries. After the adoption of double sorting approach to control market, size and value, the volatility effect persists respectively in all G7 countries. Singnificant outperformance of low indiosyncratic volatility portfolio indicates that indiosyncratic volatility may serves as a pricing factor behind this effect.

Baker and Haugen (2012) seems to provide compelling evidence for low volatility anomaly. In their study period from 1990 to 2011, they cover stock markets in 21 developed countries and 12 emerging countries, with more than 99.5% of market capitalization of each countries included in the dataset. The approach is transparent and easy to reproduce, which mainly sorting stocks in each country into quintiles based on past 24-month volatility and then calculating the return of subsequent one month. An rolling window of 24 months is implemented on each of the month in the sample period, and the results show that the relation between volatility and return is negative. They conclude that bearing addition risk in equity markets around the world can actually give rise to lower average return.

Ang, Hodrick et al. (2006) shed light on how volatility can affect the cross-session of expected return by including volatility as a systematic risk factor and assessing the risk permium of volatility. Time-varing aggregate market volatility gives rise to investors' differnent expectations on the future expected return and accordingly affects the tradeoff between risk and return. If market volatility becomes a systematic risk factor, as implied by aribitrage pricing theory, volatility should be priced in the cross session of the expected returns, indicating that stocks with different exposure to volatility factor should have different expected returns. One of the major findings is that aggregate market volatility carries a negative factor premium of approximately -1% per annum with statistical significance after controlling other effects like size, value and momentum. In addition, they admit that given their short sample period, volatility clutering in abnormal periods has a huge impact on the final results, suggesting that a potential Peso problem would eventually revert the negative relation between exposure to aggregate volatiltiy and expected average return into positive one.

Empirical results from Ang, Hodrick et al. (2006) also prove that the cross-sectional factor price of idiosyncratic volatility is statistically significant at negative 1.07% per month, where idiosyncratic volatility is examied at a firm level relative to the Fama and French (1993) model. Their results on idiosyncratic volatility demonstrate robustness after controlling size, value, liquidity, volumn and monmentum. The effect also persists with different lengths of period for measuring idiosycratic volatility, bull-bear market, period of economic expansion and recession. Thus the idiosyncratic volatiltiy effect remains an the secret treasure waiting to be uncovered.

Bali and Cakici (2008) examines the cross-seccional relation of idiosyncratic volatility and expected returns. The results show that several methodological choices can sustantially affect the robustness of the idiosyncratic volatility effect, including data frequency to measure idiosyncratic volatility, the breakpoints to sort volatility quintile portfolios, the controlling of other effects (size, value and momentum ect), portfolio weighting approach (equally-weighted, value-weighted or volatility-based weighting). They further argue that there is no statistical robustness on the relation of idiosyncratic volatility and expected returns.

In summary, a large number of empirical studies has proven that the relation between volatility and expected returns is negative. While acdemias are having a heated debate over the negative cross-secsional price of volatility, random individual publications advocating the positive relation also emerge. More recently low volatility anomaly has been widely put forward by various papers, and it has been proven in many international markets. But even until now no standard research methods have been widely agreed upon, which stirs an ever-lasting discussion on the potential effects of methodological choices on this topic.

2.3 Possible explanations of low volatility anomaly

An increasing amount of literatures have provided theoretical explanations on low volatility anomaly. Among them the most widespread ones are explanations from behavior finance, leverage constraints, short-selling constraints and benchmarking. Each of these possible explanation will be presented in this section.

2.3.1 Behavior finance

Behavior finance models provide insights on the reasons why high volatility stocks are generally preferred by individual investors. The excessive demand on high volatility stocks bids up the stock prices and naturally decreases the expected return, which historically results in the relative underperformance of high volatility stocks. Behavior biases explain the reasons why individual investors are preferring high volatility stocks rather than low volatility stocks.

I) Preference for lotteries

Image that if there is a gamble with 50% chance of losing 50 euro and 50% chance of winning 55 euro, most people would opt for escaping from this gamble as they are afraid of losing 50 euro despite the fact that the expected return from playing this gamble is positive. Kahneman and Tversky (1979) defines this phenomenon as “loss aversion”, suggesting that people will avoid taking risk for fear of realizing losses. However, the story changes dramatically in the case of lottery. Image again that a lottery costs 2 Euro that have 0.0000001% probability of winning two million Euro. Basic understanding on probability theory will tell us that the expected return from purchasing a lottery is negative, thus people should avoid lotteries. Contrary to avoiding lotteries, even though the expected return being negative, many people are still willing to bet on the little hope of earning millions overnight.

The same phenomenon of purchasing lotteries also happens in the stock markets. High volatility stocks are mainly associated with small companies with generally low prices. These low-priced, high volatility stocks are inherently presenting the similar payoff patterns as lotteries: there is a fairly small possibility of doubling or even tripling the initial investments, which is also accompanied by a much larger chance of losing money. Barberis and Xiong (2012) argues that investors measures utility from the realized gains or losses, rather than the gains or losses shown in the account. High volatility stocks exactly satisfy this pattern of utility measurement. When the prices of high volatility stocks jump up, investors sell these stocks immediately to realize gains. But when the prices go down, they just pretend that nothing happens and refuse to realize losses by selling stocks.

II) Representativeness

Tversky and Kahneman (1974) defines representativeness as the assessment of individual sample or event associated with the existing characteristics of the group. Representativeness is mainly utilized when people make judgement on the probability that a single object or event belongs to a group or category. In other words, representativeness bias tends to guide people to analyze a single event based on the previous information from the group that it might belong to.

For instance, successful stories on internet giants such as Google, Microsoft and Facebook may unconsciously make investors believe that investing in IT companies will earn huge amount of money. But the stories of failure are never told or forgotten in the market. Baker, Bradley and Wurgler (2011) argues that investors tend to purchase IT stocks in the hope of earning higher returns, since IT stocks share similarities with those successful giants.

As for high volatility stocks, successful stories are frequently heard in the financial news, which gradually makes people to form the perception that high volatility stocks are associated with great successes on investment. Investors start to overweight high volatility stocks in their portfolios and accept to overpay for high volatility stocks. Little attention has been paid to the potential huge losses delivered by the characteristics of high volatility.

III) Overconfidence

Overconfidence is a bias that make people put too much confidence on their judgements. Overconfident investors believe that they have better stock picking skills than the average level. Baker, Bradley and Wurgler (2011) suggests that overconfident investors are likely to show their disagreement. Being overconfident means that they also agree to disagree, stubbornly insisting on their own estimation even if it is false. The extent of disagreement is even larger when uncertainty arises. High volatility stocks perfectly fit the tastes of overconfident investors. When valuing high volatility stocks especially those with high past performance, overconfident investors tend to reach unrealistic overvaluation or undervaluation. The more volatile the stock is, the larger the valuation gap might be.

Miller (1977) points out that the prices of high volatility stocks are generally determined by optimistic investors with overconfidence. High volatility stocks are associated with wider range of opinion on valuation. Due to the fact of limited accesses to short selling for individual investors, pessimistic investors can hardly short sell high volatility stocks while optimistic investors are just simply bidding up the prices by purchasing stocks in the market. Therefore the prices of high volatility stocks are determined by the optimists rather than pessimists, which leads to lower returns on high volatility stocks because of overvaluation by optimists.

2.3.2 Leverage constraints

As implied by the modern portfolio theory, the market portfolio is the tangent portfolio on the efficient frontier that maximizes the Sharpe ratio. The tangent line that crosses the optimal market portfolio and risk free rate is generally defined as the capital market line. Every investor adjusts the allocations between the market portfolio and risk free assets under their own tolerance level to risk, termed as risk aversion parameter. Due to the constraints on leverage, most of the individual investors and some institutional investors either have no access to leverage or are simply prohibited from applying leverage (Frazzini and Pedersen (2014)). Under the constraints on leverage, if certain investors want to earn expected returns higher than the return of market portfolio, the viable solution for them is to overweight high volatility stocks and underweight low volatility stocks in their portfolio. The increasing demand on high volatility stocks will generate lower expected returns, while the decreasing demand for low volatility stocks conversely gives rise to higher expected returns.

2.3.3 Short-selling constraints

Another explanation for low volatility anomaly is the constraints on short-selling. People also normally hesitate to short sell for fear of great losses. Most individual investors have little or none access to short-selling, while institutional investors like pension funds or mutual funds are lawfully not allowed to short-selling. Miller (1977) points out that the prices of high volatility stocks are generally determined by optimistic investors with overconfidence. The constraints on short-selling have greatly deterred arbitrage activities on overvalued high volatility stocks. Also, high volatility stocks are generally small companies. It might be hard to make arbitrage by borrowing huge amount of high volatility stocks from the intermediaries. Failure of short-selling on overvalued volatile stocks potentially forms the basis for low volatility anomaly.

2.3.4 Benchmarking as the limits to arbitrage

It has increasingly become an industrial practice that when investors put money into a fund, it is clearly stated in the contract that it follows a benchmark to evaluate the performance of the fund. The benchmark could be an index, a combination of several indexes or even certain assets, as most pre-defined benchmarks are value-weighted. For the portfolio managers, the priority for them is not maximizing the portfolio return by investing aggressively with large deviation from the benchmark. The managerial skills of portfolio managers are measured by the information ratio, which is the return difference from the benchmark divided by the volatility of the return difference, also called tracking error. The prime concern for portfolio managers is to maximize the return by taking advantage of the small range of freedom to deviate from the benchmark.

Baker, Bradley and Wurgler (2011) argues that low volatility anomaly would persist in the presence of benchmarking. Portfolio managers with a fixed benchmark have little incentives to overweight low beta stocks as the tracking error would be out of control. The paper mathematically explains that fund managers will not choose to overweight the undervalued low beta stocks except that these stocks deliver extreme alphas. Low beta stocks with less extreme but still substantial alpha may not attract investment managers to overweight. The same logic applies to high beta stocks. Investment managers will only start to underweight high beta stocks when they carries extreme negative alpha. A fairly negative alpha for a high beta stock will not push managers to change the overweighting decision. In conclusion, a benchmark will make investment managers less likely to take advantage of low volatility anomaly.

3. DATA AND METHODOLOGY

3.1 Data

The research data in this thesis consists of multiple datasets. The first dataset contains historical daily returns and market capitalization of 1000 US-listed companies, spanning a period from January 2nd 1990 to December 31st 2015. This dataset also includes historical returns of delisted companies to avoid survivorship bias. It is downloaded from CRSP database[1].

The second dataset consists of the daily Fama French three factors, including market, size and value (Fama and French (1993)). This dataset is downloaded from Kenneth R French factor library[2]. It is later merged with the daily stock return dataset by date.

The third dataset is the monthly returns of the above-mentioned 1000 US-listed companies from January 1990 to December 2015. The dataset also contains the market capitalization of each company at the end of each month.

The fourth dataset is made up of monthly Fama French three factors as well as momentum factor (Jegadeesh and Titman (1993)), covering a period from January 1990 to December 2015. It is also directed downloaded from Kenneth R French factor library.

The fifth dataset contains the monthly Fama French five factors, including market, size, value, profitability and investment (Fama and French (2015)). It is adapted to cover the period from January 1990 to December 2015. The source of the dataset again comes from Kenneth R French factor library.

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[1] CRSP database: http://www.crsp.com/

[2] K. French factor library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

3.2 Methodology

3.2.1 Total volatility

The methodology to measure total volatility in this thesis is calculated by the standard deviation of stock daily return within each month. The total volatility is calculated as the square root of the average variance of stock return. The formula is presented as follows:

$$\text{Total volatility} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Ret - Mean(RET))^2} \tag{3}$$

Where *Ret* is the daily return of the stock, *Mean(RET)* is the average stock daily return within each month, N is the number of observations of daily return for the stock. Through this approach, the total volatility for each stock within each month is obtained.

3.2.2 Idiosyncratic volatility

Unlike the total volatility, idiosyncratic volatility is calculated through the Fama French three-factor (hereafter FF-3) regression with the daily excess return of the stock within each month. The idiosyncratic volatility is defined as the square root of residual variance from the Fama French three-factor regression. The formula is shown below:

$$\text{Ret} - \text{Rf} = \alpha_i + \beta_{mkt}MKT + \beta_{SMB}SMB + \beta_{HML}HML + \varepsilon_i \tag{4}$$

Where Ret-Rf is the excess return of the stock, MKT, SMB and HML represent market, size and value factors respectively. ε_i is the residual term from the regression.

In this model, idiosyncratic volatility is measured by the square root of residual variance, denoted as $\sqrt{VAR(\varepsilon_i)}$.

3.2.3 Weighting approaches

Inspired by Bali and Cakici (2008), this thesis also examines the impact of three different weighting approaches: equally-weighted, value-weighted and volatility-weighted approaches.

Equally-weighted approach implies that all the stocks are alocated to the same weights when forming the portfolio. It can be denoted as:

$$W_i = \frac{1}{N} \tag{5}$$

Where W_i is the weight of stock *i* in the equally-weighted portfolio, and N is the number of stocks in the portfolio.

Value-weighted approach defines the weights of stocks based on their market capitalization, which is denoted as:

$$W_i = \frac{CAP_i}{\sum_{i=1}^{N} CAP_i} \tag{6}$$

Where W_i is the weight of stock *i* in the value-weighted portfolio, CAP_i is the market capitalization of the stock *I,* and N is the number of stocks in the portfolio.

Volatility-weighted approach defines the weights of stocks based on their volatility. Due to the fact that total volatility and idiosyncratic volatility are both examined in this thesis, volatility-weighted approach defines the weights of stocks in two scenarios.

For stocks ranked by their total volatility, volatility-weighted approach defines the weights as follows:

$$W_i = \frac{VOL_i}{\sum_{i=1}^{N} VOL_i} \tag{7}$$

Where W_i is the weight of stock i in the volatility-weighted approach, VOL_i is the total volatility of the stock *I,* and N is the number of stocks in the portfolio.

In terms of the stocks ranked by their idiosyncratic volatility, volatility-weighted approach calculates the weights as follows:

$$W_i = \frac{IVOL_i}{\sum_{i=1}^{N} IVOL_i} \tag{8}$$

Where W_i is the weight of stock i in the volatility-weighted approach, $IVOL_i$ is the idiosyncratic volatility of the stock *i,* and N is the number of stocks in the portfolio.

4. EMPIRICAL RESULTS

This chapter reports the empirical results based on analysis on the sample data. The first part presents the results from sorts on total volatility, which analyzes performance of the volatility deciles, estimates from CAPM, Carhart four-factor model and FF-5 model. The second subsection follows the same approach in analyzing the results from sorts on idiosyncratic volatility. The third component of this chapter constructs the volatility factors and subsequently calculates the factor risk premium through Fama-MacBeth procedures. The final sub-chapter presents the results on the interaction between volatility factors and other anomalous factors.

4.1 Sorts on total volatility

This section analyzes the performance of volatility deciles formed based on the sorts on total volatility. The time series return of volatility deciles are subsequently fitted into CAPM, Carhart four-factor and FF-5 model to identify the characteristics of volatility deciles and exposures to other factors.

4.1.1 Performance of portfolio deciles

i) Annualized returns

As shown in Table 1, annualized returns of portfolio deciles show very mixed results that seem to contradict the low volatility anomaly. Basically, the annualized returns are not strictly monotonously increasing from the highest volatility decile (P01) to the lowest volatility decile (P01). The maximum of annualized returns appears in the deciles with volatility in-between, which indicates that volatility are not permanently penalized.

For equally-weighted portfolio formation approach in Panel A, the annualized returns of volatility deciles range from 8.71% to 15.25%. Volatility decile P04 has the highest annualized return of 15.25%, whereas the lowest annualized return exactly happens to the highest volatility decile P01. Three lowest volatility deciles (P08, P09, and P10) on average outperform the three highest volatility deciles (P01, P02, P03). Meanwhile, the lowest volatility decile P10 outperforms the highest volatility decile by a margin of around 3%. Low volatility anomaly cannot be reflected in the case of equally-weighted approach as the annualized returns do not monotonously increase from the highest volatility to the lowest volatility decile.

With regards to the value-weighted approach in Panel B, obtained results show the similar patterns as in the equally-weighted approach. In general, low volatility deciles outperform high volatility deciles, while the maximum annualized return of 16.20% appears in the middle decile P05. However, return spread between the highest volatility decile P01 and the lowest volatility decile P10 increases substantially to the level of 16.29%, and the highest volatility deciles are punished for bearing excessive volatility, generating an annualized return of negative -4.26%. As the annualized returns of volatility deciles shows neither an increasing nor decreasing trend, the relation of return and volatility cannot be concluded.

In terms of the volatility-weighted approach, the results show a horse match among the volatility deciles. Volatility decile P04 generates the highest annualized return of 15.38%, as the volatility decile P03 has the lowest return of 11.27%. In this case, the highest decile P01 surprisingly outperforms the lowest decile P01, which may as well provide contradictory findings to low volatility anomaly, since inherently high volatility stocks carries a relatively larger weights compared with equally and value weighted approaches. Thus if low volatility anomaly exists, the spread between the lowest decile and the highest decile should be enlarged, let alone being inverted.

Table 1: Summary statistics of deciles sorted on total volatility

This table shows the summary statistics of portfolio deciles and regression results on CAPM. At the end of each month, all stocks in the sample are ranked based on their total volatility within this month. Then stocks are subsequently sorted into portfolio deciles, each decile contains the same number of stocks. Portfolios are formed with three different weighting approaches. Monthly rebalancing is conducted at the end of each month, as the portfolio deciles returns are calculated monthly at the end. Portfolio deciles are labeled with decreasing total volatility from P01 to P10, with P01 being the highest volatility decile and P10 being the lowest volatility decile. Spread portfolio P00 is calculated by P10-P01, as long lowest volatility and short highest volatility. CAMP regressions are fitted with time-series of monthly returns of decile portfolios. Return, volatility, Sharpe ratio and CAPM alpha are all in annualized numbers. Panel A shows the results with equally-weighted approach. Panel B shows the results from adopting value weighted approach, portfolio returns are calculated with weighting based on market capitalization of stocks in the same decile. Panel C presents the results with the volatility-weighted method, as the monthly volatility plays a key role in determining the weights of stocks in the decile.

Panel A: Equally-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	11.79	11.12	0.80	5.43	3.11	0.46	13.70	0.38
P09	14.65	13.80	0.85	6.21	3.71	0.73	22.81	0.63
P08	13.98	15.30	0.72	5.07	2.64	0.80	21.65	0.60
P07	14.92	17.88	0.67	4.81	2.24	0.95	23.11	0.63
P06	15.01	19.86	0.61	4.09	1.71	1.06	23.16	0.63
P05	15.12	23.68	0.52	3.19	1.03	1.19	20.10	0.57
P04	15.25	27.07	0.46	2.10	0.59	1.35	19.80	0.56
P03	11.48	28.89	0.30	-1.26	-0.30	1.30	15.97	0.45
P02	14.88	37.37	0.32	-0.10	-0.02	1.60	14.56	0.41
P01	8.71	42.13	0.14	-5.34	-0.75	1.47	10.77	0.27
P00	3.08	39.66	0.08	10.77	1.48	-1.02	-7.28	0.15

Panel B: value-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	12.03	13.28	0.69	5.64	2.50	0.46	10.68	0.27
P09	11.77	16.14	0.55	3.19	1.38	0.75	16.96	0.48
P08	11.34	18.78	0.45	1.40	0.56	0.93	19.39	0.55
P07	11.83	22.14	0.40	0.52	0.18	1.11	19.91	0.56

Table 1 to be continued

P06	14.69	24.25	0.49	2.27	0.74	1.26	21.51	0.60
P05	16.20	29.85	0.45	2.54	0.61	1.42	17.82	0.51
P04	6.80	37.14	0.11	-9.60	-1.87	1.78	18.10	0.51
P03	10.04	37.01	0.19	-5.62	-1.04	1.69	16.33	0.46
P02	3.69	52.11	0.02	-15.30	-1.86	2.13	13.52	0.37
P01	-4.26	45.40	-0.16	-19.86	-2.64	1.68	11.64	0.30
P00	16.29	45.43	0.36	25.50	3.08	-1.22	-7.67	0.16

Panel C: volatility-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	12.38	11.20	0.85	5.71	3.43	0.50	15.67	0.44
P09	14.69	13.90	0.85	6.22	3.68	0.74	22.72	0.63
P08	14.08	15.35	0.73	5.15	2.67	0.80	21.61	0.60
P07	14.97	17.91	0.67	4.85	2.25	0.95	23.17	0.63
P06	14.90	19.92	0.60	3.97	1.66	1.06	23.12	0.63
P05	15.12	23.70	0.52	3.19	1.03	1.19	20.06	0.57
P04	15.38	27.09	0.46	2.22	0.62	1.36	19.77	0.56
P03	11.27	28.97	0.29	-1.45	-0.34	1.30	15.82	0.45
P02	14.67	37.29	0.32	-0.28	-0.05	1.59	14.56	0.41
P01	12.95	47.88	0.21	-0.98	-0.12	1.46	8.97	0.21
P00	-0.57	46.06	-0.01	6.69	0.77	-0.96	-5.75	0.10

II) Sharpe ratios

The results on Sharpe ratios in table 1 seemingly provides some evidence supporting the existence of low volatility anomaly, as the Sharpe ratios generally decrease from the lowest volatility decile to the highest decile. Volatility are monotonously decreasing, because deciles are formed with stocks ranked by the level of volatility. Sharpe ratio provides a clear overview of risk-adjusted return of volatility deciles.

For the equally-weighted approach, from the highest volatility decile P01 up to volatility decile P09, the Sharpe ratios are generally increasing, and the lowest volatility deciles P10 generate slightly lower but similar results as volatility decile P09. With regards to value-weighted approach, the lowest volatility decile P10 has the highest

Sharpe ratio, and the Sharpe ratios are increasing from P01 to P10 except for volatility deciles P04 and P07. It is also worthwhile to mention that the highest volatility decile P01 presents negative risk-adjusted return. When it comes to volatility-weighted approach, the results show that Sharpe ratios are decreasing as volatility increases except in the case of volatility decile P02. The highest volatility deciles P10 and P09 jointly take the lead with the Sharpe ratio of 0.85.

In summary, as a measurement of reward to risk, Sharpe ratio presents a reliable approach to compare the risk-adjusted returns for volatility deciles. In this case, the results shows that broadly speaking, low volatility deciles have higher risk-adjusted returns than high volatility deciles, which might provide evidence for low volatility anomaly. But strictly speaking from a monotonous point of view, the conclusion of existence of low volatility anomaly cannot hold. Further inference points to the direction of statistically insignificant existence of low volatility anomaly.

III) CAPM estimates

Single-factor CAPM model is fitted by running linear regression on the time series of returns of portfolio deciles and market excess returns at a monthly frequency. Table 1 presents the results on CAPM estimates. A couple of conclusions can be drawn through analysis on the results. First of all, statistically significant alphas are comparably easier to find in low volatility deciles than high volatility deciles. Secondly, given that all loadings on market factor are significant, the betas of volatility deciles are generally increasing as the volatility of deciles increases except for the case of the highest volatility decile P01. In addition, spread portfolio P00 (constructed by P10 minus P01) is negatively correlated with market factor.

For equally-weighted approach, the four lowest volatility deciles (P07-P10) carry statistically significant positive alphas, with the t-statistics of alpha able to reject the hull hypothesis of zero alpha at 95% confidence interval, while high volatility deciles are exposed to negative alpha without statistical significance. Volatility decile P09 obtains the highest annualized alpha of 6.21%. Loadings on market factors generally increase as volatility increases. Special case is applicable to the highest volatility decile P01 as the beta actually drops.

With respect to value-weighted approach, the lowest volatility decile P10 grabs the maximum significant annualized alpha of 5.64%, while the highest volatility decile P01 presents significant negative alpha of -19.86%. It is obviously tempting to believe that volatility might be penalized by the market. Unfortunately, there is no statistical significance of alphas in other volatility deciles, which generally provides evidence against the significant existence of low volatility anomaly. Loadings on market factor follows the same pattern as equally-weighted approach, which generally increase as the volatility increases while P01 experiences abrupt drop.

In terms of volatility-weighted portfolio formation, an increasing number of significant alphas can be observed in low volatility deciles, as the alphas of high volatility deciles obtain no significance at all. Volatility decile P09 dominates the ranking with an annualized alpha of 6.22%. Market betas of volatility deciles demonstrate the same increasing pattern that as volatility goes up, while similar drop always happens in the case of the highest volatility decile P01.

In conclusion, through the majority of alphas are generally not statistically significant, it is worthwhile to mention that median volatility decile P09 tends to have the significant and highly positive alpha among all volatility deciles. Neither the lowest volatility decile P10 nor the highest volatility decile P01 have shown robust outperformance against the other. Evidence on market betas in general reflects an increasing trend, while the highest volatility decile P01 abruptly drops in the value of market beta. Up to this point, it is still far away from concluding on the existence of low volatility anomaly at least in the CAPM estimates.

4.1.2 Carhart four-factor regression

As discussed above, single-factor CAPM fails to provide neither supportive nor objective evidence on the existence of low volatility anomaly. In view of other widely acknowledged market anomalies and heavy criticism on single-factor CAPM for lack of explaining power, an extended Carhart four-factor is utilized to gain better explanatory ground. Size, value and momentum effects are taken into consideration when estimating regressions against the monthly returns of volatility deciles. The purpose of implementing Carhart four-factor regression lies in uncovering the potential connectivity of low volatility anomaly with other anomalous effects through a better-explanatory model.

I) Annualized alphas

With regard to equally-weighted approach, low volatility deciles P06-P10 gain statistical significance on their alphas, the rest of the deciles find no significance. Volatility decile P09 has the highest annualized alpha of 5.45%, as P10 carries a significant 3.93%. Concerning the value-weighted approach, statistical significance can only be observed on the lowest volatility decile P10, with annualized alpha ending up with -14.61%. When it comes to volatility weighted approach, significant alphas can be easily found in the lower-bound of volatility deciles from P06 to P10. Again, volatility decile P09 comes with the highest annualized alpha of 5.47%, showing that low volatility deciles relatively outperform high volatility deciles.

II) Factor loadings

a. Market (MKT)

Regardless of weighting approaches, loadings on market factor demonstrate the same pattern as the CAPM betas mentioned above. Market betas of volatility deciles increase as the volatility increases from the lowest volatility decile P10 to the highest volatility decile P01, while abrupt drop happens in the case of volatility deciles P03 and P01, the highest volatility decile.

b. Size effect (small minus big: SMB)

To begin with, statistical significance on size prevails in our sample. In general, loadings on size factor increase as the volatility increases in all weighting methods. As for equally-weighted and volatility-weighted approach, both demonstrate a monotonously increasing pattern as volatility increases. All loadings are significantly positive, thus all volatility deciles are tilting to small stocks, and the sensitivity to size effect gains as volatility moves up.

In terms of value-weighted approach, loadings on size factor are generally increasing with the volatility. Contrary to the other weighting approaches, low volatility deciles start with negative loadings, and high volatility ends up with positive loadings. The loadings of volatility deciles range from -0.21 to 1.49. It is clearly showing that low volatility deciles are tilting to large stocks, while high volatility deciles are tilting to small stocks.

c. Value effect (high minus low: HML)

Generally speaking, loadings on value factor in all weighting approaches are mathematically decreasing as volatility increases. Results show that loadings are starting from positive numbers in low volatility deciles, decreasing to negative in high volatility deciles. This finding supports the argument that low volatility deciles are tilting to value stocks, as high volatility deciles tilt to growth stocks.

d. Momentum effect (MOM)

Similar to value effect, loadings on momentum factor are generally decreasing as volatility drops across deciles. Loadings are generally negative, meaning the tilting to contrarian with a greater exposure. Except that in value-weighted approach, loadings are initially being positive and then inverted to negative, which indicates a shift of tilting from momentum to contrarian.

III) Spread portfolio P00 (P10 minus P01)

As for equally-weighted approach, Panel A of Table 2 shows that spread portfolio P00 has a positive alpha, but not significant, which implies that the relation between volatility and expected returns is insignificantly negative. The lowest volatility decile on average outperforms the highest volatility decile. Factor loadings are all statistically significant, with negative loadings on MKT and SMB, positive loadings on HML and MOM. It clearly shows that spread portfolio P00 is tilting to large, value and momentum stocks.

In terms of value-weighted approach, Panel B shows that significant positive alpha of 18.39% can be observed on spread portfolio P00. This leads to the significant negative relation between volatility and expected returns, which provides strong evidence supporting the low volatility anomaly. Characteristics of the value-weighted spread portfolio are similar to equally-weighted one, with a tilting to large, value and momentum stocks.

With regard to volatility-weighted approach, Panel C shows that alpha of spread portfolio P00 is insignificantly negative, offering evidence against the low volatility anomaly. Judging from the loadings on factors, it is apparent that volatility-weighted spread portfolio is tilting to large, value and momentum, consistent with findings on other weighting approaches.

Table 2: Carhart four-factor estimation on deciles sorted on total volatility

This table presents the results of regression estimates with Carhart four-factor model: time series regressions are fitted by putting the monthly returns of volatility deciles (from P01 to P10) and the spread portfolio P00 (P10 minus P01) as the dependent variables, while market excess return, size, value and momentum factors are on the independent variables, denoted by MKT, SMB, HML and MOM respectively. Alphas are multiplied by twelve to annualize. Each column of regression estimates are subsequently followed by a bracket with sign of t, which means the t-statistics of the estimate on the left, also the symbol in the bracket denotes the estimate. The right-most column shows the coefficient of determination, denoted as R^2. Different portfolio formation approach are also examined here in the regressions. Panel A shows the results of equally-weighted approach. Panel B presents the results of value-weighted approach, as weights of stocks are determined by the market capitalizations. Panel C reports the results of volatility-weighted approach, and volatility plays a key role in determining the allocated weights.

Panel A: equally-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	3.93	2.60	0.47	15.37	0.24	5.96	0.45	10.29	-0.01	-0.50	0.56
P09	5.45	3.82	0.71	24.68	0.28	7.41	0.39	9.58	-0.07	-2.98	0.74
P08	3.65	2.30	0.77	23.92	0.43	10.19	0.44	9.66	-0.03	-1.13	0.74
P07	3.52	2.01	0.90	25.41	0.54	11.78	0.44	8.70	-0.05	-1.63	0.77
P06	4.73	2.37	0.93	23.02	0.62	11.93	0.17	2.87	-0.16	-4.59	0.76
P05	4.01	1.71	1.00	21.02	0.97	15.76	0.05	0.69	-0.16	-3.84	0.77
P04	4.19	1.69	1.08	21.50	1.17	18.09	0.01	0.08	-0.29	-6.68	0.80
P03	2.85	0.90	0.95	14.82	1.20	14.47	-0.21	-2.25	-0.43	-7.71	0.71
P02	6.25	1.40	1.13	12.51	1.46	12.48	-0.40	-3.15	-0.61	-7.81	0.66
P01	-0.21	-0.03	1.00	8.21	1.72	10.89	-0.36	-2.06	-0.51	-4.88	0.51
P00	4.13	0.66	-0.53	-4.17	-1.48	-9.04	0.81	4.48	0.50	4.57	0.41

Table 2 to be continued

Panel B: value-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	3.78	1.75	0.56	12.86	-0.21	-3.73	0.28	4.48	0.11	2.91	0.37
P09	1.84	0.83	0.83	18.66	-0.19	-3.28	0.33	5.18	0.04	0.98	0.55
P08	0.61	0.25	0.98	20.13	-0.06	-0.94	0.41	5.89	-0.06	-1.34	0.61
P07	-1.05	-0.37	1.12	19.43	0.25	3.34	0.42	5.09	0.00	0.08	0.60
P06	2.57	0.83	1.22	19.46	0.16	2.00	0.13	1.42	-0.09	-1.58	0.61
P05	3.05	0.84	1.23	16.78	0.95	10.00	-0.21	-2.04	-0.03	-0.53	0.65
P04	-5.61	-1.30	1.44	16.42	1.17	10.26	-0.45	-3.64	-0.33	-4.30	0.67
P03	-2.68	-0.53	1.42	13.97	0.94	7.11	-0.26	-1.82	-0.27	-3.07	0.56
P02	-10.29	-1.41	1.68	11.35	1.49	7.75	-0.92	-4.36	-0.29	-2.26	0.53
P01	-14.61	-2.15	1.25	9.06	1.44	8.06	-0.46	-2.33	-0.47	-3.98	0.46
P00	18.39	2.54	-0.68	-4.66	-1.65	-8.67	0.74	3.53	0.59	4.60	0.39

Panel C: volatility-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	4.08	2.85	0.52	17.93	0.22	5.81	0.45	10.81	0.00	0.09	0.61
P09	5.47	3.80	0.72	24.61	0.28	7.40	0.40	9.63	-0.08	-3.08	0.74
P08	3.76	2.35	0.77	23.86	0.43	10.17	0.45	9.65	-0.03	-1.22	0.74
P07	3.54	2.01	0.91	25.44	0.54	11.75	0.44	8.64	-0.05	-1.54	0.77
P06	4.62	2.31	0.93	22.98	0.63	11.97	0.16	2.85	-0.16	-4.62	0.76
P05	3.98	1.70	1.00	20.95	0.97	15.73	0.05	0.74	-0.16	-3.82	0.77
P04	4.31	1.73	1.08	21.43	1.17	18.01	0.01	0.12	-0.29	-6.64	0.80
P03	2.72	0.85	0.95	14.57	1.20	14.27	-0.20	-2.20	-0.44	-7.74	0.71
P02	6.19	1.40	1.12	12.51	1.45	12.49	-0.42	-3.28	-0.62	-7.94	0.66
P01	4.68	0.62	0.97	6.28	1.71	8.58	-0.38	-1.72	-0.56	-4.21	0.40
P00	-0.60	-0.08	-0.45	-2.80	-1.49	-7.21	0.82	3.62	0.56	4.08	0.30

4.1.3 Fama French five-factor regression

I) Annualized alpha

In Panel A of Table 3, it can be easily observed that in equally-weighted approach, results on annualized alpha generally show no pattern, only volatility deciles P09, P05 and P02 shows significant alphas. As for the value-weighted approach, only annualized alpha of volatility decile P05 is significant. In terms of volatility-weighted approach, results show the same picture as equally-weighted approach, suggesting that neither low nor high volatility deciles outperforms its counterparts.

II) Factor loadings

a. Market (MKT)

Table 3 shows that generally all volatility deciles have highly significant beta coefficient on MKT. These market betas are slightly less than one in equally and volatility weighted approaches, whereas in value-weighted approach, betas are showing a generally increasing trend from low volatility deciles to high volatility deciles.

b. Size effect (Small minus Big: SMB)

Table 3 shows that in all weighting approaches, loadings on size are generally increasing from low volatility deciles to high volatility deciles, indicating an increasing tilting to small stocks. It is noteworthy to mention that only in value-weighted approach, significant negative loading on size appears, implying a tilting to large stocks.

c. Value effect (High minus Low: HML)

Results of loadings on value in table 3 shows that value becomes ineffective in explaining the cross section of expected returns, as only low volatility deciles in equally and volatility weighted approaches have significant loadings on value. These results provide evidence that Fama and French (2015) claims that adding profitability factor will make value become redundant.

d. Profitability effect (Robust minus weak: RMW)

As shown in Table 3, basically all loadings on profitability are significant across all weighting approaches. It is also robust that loadings on profitability are generally starting from positive numbers in low volatility deciles and decreasing to negative numbers in high volatility deciles, which strongly supports the argument that low volatility deciles are tilting to robust profitability while high volatility deciles tilting to weak profitability.

e. Investment effect (Conservative minus Aggressive: CMA)

Table 3 indicates that loadings on investment are mostly insignificant, suggesting that investment effect does not carry much influence on interpreting volatility effect.

III) Spread portfolio P00 (P10 minus P01)

The results in Table 3 show that there are no significant alphas in any weighting approaches for spread portfolio P00. In equally-weighted approach, significant negative loading on size as well as positive loadings on profitability jointly imply a tilting to large and robust profitability. As for value-weighted approach, the loadings indicate a tilting to large, robust profitability and conservative investment. Finally in terms of volatility-weighted approach, the same picture is painted by the tilting to large and robust profitability.

Table 3: Fama French five-factor estimation on deciles sorted on total volatility

This table presents the results of regression estimates with FF-5 model: time series regressions are fitted by putting the monthly returns of volatility deciles (from P01 to P10) and the spread portfolio P00 (P10 minus P01) as the dependent variables, while market excess return, size, value, profitability and investment factors are on the independent variables, denoted by MKT, SMB, HML, RMW and CMA respectively. Alphas are multiplied by twelve to annualize. Each column of regression estimates are subsequently followed by a bracket with sign of t, which means the t-statistics of the estimate on the left, also the symbol in the bracket denotes the estimate. The right-most column shows the coefficient of determination, denoted as R^2. Different portfolio formation approach are also examined here in the regressions. Panel A shows the results of equally-weighted approach. Panel B presents the results of value-weighted approach, as weights of stocks are determined by the market capitalizations. Panel C reports the results of volatility-weighted approach, and total volatility plays a key role in determining the allocated weights.

Panel A: equally-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	2.61	1.69	0.52	15.33	0.28	6.35	0.32	5.50	0.15	2.46	0.13	1.52	0.57
P09	3.07	2.14	0.79	25.08	0.36	8.80	0.28	5.19	0.23	4.20	0.08	1.06	0.76
P08	2.36	1.48	0.81	22.97	0.51	11.02	0.36	5.86	0.18	2.99	-0.04	-0.47	0.75
P07	2.43	1.37	0.94	23.88	0.61	11.99	0.36	5.34	0.12	1.81	-0.05	-0.53	0.78
P06	3.00	1.44	0.98	21.26	0.66	10.94	0.13	1.61	0.03	0.43	-0.05	-0.44	0.75
P05	5.41	2.28	0.94	17.95	0.82	12.04	0.08	0.90	-0.45	-4.95	-0.09	-0.68	0.77
P04	4.85	1.83	1.04	17.77	1.03	13.57	0.16	1.59	-0.45	-4.40	-0.34	-2.34	0.78
P03	4.22	1.24	0.88	11.69	0.96	9.81	0.15	1.15	-0.72	-5.47	-0.54	-2.94	0.69
P02	9.74	2.11	0.99	9.75	0.95	7.16	0.11	0.60	-1.42	-8.00	-0.46	-1.83	0.66
P01	4.37	0.73	0.85	6.41	1.11	6.41	-0.09	-0.37	-1.63	-7.03	0.01	0.02	0.54
P00	-1.76	-0.29	-0.33	-2.43	-0.83	-4.67	0.41	1.74	1.77	7.48	0.12	0.36	0.46

Table 3 to be continued

Panel B: value-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	3.24	1.46	0.60	12.25	-0.19	-2.93	0.09	1.03	0.12	1.35	0.37	3.10	0.37
P09	-1.25	-0.57	0.94	19.63	-0.01	-0.12	0.15	1.85	0.48	5.78	0.19	1.64	0.59
P08	-2.12	-0.88	1.06	19.88	0.08	1.20	0.36	3.95	0.36	3.86	-0.02	-0.19	0.63
P07	-2.97	-1.03	1.19	18.57	0.37	4.43	0.26	2.32	0.27	2.44	0.12	0.78	0.61
P06	0.62	0.20	1.27	18.21	0.27	2.93	0.14	1.13	0.24	2.00	-0.15	-0.90	0.61
P05	7.34	2.04	1.08	13.53	0.75	7.20	-0.12	-0.91	-0.67	-4.82	-0.19	-0.97	0.67
P04	-3.38	-0.75	1.35	13.63	0.91	7.08	-0.18	-1.08	-0.76	-4.39	-0.39	-1.59	0.67
P03	-0.28	-0.06	1.34	11.79	0.63	4.24	-0.09	-0.47	-0.80	-4.04	-0.11	-0.40	0.56
P02	-2.23	-0.30	1.39	8.54	0.97	4.58	-0.38	-1.37	-1.46	-5.16	-0.79	-1.98	0.55
P01	-9.40	-1.36	1.04	6.82	0.98	4.91	0.10	0.36	-1.34	-5.02	-0.73	-1.96	0.48
P00	12.64	1.71	-0.44	-2.70	-1.16	-5.47	-0.01	-0.03	1.45	5.10	1.11	2.76	0.40

Panel C: volatility-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	2.59	1.80	0.58	18.14	0.28	6.78	0.29	5.37	0.19	3.34	0.16	2.03	0.63
P09	3.00	2.07	0.80	25.10	0.37	8.81	0.28	5.17	0.24	4.28	0.09	1.13	0.76
P08	2.44	1.52	0.81	22.91	0.51	10.98	0.36	5.88	0.18	2.96	-0.04	-0.48	0.75
P07	2.53	1.43	0.93	23.81	0.61	11.91	0.36	5.35	0.12	1.70	-0.06	-0.60	0.78
P06	2.90	1.39	0.98	21.20	0.66	10.97	0.13	1.65	0.03	0.42	-0.06	-0.51	0.75
P05	5.38	2.26	0.94	17.91	0.82	12.00	0.08	0.90	-0.45	-4.95	-0.08	-0.62	0.77
P04	4.89	1.84	1.04	17.75	1.03	13.51	0.15	1.52	-0.44	-4.31	-0.32	-2.21	0.78
P03	4.03	1.17	0.88	11.50	0.96	9.65	0.16	1.18	-0.72	-5.42	-0.55	-2.91	0.68
P02	9.58	2.08	0.99	9.76	0.94	7.11	0.09	0.51	-1.42	-8.04	-0.44	-1.78	0.66
P01	9.00	1.18	0.83	4.93	1.07	4.84	-0.12	-0.42	-1.73	-5.86	0.14	0.33	0.42
P00	-6.41	-0.81	-0.26	-1.48	-0.79	-3.47	0.41	1.39	1.92	6.32	0.02	0.05	0.34

4.1.4 Summary on volatility deciles sorted on total volatility

Table 1 shows that the annualized returns of volatility deciles are not monotonously decreasing as the volatility increases. Highest annualized return generally appears among the in-between volatility deciles. The results on Sharpe ratio in Table 1 provide evidence supporting the negative relation between the volatility and expected returns. Sharpe ratios of volatility deciles are generally decreasing as volatility increases, which means that the low volatility deciles generally outperform high volatility deciles in terms of risk-adjusted returns. Results from CAPM estimates show that significant positive alphas are comparably easier to observe in low volatility deciles than high volatility deciles. In a word, there are no strong evidences that prove the significant existence of low volatility anomaly in Table 1. But generally speaking, low volatility deciles outperform high volatility deciles on average.

Table 2 shows that in terms of Carhart four-factor alphas, weighting approaches play a vital role as the patterns of alpha vary greatly across weightings. Results of loadings on factors actually uncover the potential connection between low volatility anomaly and other anomalous effects. It is robust that regardless of different weighting approaches, there is a monotonous trend in factor loadings as volatility increases. An increasing trend can be observed in MKT and SMB factors, while a decreasing trend can be found in HML and MOM factors. Loadings on SMB increase as volatility increases, indicating that low volatility deciles are less tilting to small stocks than high volatility deciles. Also, loadings on HML decreases as volatility increases, but there is a clear distinction: low volatility deciles have positive tilting to value while high volatility deciles do the opposite with a tilting to growth stocks. As for momentum, similar conclusion can be drawn that high volatility deciles are comparably more tilting to contrarian stocks than low volatility deciles. Empirical results show that there is no evidence that points to the significant existence of low volatility anomaly. A mixed combination of size, value and momentum effects lies in the potential explanation to relative outperformance of low volatility deciles.

Table 3 shows that compared to Carhart four-factor model, there are less significant alphas in FF-5 model. These significant alphas are randomly distributed in the in-between deciles, presenting no monotonous pattern. Loadings on factors suggests an increased degree of tilting to small and robust profitability from the lowest to highest volatility deciles. Results on spread portfolio conclude that there is no evidence supporting the volatility effect, but a tilting to large and robust profitability can be observed in equally and volatility weighted approach, whereas in value-weighted approach, it is a tilting to large, robust profitability and conservative investment.

4.2 Sorts on idiosyncratic volatility

This section analyzes the performance of volatility deciles formed based on the sorts on idiosyncratic volatility. The time series return of volatility deciles are subsequently fitted into CAPM, Carhart four-factor and FF-5 model to identify the characteristics of volatility deciles and exposures to other factors.

4.2.1 Performance of portfolio deciles

I) Annualized returns

In equally-weighted approach, results in Panel A of Table 4 shows that annualized returns of volatility deciles range from 9.63% to 15.90%. Volatility deciles P06 outperforms the other volatility deciles with an annualized return of 15.90%. The title of worst performing deciles falls to the highest volatility decile P10. On average, it is evident that low volatility deciles are outperforming high volatility deciles, but there is no monotonous trend that is convincing enough to draw the conclusion of low volatility anomaly.

As for value-weighted approach, Panel B of Table 4 presents that there is significant underperformance in high volatility deciles, and highest volatility decile P01 surprisingly generates a negative annualized return of -7.48%, while its counterpart, the lowest volatility decile P10 is among one of the top outperforming volatility deciles. Volatility decile P06 dominates the ranking with an annualized return of 15.41%. Again, it is even more apparent that low volatility deciles on average outperform high volatility deciles.

In terms of volatility-weighted approach, results in Panel C of Table 4 shows that the highest volatility decile P01 distinctively outperforms the lowest volatility decile P10, and the evidence seems to support that high volatility deciles are on average outperforming low volatility deciles, blurring the picture of a potential low volatility anomaly. The highest annualized returns are held by volatility deciles P05 & P06, both with an annualized return of 15.65%.

In summary, the results in Table 3 indicate that there is no evidence that strongly supports the existence of low volatility anomaly. For equally-weighted and value-weighted approaches, relative outperformance of low volatility deciles can be observed, while volatility-weighted approach provides evidence against the low volatility anomaly. Most importantly, no monotonous trend can be found in any weighing approach, which directly implies that low volatility anomaly is not significant in volatility deciles sorted on idiosyncratic volatility.

II) Sharpe ratio

Table 3 shows that in general, low volatility deciles have higher risk-adjusted returns than high volatility deciles. The results in Panel A & C of Table 4 show that equally-weighted and volatility-weighted approaches share similarities in the patterns of Sharpe ratio across deciles. An increase from the lowest volatility decile P10 to second lowest decile P09, and then accompanied by a subsequent decrease in Sharpe ratio as volatility increases, but an abrupt increase happens to volatility decile P02, finally ending up with a drop again in highest volatility decile P01. In terms of value-weighted approach, Sharpe ratios are decreasing as volatility increase except for the case of volatility decile P06. Compared to the performance measurement of annualized returns, Sharpe ratios offer a better view of the whole picture of low volatility anomaly. As generally a decreasing trend can be observed, this provides evidence for the potential existence of low volatility anomaly with statistical insignificance.

III) CAPM estimates

Concerning the equally-weighted approach, Panel A of Table 4 shows that statistically significant CAPM alphas can only be observed in low volatility deciles (P07, P08, and P09). Volatility decile P09 has the highest annualized alpha of 5.96%. Generally, low volatility deciles are outperforming high volatility deciles in terms of CAPM alpha. Loadings on market factor show an increasing trend as volatility increases.

With regard to value-weighted approach, results in Panel B shows that only the lowest and the highest volatility deciles P01 and P10 possess significant CAPM alphas. The lowest volatility decile P01 has the highest CAPM alpha of 6.27% while the highest volatility decile P10 ends up with a stunning negative alpha of -20.59%. The relative underperformance of high volatility deciles provides evidence to the negative relation between idiosyncratic volatility and expected returns. Loadings on market factor increase as the volatility increases until a drop on the highest volatility decile P01.

When it comes to volatility-weighted approach, similar to the equally-weighted one, significant alphas are only observed in few low volatility deciles (P07, P08, and P09). Volatility decile P09 dominates the ranking with an annualized alpha of 5.98%. In general, high volatility deciles are underperforming low volatility deciles, which provides evidence supporting low volatility anomaly. Loadings on market factor demonstrate an increasing trend as volatility increases.

In summary, CAPM estimates in Table 4 show that regardless of weighting approaches, significant positive alphas are comparably easier to find in low volatility deciles. Furthermore, low volatility deciles are generally outperforming high volatility deciles in terms of CAPM alphas. However, no monotonous trend has been detected in our results on alpha, meaning that there is no evidence supporting the existence of low volatility anomaly with statistical significance. Loadings on market factor imply a positive relation between market exposure and idiosyncratic volatility.

Table 4: Summary statistics of deciles sorted on idiosyncratic volatility

This table shows the summary statistics of portfolio deciles and regression results on CAPM. At the end of each month, all stocks in the sample are ranked based on their idiosyncratic volatility within this month. Then stocks are subsequently sorted into portfolio deciles, each decile contains the same number of stocks. Portfolios are formed with three different weighting approaches. Monthly rebalancing is conducted at the end of each month, as the portfolio deciles returns are calculated monthly at the end. Portfolio deciles are labeled with decreasing idiosyncratic volatility from P01 to P10, with P01 being the highest volatility decile and P10 being the lowest volatility decile. Spread portfolio P00 is calculated by P10-P01, as a position of long lowest volatility and short highest volatility. CAPM regressions are fitted with time-series of monthly returns of decile portfolios. Return, volatility, Sharpe ratio and CAPM alpha are all in annualized numbers. Panel A shows the results with equally-weighted approach. Panel B shows the results from adopting value weighted approach, portfolio returns are calculated with weighting based on market capitalization of stocks in the same decile. Panel C presents the results with the volatility-weighted method, as the idiosyncratic volatility plays a key role in determining the weights of stocks in the decile.

Panel A: Equally-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	10.36	12.06	0.62	3.20	1.86	0.56	17.11	0.49
P09	14.69	14.41	0.82	5.96	3.45	0.77	23.31	0.64
P08	14.05	16.33	0.68	4.49	2.34	0.88	23.94	0.65
P07	14.95	17.64	0.68	4.89	2.33	0.95	23.56	0.64
P06	15.90	21.48	0.61	4.35	1.68	1.14	23.01	0.63
P05	15.58	23.67	0.54	3.37	1.14	1.23	21.68	0.60
P04	14.93	27.66	0.43	2.18	0.56	1.30	17.35	0.49
P03	9.78	27.51	0.25	-2.67	-0.67	1.26	16.56	0.47
P02	15.59	34.87	0.36	2.08	0.38	1.40	13.21	0.36
P01	9.63	42.04	0.16	-3.87	-0.53	1.40	10.09	0.25
P00	0.73	39.57	-0.05	7.08	0.95	-0.84	-5.86	0.10

Panel B: Value-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	13.27	13.15	0.79	6.27	3.04	0.54	13.78	0.38
P09	10.83	17.04	0.47	1.82	0.76	0.81	17.63	0.50
P08	11.26	19.80	0.42	0.69	0.27	1.01	20.86	0.58
P07	10.63	21.10	0.37	-0.50	-0.19	1.09	21.21	0.59
P06	15.41	28.67	0.44	1.62	0.43	1.44	19.92	0.56
P05	14.83	31.92	0.37	0.37	0.08	1.53	17.99	0.51

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P04	9.76	39.95	0.17	-6.49	-1.09	1.76	15.43	0.44
P03	6.68	40.64	0.09	-10.63	-1.84	1.90	17.22	0.49
P02	4.40	40.95	0.04	-11.64	-1.85	1.74	14.39	0.40
P01	-7.48	47.99	-0.22	-20.59	-2.38	1.35	8.13	0.18
P00	20.75	48.85	0.37	26.86	2.85	-0.81	-4.47	0.06

Panel C: Volatility-weighted approach

Portfolio	Return	Volatility	SR	CAPM α	t(α)	β	t(β)	R^2
P10	10.65	12.17	0.64	3.12	1.95	0.61	20.01	0.56
P09	14.72	14.47	0.82	5.98	3.44	0.77	23.21	0.64
P08	13.99	16.35	0.68	4.41	2.30	0.88	23.99	0.65
P07	14.97	17.70	0.68	4.90	2.32	0.95	23.45	0.64
P06	15.65	21.51	0.59	4.10	1.57	1.14	22.91	0.63
P05	15.65	23.81	0.54	3.40	1.14	1.23	21.56	0.60
P04	14.83	27.64	0.43	2.08	0.53	1.30	17.39	0.49
P03	9.79	27.66	0.25	-2.65	-0.66	1.26	16.36	0.46
P02	15.40	35.03	0.36	1.87	0.34	1.40	13.14	0.36
P01	14.45	47.85	0.24	0.83	0.10	1.42	8.66	0.20
P00	-3.81	45.97	-0.15	2.28	0.26	-0.80	-4.76	0.07

4.2.2 Carhart four-factor regression

Given the fact that single-factor CAPM cannot provide a proper explanation for other anomalous effects, such as size, value and momentum. Carhart four-factor model has to be adopted to detect the characteristics of factor exposures for volatility deciles. The purpose of implementing Carhart four-factor regression is to uncover the potential connection between low volatility anomaly and other well-documented anomalous effects.

I) Annualized alphas

Regarding the equally-weighted approach, results in Table 5 show that positive alphas with statistical significance can only be observed in low volatility deciles. Neither the lowest volatility decile P10 nor the highest volatility decile P01 is statistically significant. The highest significant alpha falls to volatility decile P06 of 4.81%. In terms of value-weighted approach, the lowest volatility decile P10 ends up being the only decile with statistically significant alpha, outperforming others with an annualized alpha of 4.58%. When it comes to volatility-weighted approach, low volatility deciles (P07, P08 and P09) are the only three volatility deciles that maintain statistically significant alphas. Volatility decile P09 leads the ranking with an alpha of 4.58%. As a conclusion, although relative outperformance of low volatility deciles can be found, the lack of strong evidence cannot draw the conclusion of a significant existence of low volatility anomaly.

II) Factor loadings

a. Market (MKT)

Table 5 shows that all loadings on market factor are highly significant. A majority of volatility deciles have a market exposure of less than one in equally-weighted and volatility-weighted approach. As for the value-weighted approach, most of the volatility deciles come with a market beta larger than one. In all weighting approaches, it is evident that high volatility deciles generally have larger market exposure than low volatility deciles.

b. Size effect (small minus big: SMB)

The results presented in Table 5 show that except for the volatility decile P08 in value-weighted portfolios, all the loadings on size are statistically significant. Equally-weighted approach and volatility-weighted approach share the similar pattern, in which loadings on size start from around 0.20, and then generally increase as volatility increases, finally ends up with a value around 1.70. This implies that low volatility deciles are less tilting to small stocks than high volatility deciles. An increasing pattern can also be seen in value-weighted approach, but there is a clear difference with the starting and ending values. Low volatility deciles have negative loadings on size while high volatility deciles do the opposite, indicating that low volatility deciles are tilting to large stocks as high volatility deciles tilt to small stocks.

c. Value effect (high minus low: HML)

Results in Table 5 show that most of the loadings on value are statistically significant. Regardless of different weighting approaches, a generally decreasing trend as volatility increases can be spotted. Low volatility deciles have negative loadings on value, meaning that they are actually tilting to growth stocks. Contrary to high volatility deciles, low volatility deciles keep positive loadings on value, which indicates that they are in fact tilting to value stocks.

d. Momentum effect (MOM)

As shown in Table 5, similar decreasing trend as volatility increases also happens to the case of momentum effect. Apart from the insignificant positive loadings in the lowest volatility decile, significant negative loadings on momentum show that in general, volatility deciles are tilting to contrarian stocks. High volatility deciles are more tilting to contrarian stocks than low volatility deciles.

III) Spread portfolio P00 (P10 minus P01)

As can be seen from Table 5, for equally-weighted approach, the spread portfolio P00 has an insignificantly negative alpha of -0.12%, indicating the lowest volatility decile might underperforms the highest volatility decile. As for the loadings on factors, negative loadings on market and size while positive loadings on value and momentum can be seen from the results. Therefore, spread portfolio P00 are tilting to large, value and momentum stocks.

In the case of value-weighted approach, an alpha of 19.43% with statistical significance proves that the lowest volatility decile P10 significantly outperforms the highest volatility decile P01, offering strong evidence supporting the low volatility anomaly. Loadings on factors also echo the tilting to large, value and momentum stocks.

With respect to volatility-weighted approach, the spread portfolio P00 has an insignificantly negative alpha of -5.31%, which means that the lowest volatility decile P10 generally underperforms the highest volatility decile P01. The results on loadings on factors put forward that the spread portfolio P00 tilts to large, value and momentum stocks.

Table 5: Carhart four-factor estimation on deciles sorted on idiosyncratic volatility

This table presents the results of regression estimates with Carhart four-factor model: time series regressions are fitted by putting the monthly returns of volatility deciles (from P01 to P10) and the spread portfolio P00 (P10 minus P01) as the dependent variables, while market excess return, size, value and momentum factors are on the independent variables, denoted by MKT, SMB, HML and MOM respectively. Alphas are multiplied by twelve to annualize. Each column of regression estimates are subsequently followed by a bracket with sign of t, which means the t-statistics of the estimate on the left, also the symbol in the bracket denotes the estimate. The right-most column shows the coefficient of determination, denoted as R^2. Different portfolio formation approach are also examined here in the regressions. Panel A shows the results of equally-weighted approach. Panel B presents the results of value-weighted approach, as weights of stocks are determined by the market capitalizations. Panel C reports the results of volatility-weighted approach, and idiosyncratic volatility plays a key role in determining the allocated weights.

Panel A: Equally-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	1.49	1.01	0.59	19.80	0.21	5.53	0.48	11.23	0.00	0.06	0.65
P09	4.46	3.28	0.77	28.07	0.31	8.66	0.51	13.04	-0.04	-1.63	0.79
P08	3.51	2.38	0.83	27.88	0.48	12.51	0.46	10.88	-0.09	-3.39	0.80
P07	4.26	2.59	0.87	26.07	0.59	13.60	0.38	7.90	-0.10	-3.43	0.79
P06	4.81	2.30	1.00	23.57	0.74	13.40	0.18	2.94	-0.15	-4.16	0.77
P05	4.15	1.80	1.05	22.42	0.87	14.43	-0.04	-0.66	-0.12	-2.89	0.77
P04	4.94	1.80	0.99	17.79	1.25	17.39	-0.04	-0.49	-0.35	-7.23	0.76
P03	0.57	0.18	0.96	15.23	1.08	13.15	-0.18	-1.95	-0.34	-6.23	0.69
P02	8.12	1.85	0.96	10.79	1.37	11.86	-0.40	-3.14	-0.58	-7.45	0.62
P01	1.61	0.26	0.91	7.40	1.74	10.93	-0.34	-1.92	-0.56	-5.24	0.50
P00	-0.12	-0.02	-0.32	-2.51	-1.53	-9.18	0.81	4.43	0.56	5.03	0.38

Table 4 to be continued

Panel B: Value-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	4.58	2.34	0.64	16.14	-0.23	-4.48	0.23	3.98	0.11	3.25	0.47
P09	0.38	0.17	0.89	19.67	-0.14	-2.42	0.44	6.78	0.01	0.17	0.59
P08	-0.18	-0.07	1.06	21.34	-0.07	-1.10	0.37	5.27	-0.04	-0.82	0.63
P07	-0.27	-0.10	1.03	19.30	0.29	4.15	0.20	2.60	-0.11	-2.39	0.63
P06	3.65	1.00	1.29	17.32	0.48	4.96	-0.17	-1.65	-0.18	-2.81	0.61
P05	-0.09	-0.02	1.39	16.92	0.79	7.44	-0.24	-2.08	0.09	1.24	0.61
P04	-2.33	-0.45	1.39	13.28	1.36	10.02	-0.16	-1.09	-0.46	-5.08	0.60
P03	-6.64	-1.25	1.60	14.78	0.96	6.89	-0.43	-2.76	-0.33	-3.48	0.59
P02	-8.47	-1.50	1.41	12.33	1.20	8.13	-0.57	-3.48	-0.20	-2.06	0.55
P01	-14.85	-1.86	0.90	5.54	1.43	6.83	-0.74	-3.20	-0.43	-3.05	0.34
P00	19.43	2.30	-0.26	-1.50	-1.66	-7.49	0.96	3.95	0.54	3.63	0.28

Panel C: Volatility-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
P10	1.28	0.94	0.65	23.49	0.18	5.04	0.46	11.61	0.02	1.00	0.70
P09	4.49	3.27	0.77	27.85	0.31	8.57	0.51	12.93	-0.04	-1.68	0.78
P08	3.43	2.32	0.84	27.92	0.49	12.54	0.46	10.83	-0.09	-3.32	0.80
P07	4.28	2.58	0.87	25.86	0.59	13.53	0.37	7.80	-0.10	-3.42	0.79
P06	4.56	2.17	1.00	23.39	0.74	13.34	0.17	2.80	-0.15	-4.08	0.77
P05	4.21	1.81	1.05	22.25	0.88	14.39	-0.04	-0.67	-0.12	-2.95	0.77
P04	4.82	1.76	0.99	17.86	1.25	17.42	-0.04	-0.51	-0.35	-7.22	0.77
P03	0.69	0.22	0.96	14.95	1.08	13.01	-0.18	-2.02	-0.35	-6.28	0.69
P02	8.14	1.83	0.96	10.65	1.36	11.64	-0.41	-3.22	-0.59	-7.61	0.62
P01	6.59	0.86	0.92	5.95	1.71	8.53	-0.34	-1.55	-0.58	-4.35	0.39
P00	-5.31	-0.67	-0.28	-1.72	-1.53	-7.34	0.80	3.47	0.61	4.35	0.28

4.2.3 Fama French five-factor regression

I) Annualized alpha

As shown in Table 6, significant annualized alphas only appear in volatility deciles P09, P05 and P02 in equally and volatility weighted approaches. Also there is no monotonous trend in the annualized alphas as idiosyncratic volatility increases. In terms of value-weighted approach, none of the alphas are statistically significant. Therefore no conclusion on the relation between idiosyncratic volatility and expected returns can be reached based on the criteria of annualized alpha.

II) Factor loadings

a. Market (MKT)

Results on Table 6 reveal that all beta coefficients on MKT are statistically significant. For equally and volatility weighted approaches, betas are generally less than one, meaning less volatile than the market. As for value-weighted approach, market betas initially increase as idiosyncratic volatility increases but later abruptly decrease. Most of the value-weighted volatility deciles have a market beta larger than one.

b. Size effect (Small minus Big: SMB)

Table 6 shows that size remains a reliable factor in explaining the cross section of expected returns for volatility deciles, as all loadings on size are significant. Moreover, loadings on size are generally increasing as idiosyncratic volatility increases from low volatility to high volatility deciles across all weighting approaches. High volatility deciles have higher degree of tilting to small stocks compared to low volatility deciles.

c. Value effect (High minus Low: HML)

As can be seen from the Table 6, loadings on value only carry statistical significance in low volatility deciles for equally and volatility weighted approach. The case is even worse in value-weighted approach, only volatility deciles P09 and P08 have significant loadings on value. Thus value is no longer an efficient measurement.

d. Profitability effect (Robust minus Weak: RMW)

Table 6 presents that most of the loadings on profitability are statistically significant. Also it is apparent that loadings on profitability are generally decreasing from positive numbers in low volatility deciles to negative numbers in high volatility deciles, which marks a clear difference: low volatility deciles are tilting to robust profitability while the high volatility deciles tilt to the opposite.

e. Investment effect (Conservative minus Aggressive: CMA)

Table 6 tells that investment factor cannot explain the cross section of expected returns in most cases, as the most of loadings on investment carry no significance. However, it is surprising to discover that the lowest volatility deciles (P10) always have significant positive loadings on investment, suggesting that P10 is tilting to conservative investment.

III) Spread portfolio P00 (P10 minus P01)

The results on spread portfolio P00 in Table 6 show that none of the alphas in all weighting approaches have statistical significance, indicating that there is no significant outperformance between lowest volatility deciles (P10) and highest volatility decile (P01). Beta coefficients on market are always insignificant and close to zero, signaling its neutrality to market. In equally and volatility weighted approaches, only size and profitability maintain significant loadings, which provides evidence for a tilting to large and robust profitability. As for value-weighted approach, investment joins size and profitability to become the significant factors, proving that low volatility strategy has a tilting to large, robust profitability and conservative investment.

Table 6: Fama French five-factor estimation on deciles sorted on idiosyncratic volatility

This table presents the results of regression estimates with Fama French five-factor model: time series regressions are fitted by putting the monthly returns of volatility deciles (from P01 to P10) and the spread portfolio P00 (P10 minus P01) as the dependent variables, while market excess return, size, value, profitability and investment factors are on the independent variables, denoted by MKT, SMB, HML, RMW and CMA respectively. Alphas are multiplied by twelve to annualize. Each column of regression estimates are subsequently followed by a bracket with sign of t, which means the t-statistics of the estimate on the left, also the symbol in the bracket denotes the estimate. The right-most column shows the coefficient of determination, denoted as R^2. Different portfolio formation approach are also examined here in the regressions. Panel A shows the results of equally-weighted approach. Panel B presents the results of value-weighted approach, as weights of stocks are determined by the market capitalizations. Panel C reports the results of volatility-weighted approach, and idiosyncratic volatility plays a key role in determining the allocated weights.

Panel A: Equally-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	-0.23	-0.16	0.65	20.21	0.28	6.59	0.30	5.43	0.21	3.66	0.20	2.45	0.67
P09	2.92	2.16	0.82	27.49	0.39	10.07	0.43	8.39	0.20	3.75	-0.01	-0.13	0.80
P08	1.41	0.94	0.91	27.18	0.55	12.57	0.34	5.87	0.17	2.84	0.08	1.01	0.81
P07	3.07	1.82	0.90	24.19	0.62	12.78	0.31	4.85	0.04	0.64	-0.02	-0.27	0.79
P06	3.25	1.48	1.04	21.59	0.77	12.16	0.13	1.54	0.04	0.47	-0.08	-0.69	0.77
P05	6.49	2.82	0.96	18.86	0.73	11.00	0.05	0.61	-0.49	-5.49	-0.21	-1.66	0.79
P04	5.61	1.90	0.95	14.59	1.07	12.59	0.15	1.30	-0.57	-4.97	-0.31	-1.96	0.74
P03	2.19	0.66	0.89	12.26	0.84	8.86	0.09	0.72	-0.67	-5.27	-0.40	-2.22	0.68
P02	11.03	2.40	0.84	8.25	0.94	7.09	0.10	0.60	-1.22	-6.90	-0.55	-2.20	0.61
P01	5.95	0.97	0.77	5.69	1.13	6.45	-0.02	-0.10	-1.65	-7.00	-0.04	-0.13	0.52
P00	-6.19	-0.98	-0.11	-0.82	-0.86	-4.74	0.32	1.36	1.86	7.67	0.24	0.70	0.43

Table 6 to be continued

Panel B: Value-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	3.33	1.68	0.70	15.98	-0.17	-2.98	-0.01	-0.09	0.21	2.71	0.43	3.94	0.48
P09	-2.83	-1.29	1.00	20.52	0.03	0.41	0.26	3.14	0.45	5.25	0.21	1.73	0.62
P08	-2.24	-0.90	1.12	20.40	0.04	0.62	0.34	3.62	0.29	3.04	-0.03	-0.26	0.64
P07	-2.57	-0.94	1.11	18.41	0.36	4.56	0.12	1.21	0.17	1.66	0.04	0.28	0.62
P06	4.80	1.28	1.22	14.68	0.41	3.80	0.10	0.69	-0.27	-1.84	-0.53	-2.58	0.61
P05	4.19	1.01	1.25	13.65	0.64	5.35	-0.27	-1.71	-0.51	-3.19	-0.06	-0.28	0.62
P04	-1.11	-0.20	1.33	11.13	1.06	6.83	0.13	0.61	-0.84	-4.02	-0.34	-1.16	0.59
P03	-3.37	-0.62	1.49	12.40	0.58	3.70	-0.16	-0.79	-1.03	-4.94	-0.14	-0.49	0.60
P02	-1.93	-0.34	1.15	9.25	0.89	5.50	-0.08	-0.35	-1.03	-4.74	-0.88	-2.88	0.57
P01	-7.60	-0.94	0.62	3.49	0.89	3.86	-0.11	-0.37	-1.54	-4.96	-0.84	-1.92	0.36
P00	10.93	1.29	0.08	0.43	-1.06	-4.35	0.11	0.33	1.75	5.33	1.27	2.74	0.31

Panel C: Volatility-weighted approach

Portfolio	$\alpha \times 12$	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
P10	-0.62	-0.47	0.72	24.52	0.27	7.03	0.25	5.02	0.26	5.07	0.23	3.16	0.73
P09	2.91	2.13	0.82	27.29	0.39	9.96	0.43	8.29	0.20	3.75	-0.01	-0.10	0.80
P08	1.39	0.92	0.91	27.13	0.55	12.56	0.34	5.88	0.16	2.75	0.08	0.93	0.81
P07	3.08	1.81	0.90	24.00	0.62	12.67	0.31	4.73	0.04	0.57	-0.02	-0.17	0.79
P06	3.05	1.39	1.04	21.42	0.77	12.14	0.12	1.48	0.04	0.46	-0.09	-0.75	0.76
P05	6.50	2.79	0.96	18.72	0.73	10.94	0.05	0.55	-0.49	-5.43	-0.20	-1.58	0.78
P04	5.53	1.88	0.95	14.64	1.07	12.61	0.14	1.30	-0.57	-5.01	-0.32	-1.98	0.74
P03	2.25	0.67	0.89	12.03	0.84	8.74	0.08	0.66	-0.67	-5.22	-0.40	-2.18	0.67
P02	10.94	2.35	0.84	8.15	0.92	6.85	0.10	0.57	-1.24	-6.88	-0.55	-2.17	0.60
P01	10.81	1.40	0.79	4.61	1.08	4.85	-0.05	-0.16	-1.73	-5.78	0.07	0.17	0.41
P00	-11.43	-1.44	-0.07	-0.39	-0.81	-3.54	0.30	0.99	1.99	6.47	0.16	0.36	0.32

4.2.4 Summary on volatility deciles sorted on idiosyncratic volatility

Table 4 shows that the results on annualized returns vary across different weighting approaches. Evidence supporting a potential low volatility anomaly can be found in equally-weighted and value-weighted approach, while volatility-weighted approach provides evidence contradicting to low volatility anomaly. However, there is no monotonous trend that is strong enough to prove the significance of low volatility anomaly in all weighting approaches. CAPM estimates echo the conclusion of insignificant existence of low volatility anomaly. Although significant positive CAPM alphas can be easier to observe in low volatility deciles than high volatility deciles, no monotonous trend has been found.

It can be seen from the data in Table 5 that in terms of alphas from Carhart regression, although relative outperformance of low volatility deciles is documented, it is not strong enough to support the significant existence of low volatility anomaly. In terms of the factor loadings, it can be easily observed that there are generally monotonous trends in the loadings on factor. Compared to high volatility deciles, low volatility deciles are tilting to large, value and momentum stocks. This conclusion is robust in all the weighting approaches. The analysis based on the spread portfolio P00 also supports this conclusion with significantly negative loading on size while positive loadings on value and momentum, which implies the tilting of large, value and momentum. It is noteworthy to mention that in value-weighted approach, a relatively strong decreasing trend can be spotted in the results of Sharpe ratio. More importantly, a significant Carhart four-factor alpha emerges in the spread portfolio P00, which indicates that the long short strategy cannot be fully explained by the Carhart four factors. A potential implication from the phenomenon is that idiosyncratic volatility might be a priced factor that helps explain the cross section of expected return.

Results from FF-5 regression provide no evidence of a potential existence of low volatility anomaly. No clear pattern can be seen in terms of the annualized alphas, as most of the alphas are statistically insignificant. Size and profitability remains the most convincing factors, indicating a tilting to large and robust profitability in low volatility deciles compared to high volatility deciles. Value totally lose its reliability in value-weighted approach, while still maintain some influence in interpreting the low volatility deciles in equally and volatility weighted deciles. Significant loadings on investment can only be found in the lowest volatility deciles P10. Analysis based on spread portfolio P00 suggest that it is a market neutral strategy without significant alpha. Large, robust profitability and conservative investment lies behind the long-short strategy.

4.3 Volatility and cross section of expected returns

This section examines the relation between volatility and cross section of expected return. The first subsection introduces the Fama-MacBeth procedures to calculate factor risk premium. The second part introduces the methodologies in constructing the total volatility (VOL) factor and idiosyncratic volatility (IVOL) factor. The final part presents the results on risk premium of VOL and IVOL factors.

4.3.1 Fama-MacBeth Procedures

Asset pricing theories traditionally utilize risk factors to explain the cross section of stock returns. There are already a "zoo" of factors that result from all kinds of potential anomalous effects, most prominent ones including size, value and momentum. Fama and MacBeth (1973) provides a framework of estimating the betas and risk premium for any risk factors that are potentially playing roles in the cross section of expected returns. It has been widely adopted by academia to testify the significance of these risk factors, and subsequently determine the risk premium of these factors accordingly. Fama and MacBeth (1973) framework consists of a two-step regression. In the first step, each portfolio's return are running a regression against one or more factor time series. The goal is to obtain the factor exposure of each portfolio. In the second step, factor exposures are regressed against the cross section of portfolio returns at each time endpoint to gain a time series of risk premium for each factor. The pioneering innovation of Fama-MacBeth procedures is to calculate the average of the time series of premium as the risk premium for the factor, representing the expected return from one unit of factor exposure.

In the form of mathematical equation, for the number of n portfolios, the first step regression obtains the time series of βs (factor exposure), shown as follows (for each equation stands for a regression):

$$\begin{aligned} R_{1,t} &= \alpha_1 + \beta_1 F_t + \varepsilon_1 \\ R_{2,t} &= \alpha_2 + \beta_2 F_t + \varepsilon_2 \\ &\vdots \\ R_{n,t} &= \alpha_n + \beta_n F_t + \varepsilon_n \end{aligned} \tag{9}$$

Where $R_{n,t}$ stands for the return of the nth portfolio at time t, F_t is the risk factor at time t, α_n is the intercept of the nth regression, ε_n is the nth regression residual. The value of t ranges from 1 to T, as T represents the length of the time series for portfolio returns. β_n is the factor exposure of the nth portfolio. After conducting n regressions, a vector of n βs can be extracted. Then the vector of factor exposure will soon proceed to cross sectional regression in the second step.

The second step is to calculate the cross sectional regressions on the vector of βs from the first step at each time endpoints. The total number of regressions in the second step is T, equal to the length of time series. The equations are shown below (for each equation stands for a regression):

$$\begin{aligned} R_{i,1} &= \delta_1 + \gamma_1 \beta_i + \epsilon_1 \\ R_{i,2} &= \delta_2 + \gamma_2 \beta_i + \epsilon_2 \\ &\vdots \\ R_{i,T} &= \delta_i + \gamma_i \beta_i + \epsilon_i \end{aligned} \tag{10}$$

Where $R_{i,T}$ is the ith portfolio return at time T, β_i is the factor exposure of the ith portfolio obtained from the step one regression. δ_i is the intercept of the ith regression. ϵ_i is the residual of the ith regression. The value of i ranges from 1 to n, as n is the total number of portfolios. A vector of γs can be obtained to later compute the risk premium for the risk factor.

As mentioned above, the factor risk premium is the average of the vector of γs, denoted as $\bar{\gamma}$. The vector of γs has a length of T variables, and standard deviation can be represented by $\sigma(\gamma)$. The t-statistics to reject the null hypothesis of γ=0 is calculated by the following formula:

$$\frac{\bar{\gamma}}{\sigma(\gamma)/\sqrt{T}} \tag{11}$$

If the t-statistics rejects the null hypothesis of $\gamma=0$, it means that the risk factor earns statistically significant risk premium, and it can help describe the cross section of portfolio returns. If not, then the risk factor is not a factor that generates consistent risk premium.

4.3.2 Constructing the volatility factors

In the Kenneth French library[3] for factors, he provides a concise description on the construction method of 6 (2x3) value-weighted portfolios formed on size and book-to-market ratio. Stocks are firstly sorted on size into small and large, and then within the small and large quintiles, a second sort based on book-to-market ratio will divide into three categories, value, neutral and growth. As they elaborate, portfolios are constructed at the end of each June sorted on the market capitalization of stocks and NYSE breakpoints. Stocks with negative market capitalization are excluded from the sample. Based on the results from sorting on book-to-market ratio, the bottom 30% are assigned to growth, and the middle 40% being the neutral while the top 30% as value. Factors are constructed based on the intersection of these six value-weighted portfolios. For instance, SMB (Small minus Big) factor are constructed based on the average return of three small portfolios minus the average return of three large portfolios:

$$\begin{aligned}\text{SMB} = &\frac{1}{3}(Small\ Value + Small\ Neutral + Small\ Growth)\\ &-\frac{1}{3}(Big\ Value + Big\ Neutral + Big\ Growth)\end{aligned} \tag{12}$$

As for the HML (High minus Low) factor, it is computed by the average return of two value portfolios minus the average return of two growth portfolios, denoted as follows:

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3 Kenneth French factor library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

$$\text{HML} = \frac{1}{2}(Small\ Value + Big\ Value) - \frac{1}{2}(Small\ Growth + Big\ Growth) \quad (13)$$

Asness et al (2014) follows the same approach of double-sorting in their construction of a "Quality minus Junk" (QMJ) factor. The QMJ factor is constructed by a position of long on the top 30% high quality stocks and short on the bottom 30% junk stocks within the small and large quintiles. They also adopt the value-weighted approach for the portfolio formation. The formula of computing QMJ factor is also dependent on the intersection of six value-weighted portfolios:

$$\text{QMJ} = \frac{1}{2}(Small\ Quality + Big\ Quality) - \frac{1}{2}(Small\ Junk + Big\ Junk) \quad (14)$$

Inspired by previous researches, in this thesis the construction of volatility factor will follow the same methodology of double-sorting on size and volatility. The criteria of sorting on size is the median market capitalization of stocks in the sample. A conditional sort on volatility will then divide the portfolios into high, middle and low deciles. The breakpoints are the 30% and 70% quintiles. Monthly rebalancing is adopted, meaning that the double-sorting is repeated in each month. Meanwhile, both total volatility and idiosyncratic volatility will be separately examined. For the portfolios formed on conditional sorting on total volatility, the construction of a total volatility factor (VOL) will be the average return of two low volatility portfolios minus the average return of two high volatility portfolios:

$$\text{VOL} = \frac{1}{2}(Small\ Low + Big\ Low) - \frac{1}{2}(Small\ High + Big\ High) \quad (15)$$

In terms of the idiosyncratic volatility (IVOL) factor, this factor is constructed by a double-sorting on size and idiosyncratic volatility, everything else remains unchanged compared to the construction of VOL factor. The equation form is as follow:

$$\text{IVOL} = \frac{1}{2}(Small\ Low + Big\ Low) - \frac{1}{2}(Small\ High + Big\ High) \quad (16)$$

Previous findings in this thesis has already put forward that the choice on weighting approach would potentially affect the final results, despite the fact that value-weighted approach is the commonly accepted approach for major benchmarks. Thus unlike previous studies on the single choice of value-weighted approach, in the factor construction processes of this thesis, equally, value and volatility weighted approaches are all tested respectively when constructing the 2x3 portfolios.

4.3.3 Risk premium

I) Volatility (VOL) factor premium

As shown in table 5, risk premium of VOL factor regardless of different weighting approaches are consistently positive, meaning that low volatility deciles are generally outperforming the high volatility deciles. However, the results on t-statistics fail to provide any evidence to reject the null hypothesis of risk premium equaling zero. Thus risk premium of VOL factor are not statistically significant at 95% confidence level, which means that total volatility is not a priced factor that can explain the cross section of expected returns.

When comparing the results from different weighting approaches, value-weighted approach generates the highest risk premium of 4.71% per annum, indicating that despite a double sorting to possibly eliminate the influence of size effect, low volatility anomaly is still hugely impacted by size. The comparison between equally-weighted and volatility weighted approach shows that more weights allocated to high volatility stocks actually lowers the risk premium, which implicitly provides evidence that low volatility stocks generally earn higher return than high volatility stocks.

II) Idiosyncratic volatility (IVOL) factor premium

In terms of the results of IVOL factor premium in table 5, except for the negative risk premium in volatility-weighted approach, risk premium for idiosyncratic volatility is positive in other two weighting approach. Again, the value-weighted approach gives rise to the highest risk premium of 5.46% per annum. All risk premiums across weighting approaches are not statistically significant at 95% confidence level, meaning that idiosyncratic volatility cannot contribute to explaining the cross section of expected returns.

It is also worthwhile to mention that value-weighted approach generates not only the highest positive risk premium, but also the highest t-statistics though insignificant. Surprisingly, negative risk premium is observed in the volatility-weighted approach, leading to the positive relation between idiosyncratic volatility and expected return. Different results on weighting approaches provide evidence that the choice of weighting approach should be taken into consideration when implementing Fama-MacBeth procedures.

Table 7: VOL and IVOL Risk Premium from Fama-MacBeth two-step regression

This table presents the results of risk premiums followed by the Fama-MacBeth two-step regression. Volatility factors are mimicked by the widely-accepted double sorting approach, first sorted on size and accompanied by a subsequent sort on total volatility (VOL) or idiosyncratic volatility (IVOL). Factors are constructed by long on the lowest 30% volatility deciles and short on the highest 30% volatility deciles within the initially sorted small and large quintiles. Monthly rebalancing is adopted to sort stocks based on their size and volatility at the end of each month. The time-series of monthly volatility factor is regressed against the time series return of each volatility decile to gain the factor exposure of respective deciles. Then the second-step cross sectional regression obtains a vector of risk prices. Volatility risk premium is calculated as the average of the risk prices, also t-statistics for rejecting the null hypothesis of risk premium equaling zero are showed below. Three different weighting approaches are implemented as always to examine the impact of portfolio formation methods in this research. Risk premiums are in annualized percentage number.

	VOL factor		**IVOL factor**	
	Risk Premium	t-stat	Risk Premium	t-stat
Equally-weighted	1.13	0.38	0.84	0.28
Value-weighted	4.71	1.33	5.46	1.62
Volatility-weighted	0.36	0.11	-0.33	-0.10

4.4 Interaction with other anomalous factors

In this section, the interaction between above-mentioned volatility factors (VOL and IVOL) and other well-known anomalous factors, such as Carhart four factors and recent FF-5 factors, will be separately examined. Volatility factors will be the dependent variables and regressed against other factors, for the purpose of detecting potential explanatory powers of other factors on the low volatility anomaly.

4.4.1 Volatility factors versus Carhart four factors

I) Total volatility (VOL) factor

In terms of equally-weighted approach, a significant alpha of 0.58% can be observed in Panel A of Table 8, explicitly showing that Carhart four-factor model cannot fully explain the VOL factor. Loadings on factors are all statistically significant except market. Beta coefficient of 0.02 on MKT indicates that VOL factor is close to market neutral. Negative loading on SMB supports the argument that low volatility strategy is tilting to large, whereas positive loadings on HML and MOM add that it also tilting to value and momentum.

As for the value-weighted approach, similar findings can also be found. Significant alpha of 0.85% indicates that VOL factor cannot be fully explain by Carhart four factors. The beta coefficient on MKT loses significance again. Also, significantly negative loading on SMB and positive loadings on HML and MOM jointly provide evidence that low volatility strategy is tilting to large, value and momentum.

Results from volatility-weighted approach also draw the same conclusion that VOL factor cannot be fully explained by Carhart four factors as well as the tilting to large, value and momentum.

II) Idiosyncratic volatility (IVOL) factor

As shown in Panel B of Table 6, statistically significant alpha appears in the results from equally-weighted approach, proving that IVOL factor cannot be fully explained by Carhart four factors. Other potential factors lie in the explanation of IVOL factor. Beta coefficient of 0.1 on MKT implies that IVOL factor is nearly market neutral. All loadings on SMB, HML and MOM are statistically significant, while negative loading on SMB and positive loadings on HML and MOM jointly suggest that idiosyncratic volatility strategy is tilting to large, value and momentum.

When it comes to value-weighted approach, the results echo the finding that idiosyncratic volatility factor cannot be fully explained by Carhart four factors and it is market neutral with tilts to large, value and momentum.

Similar conclusions can also be drawn in the results of value and volatility weighted approaches, proving again that regardless of different weighting approach, IVOL factor demands further explanation outside of Carhart four-factor framework. Idiosyncratic volatility is a market neutral strategy and has tilts to large, value and momentum.

Table 8: Time series regression of volatility factors with Carhart four factors

This table presents the regression results of volatility (VOL and IVOL) factors with Carhart four factors, including market (MKT), size (SMB), value (HML) and momentum (MOM) factors. VOL and IVOL factors are examined separately. Volatility factors act as dependent variable and being regressed against Carhart four factors. The impact of three different weighting approaches in constructing the volatility factors on regression results is presented as well.

Panel A: VOL factor

Equally-weighted approach:

α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.58	3.60	0.02	0.58	-0.36	-7.11	0.61	10.92	0.18	5.26	0.45

Table 8 to be continued.

Value-weighted approach:										
α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.85	4.45	-0.04	-0.87	-0.54	-8.97	0.50	7.55	0.21	5.27	0.42
Volatility-weighted approach:										
α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.60	3.05	0.07	1.54	-0.38	-6.17	0.63	9.29	0.19	4.71	0.36
Panel B: IVOL factor										
Equally-weighted approach:										
α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.51	3.2	0.1	2.67	-0.34	-6.83	0.65	11.94	0.18	5.28	0.45
Value-weighted approach:										
α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.77	4.17	0.02	0.52	-0.52	-8.94	0.53	8.23	0.22	5.53	0.42
Volatility-weighted approach:										
α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	MOM	t(MOM)	R^2
0.49	2.54	0.16	3.33	-0.36	-5.87	0.67	10.04	0.19	4.7	0.37

4.4.2 Volatility factors versus Fama French five factors

Due to the significant alphas in the regression results under the Carhart four-factor framework, FF-5 model is fitted in this section, for the purpose of gaining sufficient explanatory power on the volatility factors through eliminating the significance of alphas.

I) Total volatility (VOL) factor

Panel A of table 7 shows that in equally-weighted approach, a significant alpha of 0.34% shows that FF-5 factors cannot fully explain the VOL factor. All loadings on FF-5 factors are statistically significant, indicating that large, value, robust profitability and conservative investment lie behind the low volatility anomaly.

As for the value-weighted approach, the significant alpha of 0.61% implies that other potential factors lie in the explanation of VOL factor. Beta coefficient of 0.06 on market suggests neutrality to market. Significantly negative loading on size and positive loadings on value, profitability and investment provide evidence of a tilting to large, value, robust profitability and conservative investment.

In terms of the volatility-weighted approach, an insignificant alpha emerges. Only loadings on market, value and profitability are significant at 95% confidence level. Positive loadings on value and profitability suggest a tilting to value and robust profitability.

II) Idiosyncratic volatility (IVOL) factor

Results in Panel B of Table 9 show that equally and volatility weighted approaches present similar patterns. An insignificant alpha suggests that in Fama French five-factor model, IVOL factor can be properly explained by these factors. Significant positive loadings on value and profitability point out the tilting to value and strong profitability. Size and investment factors fail to explain the VOL factor.

When it comes to value-weighted approach, the reappearance of significant alpha reaffirms the fact that other potential factors lie in explanation of IVOL factor. All loadings on factors are statistically significant at 95% confidence level. Apart from the negative loading on size, loadings on the other factors are positive. Beta coefficient on market is close to zero, signaling the neutrality to market. In summary, the conclusion can be drawn that IVOL factor is market-neutral with the tilting to large, value and strong profitability and conservative investment.

Table 9: Time series regression of volatility factors with Fama French five factors

This table presents the regression results of volatility (VOL and IVOL) factors with Fama French five factors, including market (MKT), size (SMB), value (HML), profitability (RMW) and investment (CMA) factors. VOL and IVOL factors are examined separately. Volatility factors act as dependent variable and being regressed against Fama French five factors. The impact of three different weighting approaches in constructing the volatility factors on regression results is presented as well.

α	t(α)	MKT	t(MKT)	SMB	t(SMB)	HML	t(HML)	RMW	t(RMW)	CMA	t(CMA)	R^2
Panel A: VOL factor												
Equally-weighted approach:												
0.34	2.22	0.13	3.20	-0.11	-2.08	0.35	5.10	0.71	10.07	0.20	2.03	0.54
Value-weighted approach:												
0.61	3.26	0.06	1.30	-0.28	-4.36	0.23	2.72	0.72	8.27	0.26	2.15	0.47
Volatility-weighted approach:												
0.34	1.83	0.18	3.65	-0.09	-1.38	0.38	4.53	0.79	9.25	0.12	0.99	0.46
Panel B: IVOL factor												
Equally-weighted approach:												
0.28	1.87	0.2	5.02	-0.1	-1.91	0.41	6.03	0.67	9.67	0.18	1.82	0.53
Value-weighted approach:												
0.57	3.08	0.12	2.38	-0.28	-4.42	0.28	3.31	0.67	7.93	0.24	1.99	0.46
Volatility-weighted approach:												
0.25	1.37	0.25	5.18	-0.08	-1.26	0.44	5.19	0.75	8.8	0.11	0.9	0.45

4.4.3 Summary on interaction with other anomalous factors

In the first subsection, the interaction between volatility (VOL and IVOL) factors and Carhart four factors are examined. It is robust that regardless of different weighting approaches, the consistent existence of significant alpha demonstrates that other potential factors outside of the Carhart four-factor framework lie in the explanation of volatility factors. Meanwhile, evidence shows that both VOL and IVOL factor are market-neutral, with a tilting to large, value and momentum.

Due to the fact that significant alphas persist in the Carhart regression, FF-5 model is adopted for the purpose of gaining further insights in explaining the volatility factors. Compared to Carhart four-factor model, FF-5 model shows stronger explanatory power. In terms of VOL factor, results from equally and value weighted approach show that significant alphas persist, all loadings on FF-5 factors are significant, indicating a tilting of large, value, robust profitability and conservative investment, while the alpha in volatility weighted approach shows statistical insignificance with a tilting to value and robust profitability. With regard to IVOL factor, in equally and volatility-weighted approaches, as alphas in these two weighting approaches appear insignificantly. Size fails to maintain its significance, while value and profitability carry highly significant positive loadings, suggesting a tilting to value and robust profitability. As for value-weighted approach, significant alphas return back, signaling that other potential factors outside of the FF-5 model also contribute to the explanation of volatility factors. Beta coefficient on market shows neutrality to market. Loadings on FF-5 factors are statistically significant, implying that large, value, robust profitability and conservative investment partially explain the IVOL factor.

5. CONCLUSIONS

The first aim of this thesis is to empirically test the existence of low volatility anomaly in different weighting approaches. Firstly, there are substantial evidences proving that positive signals supporting the potential existence of low volatility anomaly are way easier to find in value-weighted than equally and volatility weighted approach. Secondly, the results show that low volatility deciles are generally outperforming the high volatility deciles in terms of risk-adjusted return. Thirdly, no monotonicity can be found under the criteria of annualized return, CAPM alpha, Carhart alpha and FF-5 alpha. Except that significant Carhart alpha on spread portfolios are found in the value-weighted approach, no evidence can be found which points to the significant existence of low volatility anomaly.

Another purpose of this thesis is to measure the risk premium of volatility factors through the Fama-MacBeth procedures. Again, it can be observed that the t-statistics are much higher in value-weighted than equally and volatility weighted approaches. Results implicitly show that neither total volatility (VOL) factor nor idiosyncratic volatility (IVOL) factor earns significant risk premium across all weighting approaches. Although the risk premiums are positive except in the case of volatility weighted approach for IVOL factor, the conclusion is drawn that total volatility and idiosyncratic volatility are not priced factors that explain the cross section of expected returns.

Analysis of volatility deciles sorted on total volatility or idiosyncratic volatility shows that large, value and momentum may lie behind the low volatility anomaly. Results from the FF-5 regression add robust profitability and sometimes conservative investment. The regression between volatility factors (VOL and IVOL) and Carhart four factors confirms the findings that apart from large, value and momentum, other factors are consistently needed to explain volatility factors. Regression results under the FF-5 framework suggest that value and robust profitability are the biggest reasons behind low volatility anomaly.

BIBLIOGRAPHY

Ang, A. (2014). *Asset management: A systematic approach to factor investing*. Oxford University Press.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2006). The cross-section of volatility and expected returns. *The Journal of Finance*, *61*(1), 259-299.

Ang, A., Hodrick, R. J., Xing, Y., & Zhang, X. (2009). High idiosyncratic volatility and low returns: International and further US evidence. *Journal of Financial Economics*, *91*(1), 1-23.

Asness, C. S., Frazzini, A., & Pedersen, L. H. (2014). Quality minus junk. *Review of Accounting Studies*, 1-79.

Baker, N. L., & Haugen, R. A. (2012). Low risk stocks outperform within all observable markets of the world. *Available at SSRN 2055431*.

Baker, M., Bradley, B., & Wurgler, J. (2011). Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts Journal*, *67*(1), 40-54.

Bali, T. G., & Cakici, N. (2008). Idiosyncratic volatility and the cross section of expected returns. *Journal of Financial and Quantitative Analysis*, *43*(1), 29-58.

Barberis, N., & Xiong, W. (2012). Realization utility. *Journal of Financial Economics*, *104*(2), 251-271.

Beveratos, A., Bouchaud, J. P., Ciliberti, S., Laloux, L., Lempérière, Y., Potters, M., & Simon, G. (2017). Deconstructing the low-vol anomaly. Journal of Portfolio Management, 44(1), 91.

Black, F., Jensen, M. C., & Scholes, M. (1972). The capital asset pricing model: Some empirical tests. *Studies in the theory of capital markets*, *81*(3), 79-121.

Blitz, D. C. & P. Van Vliet .(2007). The volatility effect. *Journal of Portfolio Management 34(1):* 102-113.

Carhart, M. M. (1997). On persistence in mutual fund performance. *The Journal of finance*, *52*(1), 57-82.

Cochrane, J. H. (2011). Presidential address: Discount rates. The Journal of finance, 66(4), 1047-1108.

Malkiel, B. G., & Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *The journal of Finance*, *25*(2), 383-417.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of financial economics*, *33*(1), 3-56.

Fama, E. F., & MacBeth, J. D. (1973). Risk, return, and equilibrium: Empirical tests. *Journal of political economy*, *81*(3), 607-636.

Fama, E. F., & French, K. R. (2015). A five-factor asset pricing model. *Journal of financial economics*, *116*(1), 1-22.

Frazzini, A., & Pedersen, L. H. (2014). Betting against beta. *Journal of Financial Economics*, *111*(1), 1-25.

Fu, F. (2009). Idiosyncratic risk and the cross-section of expected stock returns. *Journal of financial Economics*, *91*(1), 24-37.

Haugen, R. A., & Heins, A. J. (1975). Risk and the rate of return on financial assets: Some old wine in new bottles. *Journal of Financial and Quantitative Analysis*, *10*(5), 775-784.

Hou, K., Xue, C., & Zhang, L. (2017). *Replicating anomalies*(No. w23394). National Bureau of Economic Research.

Jegadeesh, N., & Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency. *The Journal of finance*, *48*(1), 65-91.

Kahneman, D. (1979). Tversky A.(1979). *Prospect theory: an analysis of decision under risk*, 263-292.

Lintner, J. (1975). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. In *Stochastic optimization models in finance* (pp. 131-155). Academic Press.

Markowitz, H. (1959). Portfolio selection. *Investment under Uncertainty*.

Martellini, L. (2008). Toward the design of better equity benchmarks: Rehabilitating the tangency portfolio from modern portfolio theory. *Journal of Portfolio Management*, *34*(4), 34.

Miller, E. M. (1977). Risk, uncertainty, and divergence of opinion. *The Journal of finance*, *32*(4), 1151-1168.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The journal of finance*, *19*(3), 425-442.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *science*, *185*(4157), 1124-1131.

Van Vliet, P., Blitz, D., & van der Grient, B. (2011). Is the relation between volatility and expected stock returns positive, flat or negative?.